TWMS J. App. and Eng. Math. V.13, N.2, 2023, pp. 696-709

APPLICATIONS OF THE OPERATOR ${}_r\Phi_s$ IN q-POLYNOMIALS

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Abstract. We establish ${}_r\Phi_s$ as a general operator for many q-operators. A new polynomials $h_n(a_1, \cdots, a_r; b_1, \cdots, b_s; x, y; q)$ are described as an extension of the bivariate Rogers-Szegö polynomial $h_n(x, y|q)$ and the generalized Al-Salam–Carlitz q-polynomials $\phi_n^{(\mathbf{a},\mathbf{b})}(x, y|q)$. With the use of the operator ${}_r\Phi_s$, we provide an operator proof of the generating function and its extension, Mehler's formula and its extension and Rogers formula and its extension to the polynomials $h_n(a_1, \cdots, a_r; b_1, \cdots, b_s; x, y; q)$. The generating function and its extension, Mehler's formula and its extension and Rogers formula and its extension for $h_n(x, y|q)$ and $\phi_n^{(\mathbf{a},\mathbf{b})}(x, y|q)$ are deduced by giving special values to parameters of a new polynomial $h_n(a_1, \cdots, a_r; b_1, \cdots, b_s; x, y|q)$.

Keywords: the q-operators, the bivariate Rogers-Szegö polynomials, the generalized Al-Salam–Carlitz q-polynomials, generating function, Mehler's formula, Rogers formula.

AMS Subject Classification: 05A30, 33D45.

1. Introduction

In this paper, the notations that was used in [9] is followed, and we assume that $|q| < 1$. We're going to mention to a few notations for the q-series that we depend on during this paper.

Let $a \in \mathbb{C}$. The q-shifted factorial is given as follows [9]:

$$(a; q)_0 = 1, \quad (a; q)_m = \prod_{k=0}^{m-1} (1 - aq^k), \quad (a; q)_\infty = \prod_{k=0}^{\infty} (1 - aq^k),$$

and for the multiple q-shifted factorials by:

$$(a_1, a_2, \ldots, a_r; q)_m = (a_1; q)_m (a_2; q)_m \cdots (a_r; q)_m,$$

where $m \in \mathbb{Z}$ or ∞.

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§ Manuscript received: December 11, 2020; accepted: March 27, 2021.