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SOME ANALYTICAL RESULTS ON THE $\Delta-$FRACTIONAL DYNAMIC EQUATIONS

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Abstract. In this paper, we successfully solve some linear $\Delta-$fractional dynamic equations ($\Delta-$FDE) with Caputo $\Delta-$derivative analytically by solving an auxiliary linear $\Delta-$differential equation ($\Delta-$DE) with an integer order. The idea of the proposed method is based on transforming the given $\Delta-$FDE into a linear $\Delta-$DE with an integer order. This transformation removes certain terms of the solution of the considered $\Delta-$FDE, resulting in the remaining terms being a solution to the auxiliary equation. To demonstrate the ability and efficacy of this idea, several examples have been provided.

Keywords: Time Scale Calculus, Fractional time scale calculus, Caputo fractional $\Delta-$derivatives.

AMS Subject Classification: 34-XX and MSC 35R07 and MSC 34A08.

1. Introduction

Time scale calculus was presented by Hilger [26, 27] to generalise and unify the study of theories of discrete and continuous differential equations, as well as to stretch these theories to other sorts of equations called dynamic equations, which have lately attracted a lot of attention. The two principal characteristics of time scale calculus are the unification and extension of discrete and continuous equations. There are numerous results concerning continuous dynamic equations that transfer over pretty readily to analogous results for discrete dynamic equations, whereas discrete dynamic equations' results may appear diametrically opposed to their continuous dual. On time scales, studying dynamic equations reveals these inconsistencies, allowing one to avoid having to repeat the proof of results twice for discrete and continuous dynamic equations. The beauty of time scale calculus is that it allows one to solve a given dynamic equation on any time scale set $\mathbb{T}$, and then this set will be selected later based on the type of dynamic system, such as $\mathbb{T} = \mathbb{R}$ when studying differential equations, and $\mathbb{T} = \mathbb{Z}$ when studying difference equations, and so on. This method yields results that are not only in connection with $\mathbb{Z}$ or $\mathbb{R}$, but also for any non-empty closed subset of $\mathbb{R}$. Since Hilger studied in his doctoral thesis on this topic, many scholars were interested in the extension of it and adapted many theories and issues

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