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RIGIDITY AND FLATNESS OF THE IMAGE OF CERTAIN CLASSES OF MAPPINGS HAVING TANGENTIAL LAPLACIAN

HUSSIEN ABUGIRDA, BIRZHAN AYANBAYEV AND NIKOS KATZOURAKIS

This paper is dedicated to Gunnar Aronsson with the utmost esteem for his pioneering work.

In this paper we consider the PDE system of vanishing normal projection of the Laplacian for C^2 maps $u : \mathbb{R}^n \supseteq \Omega \to \mathbb{R}^N$:

$$[\![\mathrm{D}u]\!]^{\perp} \Delta u = 0 \quad \text{in } \Omega.$$

This system has discontinuous coefficients and geometrically expresses the fact that the Laplacian is a vector field tangential to the image of the mapping. It arises as a constituent component of the p-Laplace system for all $p \in [2, \infty]$. For $p = \infty$, the ∞-Laplace system is the archetypal equation describing extrema of supremal functionals in vectorial calculus of variations in L^{∞}. Herein we show that the image of a solution u is piecewise affine if either the rank of $\mathrm{D}u$ is equal to one or $n = 2$ and u has additively separated form. As a consequence we obtain corresponding flatness results for p-Harmonic maps for $p \in [2, \infty]$.

1. Introduction

Suppose that n, N are integers and Ω an open subset of $\mathbb{R}^n$. In this paper we study geometric aspects of the image $u(\Omega) \subseteq \mathbb{R}^N$ of certain classes of C^2 vectorial solutions $u : \mathbb{R}^n \supseteq \Omega \to \mathbb{R}^N$ to the following nonlinear degenerate elliptic PDE system:

$$[\![\mathrm{D}u]\!]^{\perp} \Delta u = 0 \quad \text{in } \Omega. \tag{1-1}$$

Here, for the map u with components $(u_1, \dots, u_N)^{\top}$ the notation $\mathrm{D}u$ symbolises the gradient matrix

$$\mathrm{D}u(x) = \big(\mathrm{D}_i u_{\alpha}(x)\big)_{i=1,\dots,n}^{\alpha=1,\dots,N} \in \mathbb{R}^{N \times n}, \qquad \mathrm{D}_i \equiv \partial/\partial x_i,$$

Δu stands for the Laplacian

$$\Delta u(x) = \sum_{i=1}^{n} \mathrm{D}_{ii}^2 u(x) \in \mathbb{R}^N$$

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