The Numerical Solution of Volterra Integral Equation of Second Kind Using Quartic Non-Polynomial Spline Function

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ABSTRACT

In this study, the non-polynomial spline function of fourth order is given to solve the second kind of the nonlinear Volterra integral equation. Numerical examples are show to clarify the applications of the proposed method and to compare the calculated results with other methods. In addition, there is a comparison of the absolute value of the error that estimated by of the suggested method is compared by those that estimated through other methods and the exact solution. The obtained results are very encouraging compared by other numerical methods.

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1. Introduction

Differential and integral equations have huge importance thus; these equations are used in many fields such as control, economics, electrical engineering, and medicine,.etc. Munaty and Al-Humedi [1] in 2022 are used Laguerre-Chebyshev Petrov-Galerkin method to solve integral equations. Al-Humedi and Jameel [2] in 2020 presented the cubic B-spline least-square method that combine with a quadratic weight function to solve the integro-differential equations. Volterra integral equations (VIEs) is a type of integral equations which used for various scientific applications such as the population dynamics, spread of epidemics, and semiconductor devices. Th various types ofVolterra integral equation (VIE) are solved using in different ways. For example, the superconvergence of collocation methods for a class of weakly singular (VIE) was suggested by Diogo and Lima [3] in 2008. Kazemi [4] in 2021 discuss the triangular functions for numerical solution of the nonlinear Volterra integral equations while Burova [5] in 2021 was presented an application for the local polynomial and non-polynomial splines of the third order of approximation for the construction of the numerical solution for the second kind of the VIE . Jenaliyev et al. [6] in 2015 studied the Volterra equation of the second kind with incompressible kernel. Barycentric - Maclaurin interpolation method is used to solve second kind of VIEs by Shoukralla et al. [7] in 2020. Aggarwal et.al [8] in 2018 introduced a solution for linear VIEs of the second kind using Mohand transform whereas Agram and Djehiche [9] in 2021 studied the class of reflected backward stochastic of VIEs and related time-inconsistent optimal stopping problems. Furthermore, Burova and Alcybeev [10] in 2021 are introduced the application of splines for the second order approximation of VIEs of the second kind.

Over the last years, the interest of using a non-polynomial spline to find numerical solution for VIEs and other equations is increased through through publishing many articles. Islam et Al. [11] in 2005 presented non polynomial spline approach to find a solution to a system of third-order boundary-value while Taiwo and Ogunlaran [12] in 2011 were studied a non-polynomial spline method to solve linear fourth-order boundary-value problems. Li and Zou [13] in 2013 are suggested a random integral quadrature method for numerical analysis of the second kind of Volterra integral equations. After that, Taha and Ahmed [14] gave a numerical solution of third order BVPs using non-polynomial spline with finite difference method. Moreover, Harbi et al. [15] are studied a solution of the second kind VIEs using third order of the non-polynomial spline function. Non-polynomial spline finite difference method is studied by Taha and Kadim [16] to solve the second order boundary value problem. Further, a numerical treatment of the

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first order Volterra integro-differential equation using non-polynomial spline functions also studied. Recently, Hamasalh and Headayat [18] are explained using the non-polynomial spline to the numerical solution for fractional differential equations. Khalaf and Taha in 2021 [19] are used the second order non polynomial spline function to solve a system of nonlinear Volterra integral equations. In the present manuscript, solution of nonlinear VIEs using fourth order non polynomial spline function was studied and the obtained results are discussed. Volterra integral equations of the second kind can be given:

$$\eta(x) = g(x) + \int_a^x k(x,t)\,\eta(t)dt . \qquad a \le x \le b \qquad 1$$

Consider the partition $\Delta = \{x_0, x_1, x_2, \ldots, x_n\}$ of $[a,b] \subseteq R$, let set x_i of points on the interval $[a,b]$,

$$a = x_0 < x_1 \ldots\ldots < x_n = b,\ h = (b-a)/n, x_i = x_0 + ih . \qquad 2$$

Where n is a positive integer.

2. Quartic non-polynomial spline function

The quartic non-polynomial spline function takes the form:

$$\phi_i(x_i) = a_i \cos(m(x-x_i)) + b_i \sin(m(x-x_i)) + c_i(x-x_i) + d_i(x-x_i)^2 + e_i(x-x_i)^3 + s_i(x-x_i)^4 + p_i, i = 0, \ldots, n. \qquad 3$$

Where $a_i, b_i, c_i, d_i, e_i, s_i$ and $p_i$are constants. When the following equations are observed:

$$\left.\begin{aligned} \phi_i(x_i) &= a_i + p_i \\ \phi_i{}'(x_i) &= mb_i + c_i \\ \phi_i''(x_i) &= -m^2 a_i + 2\,d_i \\ \phi_i^{(3)}(x_i) &= -m^3 b_i + 6e_i \\ \phi_i^{(4)}(x_i) &= m^4 a_i + 24 s_i \\ \phi_i^{(5)}(x_i) &= m^5 b_i \\ \phi_i^{(6)}(x_i) &= -m^6 a_i \end{aligned}\right\} i = 0, \ldots, n. \qquad 4$$

The values of $a_i, b_i, c_i, d_i, e_i, s_i$ and p_i can be found as follows:

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$$\left.\begin{aligned} a_i &= -(1/m^6)\phi_i^{(6)}(x_i) \\ b_i &= (1/m^5)\phi_i^{(5)}(x_i) \\ c_i &= \acute{\phi}_i(x_i) - mb_i \\ d_i &= (1/2)(\phi_i^{(2)}(x_i) + m^2 a_i) \\ e_i &= (1/6)(\phi_i^{(3)}(x_i) + m^3 b_i) \\ s_i &= (1/24)(\phi_i^{(4)}(x_i) - m^4 a_i) \\ p_i &= \phi_i(x_i) - a_i \end{aligned}\right\} i = 0, \ldots., n. \qquad 5$$

3. Method of quartic non-polynomial spline function to solve Volterra integral equation of the second kind

$$\eta_0' = g'(a) + k(a,a)\eta(a). \qquad 6$$

$$\eta_0'' = g''(a) + ((\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\,\eta(a) + (\frac{d}{dx}k(x,x))_{x=a}\,\eta(a) + k(a,a)\eta'(a). \qquad 7$$

$$\eta_0^{(3)} =$$
$$g^{(3)}(a) + ((\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\,\eta(a) + (\frac{d}{dx}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta(a) +$$
$$((\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta'(a) + (\frac{d^2}{dx^2}k(x,x))_{x=a}\eta(a) +$$
$$2(\frac{d}{dx}k(x,x))_{x=a}\eta'(a) + k(a,a)\eta''(a).$$
(8)

$$\eta_0^{(4)} =$$
$$g^{(4)}(a) + ((\frac{\partial^3}{\partial x^3}k(x,t))_{t=x})_{x=a}\,\eta(a) + (\frac{d}{dx}(\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta(a) +$$
$$((\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta'(a) + (\frac{d^2}{dx^2}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta(a) + 2(\frac{d}{dx}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta'(a) +$$
$$((\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta''(a) + (\frac{d^3}{dx^3}k(x,x))_{x=a}\eta(a) + 3(\frac{d^2}{dx^2}k(x,x))_{x=a}\eta'(a) +$$
$$3(\frac{d}{dx}k(x,x))_{x=a}\eta''(a) + k(a,a)\eta^{(3)}(a). \qquad 8$$

$$\eta_0^{(5)} =$$
$$g^{(5)}(a) + ((\frac{\partial^4}{\partial x^4}k(x,t))_{t=x})_{x=a}\eta(a) + (\frac{d}{dx}(\frac{\partial^3}{\partial x^3}k(x,t))_{t=x})_{x=a}\eta(a) +$$
$$((\frac{\partial^3}{\partial x^3}k(x,t))_{t=x})_{x=a}\eta'(a) + (\frac{d^2}{dx^2}(\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta(a) +$$
$$2(\frac{d}{dx}(\frac{\partial^2}{\partial x^2}k\big(x,t))_{t=x}))_{x=a}\eta'(a)\big) + ((\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta''(a) +$$
$$(\frac{d^3}{dx^3}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta(a) + 3(\frac{d^2}{dx^2}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta'(a) +$$

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$3(\frac{d}{dx}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta''(a)+ ((\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta^{(3)}(a)+(\frac{d^4}{dx^4}k(x,x))_{x=a}\eta(a)+$
$4(\frac{d^3}{dx^3}k(x,x))_{x=a}\eta'(a)+6(\frac{d^2}{dx^2}k(x,x))_{x=a}\eta''(a)+4(\frac{d}{dx}k(x,x))_{x=a}\eta'''(a)+$
$k(a,a)\eta^{(4)}(a).$ 9

$\eta_0^{(6)} =$

$g^{(6)}(a)+(\frac{d}{dx}(\frac{\partial^4}{\partial x^4}k(x,t))_{t=x})_{x=a}\ \eta(a)+((\frac{\partial^4}{\partial x^4}k(x,t))_{t=x})_{x=a}\eta'(a)+$

$(\frac{d^2}{dx^2}(\frac{d^3}{dx^3}k(x,t))_{t=x})_{x=a}\eta(a)+2(\frac{d}{dx}(\frac{d^3}{dx^3}k(x,t))_{t=x})_{x=a}\eta'(a)+$

$((\frac{\partial^3}{\partial x^3}k(x,t))_{t=x})_{x=a}\eta''(a)+(\frac{d^3}{dx^3}(\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta(a)+$

$3(\frac{d^2}{dx^2}(\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta'(a)+3(\frac{d}{dx}(\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta''(a)+$

$((\frac{\partial^2}{\partial x^2}k(x,t))_{t=x})_{x=a}\eta'''(a)+(\frac{d^4}{dx^4}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta(a)+4(\frac{d^3}{dx^3}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta'(a)+$

$6(\frac{d^2}{dx^2}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\eta''(a)+4(\frac{d}{dx}(\frac{\partial}{\partial x}k(x,t))_{t=x})_{x=a}\ \eta^{(3)}(a)+$

$((\frac{\partial}{\partial x}k(x,t)_{t=x})_{x=a}\eta^{(4)}(a)+(\frac{d^5}{dx^5}k(x,x))_{x=a}\eta(a)+5(\frac{d^4}{dx^4}k(x,x))_{x=a}\eta'(a)+$

$10(\frac{d^3}{dx^3}k(x,x))_{x=a}\eta^{(2)}(a)+10(\frac{d^2}{dx^2}k(x,x))_{x=a}\eta^{(3)}(a)+5(\frac{d}{dx}k(x,x))_{x=a}\eta^{(4)}(a)+$

$k(a,a)\eta^{(5)}(a).$ 10

4. Algorithm for quartic non-polynomial method

Step 1:

$h=\frac{b-a}{n}$ $,x_i = x_0+ih, i=0,\dots,n\ , x_0=a\ , x_n=b\ .$

$\eta_0 = g(a).$

Step 2:

Evaluate $a_0, b_0, c_0, d_0, e_0, s_0$ and $p_0$by substituting equations 6-11 in equations 5.

Step 3:

Calculate ϕ_0 by using step (2) and equation (3) at $i=0$.

Step 4:

Approximate $\eta_1 \approx \phi_0(x_1)$.

Step 5:

For $i=1$to $n-1$ do the following steps:

Step 6:

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1-Evaluate $a_{0i}, b_0, c_0, d_0, e_0, s_0$ and p_0 by using equations 6-11 and replacing $\eta'(x_i), \eta''(x_i), \eta^{(3)}(x_i), \eta^{(4)}(x_i), \eta^{(5)}(x_i)$ and $\eta^{(6)}(x_i)$ in $\phi'(x_i), \phi''(x_i), \phi^{(2)}(x_i), \phi^{(3)}(x_i), \phi^{(4)}(x_i), \phi^{(5)}(x_i), \phi^{(6)}(x_i)$.

Step 7:

Approximate $\eta_{i+1} \approx \phi_i(x_{i+1})$.

5. Numerical example

In this section, the study explains the numerical techniques which are discussed in the previous sections through some numerical examples where the study suggested a method to solve the nonlinear Volterra integral equation and compare our method with other existing methods. All calculations are implemented using Maple 18.

Example 1 [15]:

Consider the following nonlinear VIEs of the second kind:

$$\eta(x) = x3^x + \int_0^x -x3^{x-1}\,\eta(t)dt \quad 0 \le x \le 1 \quad .$$

With the exact solution $\eta(x) = 3^x(1 - e^{-x})$

Table 1 is listed a comparison between the absolute error of third order non-polynomial [15] and absolute error of quartic non-polynomial for Example 1 by applying quartic non-polynomial spline method,where $h = 0.1$ and $x_i = x_0 + ih, i = 0,1, \dots, n$. Figure 1 shows the comparison between the exact solution and numerical solution of $\eta(x)$ by applying thequartic non-polynomial spline method for Example 1.

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Table 1: A comparison between the absolute errors of our method with [15] for Example 1 for $n = 10$ at different values of x

x	Quartic non-polynomial φ(x)	Third order non-polynomial φ(x) [15]
0.1	7.1016e-11	4.5353e-9
0.2	9.2174e-9	2.9477e-7
0.3	1.5962e-7	3.4094e-6
0.4	1.2118e-6	1.9448e-5
0.5	5.8550e-6	7.5310e-5
0.6	2.1256e-5	2.2824e-4
0.7	6.3350e-5	5.8408e-4
0.8	1.6341e-4	1.3206e-3
0.9	1.1174e-3	3.4563e-3
1	7.9937e-4	5.1855e-3

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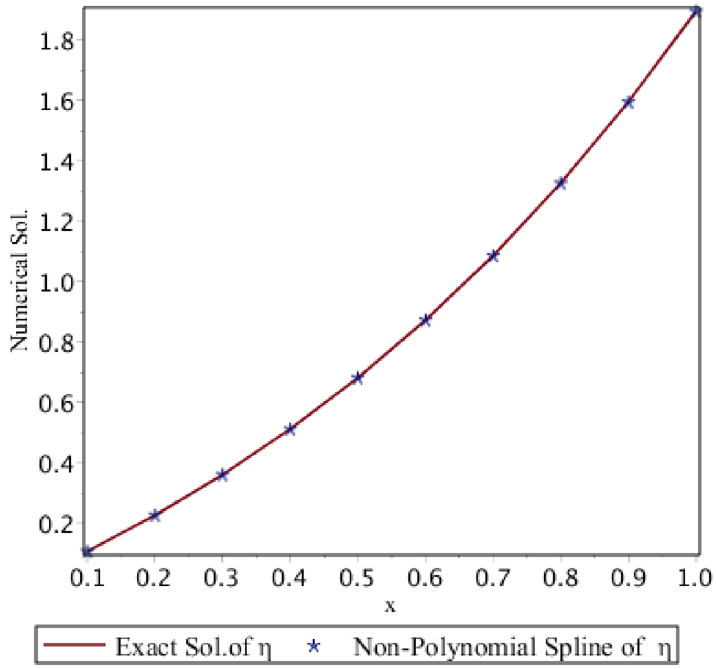

Figure 1: The exact solution and numerical solution of (η) for Example 1.

Example 2 [15]:

Consider the following nonlinear VIE of the second kind:

$$\eta(x) = 1 - x + \frac{x^2}{2} + \int_0^x (\mathrm{t} - x)\,\eta(t)dt\,. \qquad 0 \le x \le 1 \quad .$$

With the exact solution $\eta(x) = 1 - \sin(x)$

The obtained absolute error of quartic non-polynomial for Example 2 by applying quartic non-polynomial spline method compared by absolute error of third order non-polynomial that estimated by Harbi [15] are listed in Table 2 with $h = 0.1$ and $x_i = x_0 + ih, i = 0,1, \dots, n$. Figure 2 showas the comparison between the exact solution and numerical solution of $\eta(x)$ by applying quartic non-polynomial spline method for Example 2.

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Table 2: A comparison between the absolute errors of our method with [15] for Example 2 for $n = 10$ at different values of x.

x	Quartic non-polynomial φ(x)	Third order non-polynomial φ(x) [15]
0.1	0	0
0.2	0	0
0.3	0	0
0.4	1.00e-16	2.00e-16
0.5	0	1.00e-16
0.6	1.00e-16	1.00e-16
0.7	0	1.00e-16
0.8	1.00e-16	2.00e-16
0.9	0	3.00e-16
1	1.00e-16	4.00e-16

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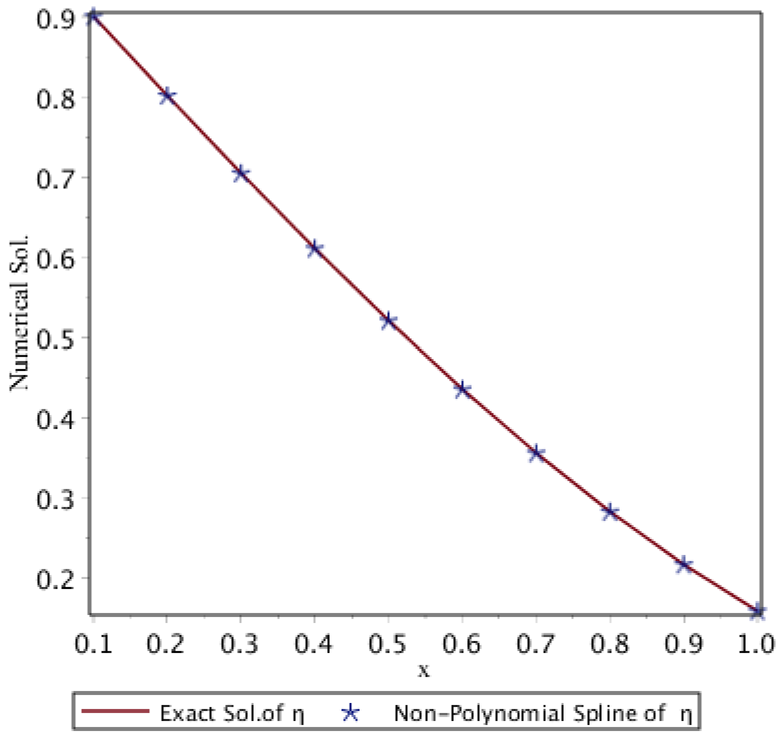

Figure 2: The exact solution and numerical solution of (η) for Example 2.

Example 3[7]:

Consider the following VIEs of the second kind:

$\eta(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (\mathrm{t} - x)\,\eta(t)dt\,.$

With the exact solution $\eta(x) = x^2$

Table 3 shows a comparison between the exact solution $\eta(x)$ and the obtained numerical solution for example by applying quartic non-polynomial spline methodwhere $h = 0.1$ and $x_i = x_0 + ih, i = 0,1, \dots, n$.

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Table 3: A comparison between the solutions of our method with exact solution for Example 3 for $n = 10$ at different values of x

x	Quartic non-polynomial φ(x)	Exact solution φ(x)
0.1	0.01	0.01
0.2	0.04	0.04
0.3	0.09	0.09
0.4	0.16	0.16
0.5	0.25	0.25
0.6	0.36	0.36
0.7	0.49	0.49
0.8	0.64	0.64
0.9	0.81	0.81
1	1	0.01

6. Conclusions

This study presents a numerical treatment for nonlinear Volterra integral equation of the second kind by the non-polynomial spline of the fourth order. As well as, the proposed method has been tested on three examples of VIEs. The numerical results which are obtained by the proposed method are very much in agreement with the exact solutions. The importance of the proposed method lies in the easy applicability, accurate and efficient to solve integral equations compared by other methods. that the obtained results of the solution of VIEs using fourth order non-

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polynomial spline function appeared more accurate and closer to the exact solution than the solution by third order non-polynomial spline function and trapezoidal method.

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الدراسة العددية لمعادلة فولتيرا التكاملية من النوع الثاني باستخدام طريقة الغير متعددات الحدود السبلاين من الدرجة الرابعة

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المستخلص

في هذا البحث ، قدمت دوال غير متعددة الحدود من الدرجة الرابعة لحل معادلة فولتيرا التكاملية غير الخطية من النوع الثاني ،حيث أظهرت الامثلة العددية توضيح لتطبيقات الطريقة المقترحة ومقارنة النتائج المحسوبة مع الطرق الاخرى .بالإضافة إلى ذلك ، قارنا القيمة المطلقة للخطأ لهذه الطريقة مع الطرق الأخرى والحل الحقيقي . لذلك فإن النتائج التي تم الحصول عليها من طريقتنا مشجعة للغاية مقارنة بالطرق العددية الاخرى.

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