

On Some Exponential Diophantine Equations

Mohammed A. Alabbod

Department of Mathematics
Faculty of Science
Basrah University
Basrah, Iraq

email: mohna_l@yahoo.com

(Received August 13, 2021, Accepted September 16, 2021)

Abstract

In this paper, we consider Exponential Diophantine equations of the form $p^X + (p + \lambda + 1)^Y = Z^2$ over the set of all positive integers, where $p > 3$, $p + \lambda + 1$ and λ are primes with λ being a non Sophie Germain prime that is congruent to -1 modulo 4. We show that such equations have no solutions in positive integers X, Y and Z .

1 Introduction

Exponential Diophantine equations of the form $p^X + q^Y = Z^2$ have been studied by many researchers [9], [10], [13], [15], [1], [2], [5], [6], [8], [11], [12], [3], [4], [16], [7], [14], [17]. In particular, Gayo and Bacani [10] studied the Diophantine equation $p^X + q^Y = Z^2$ in relation to Mersenne primes and gave a list of some unsolvable cases for this equation. Mina and Bacani [13] showed that the equation has no positive integer solutions if p and $q = p+4$ are cousin primes, and X, Y are of the same parity. In 2018, Burshtein [6] established that this equation has no solutions in positive integers X, Y, Z whenever p and $q = p + 4$ are primes. For primes $p > 3$ and $q = p + 8$, Fernando [8] showed that this equation has no solution (X, Y, Z) in the set of positive integers. In 2021, Dockan and Pakapongpun [9] proved that this equation

Key words and phrases: Diophantine, exponential, Exponential Diophantine equation, prime number, Sophie Germain.

AMS (MOS) Subject Classifications: 11D61.

ISSN 1814-0432, 2022, <http://ijmcs.future-in-tech.net>