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Review on Buckling and Bending Analysis of Functionally Graded Beam with and without Crack

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Abstract

The functionally graded beam is a wide field of research, which attracts great interest today in the field of engineering, science, and medicine society. This type of beam is made from functionally graded material that is characterized by several properties one of them is the high strength to weight ratio. In the current years, this beam has witnessed great developments in the mechanism of its composition and the materials used in its manufacture. This research provides an overview of the properties, types, advantages and challenges, and applications of the functionally graded materials. In addition, this paper review provides a summary of the analysis of bending and buckling that occurs on the functionally graded beam with and without crack effect from (2008-2021) year. Through this review, the following was noted: Firstly, a small number of researchers have worked experimentally, and the properties of a beam in most of the research are gradual towards thickness using the mixing rule. Secondly, the crack has a very severe effect on the behavior of both bending and buckling for the graded beam. This critical review can be considered a milestone in future analyzes of the graded beam and is also beneficial to designers and researchers working in this field.

Keywords: Functionally grade material, Application and types of FG material, Static deflection analysis, Buckling analysis, Vibration analysis, Crack effect.

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1. Introduction

1.1. Functionally graded material

Functionally graded materials (FG) materials are special class of composites material. Typically, an FG material is manufactured from a continuous variation of metal and ceramic mixture. Since the ceramic material has a property of heat and corrosion resistant while the metal material can provide the mechanical toughness to the produced system. The comparison between the traditional composites and FG material structure illustrated in Fig. 1. From this figure, it is very important to note that the sharp interface in composites structure is replaced with gradient variation that produce smooth transition from one material to the other. And this variation lead to eliminate the stress concentration in the interface layer and avoid the delamination failure (separation of fibers from the matrix) that found in laminated composite [1], [2]. The properties of FG materials are commonly assumed by continuous gradation through the direction of thickness by several models as follow [3]:

- Power-law
- Exponential-law
- Sigmoid law

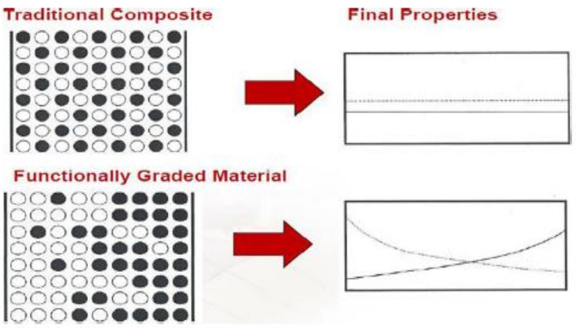

Fig. 1 Comparison between convention composite and FG material.

1.2. Types of FG materials

There are two types of FG materials:

1. Based on the material gradient forms, FG material can be divided into four types: (a) gradient in the fraction, (b) gradient in the size, (c) gradient in the orientation, and (d) gradient in the shape as shown in Fig. 2 [4].

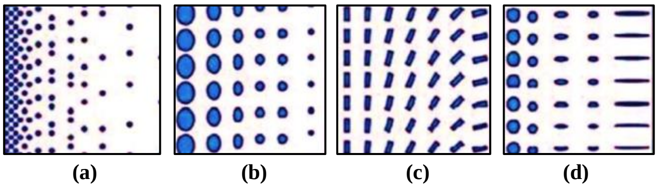

Fig. 2 FG material type based on gradient [4].

2. The basis on the structure of the material: There are two types of structure continuous and discontinuous as shown in Fig. 3. In continuous gradient, the FG material starts from one material to another. While, a material gradient is given in a layered form, in the state of discontinuous FG material [5]. In this material, functions or compositions are changing from one side to the other step wisely or continuously. Examples of stepped-wise graded structures and continuous graded structures are spark plug and bone respectively.

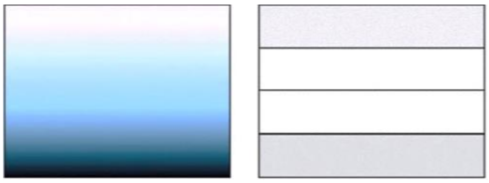
(a) (b)

Fig. 3 FG material type based on structure: **(a)** continuous and **(b)** discontinuous [6].

1.3. Advantage and challenges of FG materials

FG materials consist of two different methods as ceramic and metal. Ceramic material is characterized by high-temperature resistance while the metals can introduce support to the produced system, so the materials are adequate for applications with high temperature and special physical properties [1].

The advantage of FG materials are [2]:

1. Give multi-functionality.
2. It can design in a different complex environment.
3. It can remove and avoid stress concentration and delamination failure respectively.
4. provided from ceramic and metal to take the benefit of its such as resistance to oxidation, corrosion, and toughness.

The challenges of FG materials are:

1. High cost for production.
2. Required special type of control.
3. Mass production.

1.4. Application of FG materials

The FG materials represent the ideal solution for applications needing differing properties in the same composite where this component required high ductility inside versus high hardness on the outside. So that the FG materials with gradient properties are appropriate in many applications such as defense, aerospace, medical, structural wall, sports, coating material, energy, and automotive as shown in Fig. 4 [7].

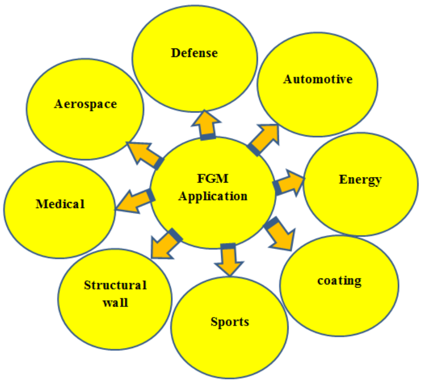

Fig. 4 FG material applications.

- *Defense:* FG material characterized by the ability to prevent crack propagation. This property makes it necessary in defense application as a diffusion-resistant material.
- *Aerospace:* FG material can withstand high thermal gradient, this makes it appropriate for use in rocket structures and space vehicles.
- *Medical:* Used extensively in the medical application for teeth and bone replacement [1-3], [7].
- *Structural wall:* including sound and thermal isolation.
- *Sports:* many tools such a clubs, golf, skis, and tennis rackets are manufactured by adding graded combination, rigidity, or elasticity.
- *Coating:* Enhanced the coating of the car's body.
- *Energy:* FG materials are used in the devices of energy conversion, also provide thermal barrier coating that is used as a protective coating on turbine blades in gas turbine engines [7].
- *Automotive:* such as diesel engine pistons, combustion chambers, drive shafts, race car brakes, and flywheels, in terms of thermal stress and strain must be graded. FG materials are adequate for these applications as shown in Fig. 5 [7].

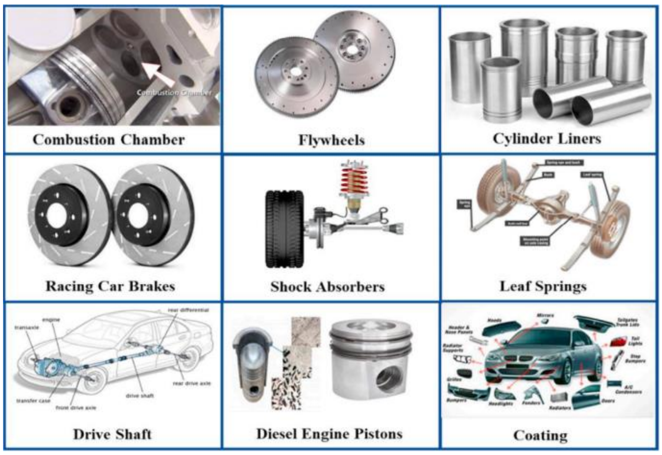

Fig. 5 Automotive applications [7].

2. Beam theory deformation

There are mainly two kinds of beam theories, the first one is classical beam theory (CBT) or Euler beam theory and the second one is shear beam theory deformation which is used to get more exact results. There are three types of shear deformation theory [1], [8]:

1. First-order or Timoshenko beam theory (FSDBT).
2. Second-order shear deformation beam theory (SSDBT).
3. High-order shear deformation theory (HSDBT).

The CBT assumed that the straight line perpendicular to the mid plane will still be straight and perpendicular after bending. This theory is neglected the transverse shear strain. This assumption is suitable for thin beam and plate and doesn't provide for thick beam and thick plate.

This limitation of CBT is removed by the Timoshenko beam theory. Timoshenko beam theory assumed that before bending, the straight line is perpendicular to the midplane will no longer remind perpendicular to this plane after bending, and on the top and bottom surface the shear condition is zero so that the shear correction factor is required in account and equal to (5/6). The SSDBT has assumed the straight line before bending is normal to the mid plane and will be changed to the form of the cubic curve after deformation. And finally, the HSDBT is assumed that the transverse shear stress and shear strain to a thickness of beam is distributed in parabolic form. From above we can conclude that in TBT, the transverse shear strain is considered while in CBT its neglected.

3. Bending and buckling analysis of FG beam with and without crack

More research was related to the functionally graded material due to the importance of this material in more mechanical applications.

In this review, the previous works are classified into four groups arranged according to the history of each one, bending and buckling with and without a crack in the FG beam.

3.1. Bending analysis of FG beam without crack

The static analysis of the simply support FG beam exposed to a uniform distributed load was investigated numerically by Simsek (2009) [9] by using the Ritz method within the basis of theories of first and high order shear deformation. The material properties varied in the direction of thickness according to the power-law form. In trigonometric functions, trial functions denoting the axial, transverse deflection, and rotation of the cross-sections of the FG beam are expressed. The effect of material distributions on the stresses and displacements of this beam is studied numerically. The results showed that the stress distributions in these beams are different from those inhomogeneous beams.

Metin (2009) [10] used a general nonlocal beam theory to analyze the static deflection of nano beams numerically. Different beam theories (Euler-Bernoulli, Timoshenko, Reddy) were used in the present formulation to study the beam length and non-locality parameters.

Huu-Tai and Thuc (2012) [11] studied for bending behavior of FG beam high order shear deformation beam theories. In thickness direction, the material properties change due to the power-law form. The effect of shear deformation and power index was studied numerically. The results showed that the including shear effect and increasing the index value leads to an increase in the deflection in the FG beam.

The static bending of function graded beam using refined shear deformation theory was presented numerically by Thuc et al. (2013) [12], who used Hamilton's principle to derive the governing equations of motion analytically. The influence of the boundary conditions, power index, and modulus ratio on the displacements were investigated numerically. The results showed that for a constant power-law index, the displacement decreases with an increasing modulus ratio.

Tharaknath et al. (2014) [13] investigated the static analysis of an FG simply-supported beam under distributed load based on HOSDT and TBT theory by using the Ritz Method. In this work, the material properties are assumed to vary in the thickness direction according to power-law form. The effect of material distribution is studied numerically. The results showed that the modulus variation has a great effect on displacement and stress distribution. Also, the choice of a suitable power index leads to a decrease the stress and displacement. Mohammadimehr and Mahmudian-Najafabadi (2014) [14] studied the static deflection of Timoshenko nano FG beam reinforced by boron nitride nanotube by using micromechanical approach inserted in an elastic medium analytically. On dimensionless deflection, the effect of material length scale parameter FG nano beam was investigated. The authors found through the results that the dimensionless deflection of FG nano beam increases with increasing of the material length scale parameter.

The finite element model (numerical model ANSYS software) of the FG beam with the active fiber composite (AFC) material acting as the dispersed actuator for the beams is presented by Tripathy et al. (2015) [15]. In FG beams the material properties are varied continuously due to a power-law distribution in a thickness direction as shown in Fig. 6. The results were presented by analyzing the simulation model which explains that with the rise in the power-law index, the bending rigidity of the beam increases and leads to decreasing the maximum deflection.

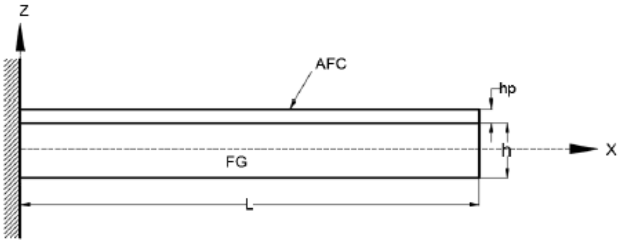

Fig. 6 Functionally Graded beam integrated with a layer of (AFC).

Symmetric Smoothed Particle Hydrodynamics (SSPH) method was used by Armagan (2016) [16] to solve the transverse deflections of FG beams under different boundary conditions and uniform load by strong formulation of the problem. The different numbers of nodes are used to perform the numerical calculations. They are distributed uniformly in the problem domain and by using different beam theories which are the EBT, TBT, and RBT. The SSPH method introduced the strong formulation to analyze the static transverse deflection of this beam dependent on different beam theories. The results showed that the use of the SSPH method can be solved the problem of FG beam by using different shear deformation beam theories.

Waleed et al. (2016) [17] presented the static behavior of orthotropic, isotropic, and FG beams analytically and numerically by a Finite Element Method. In thickness direction, the mechanical properties of this beam are distributed by the power-law form. The influence of boundary conditions and power index is studied. The results showed that the resistance to the different static loads increased with increasing the power index value. One dimension FE model for static deflection of FG beam based on zig-zag theory was presented by Ateeb et al. (2016) [18] modified rule of mixture (MROM) was used to calculate the modulus of elasticity, Poisson ratio, and density of FG beam. One dimension FE zig-zag theory for FG beam was presented by the MATLAB program. ABAQUS software based on 2D FE for the same beam was studied also the results got by the 1D FE model were acceptable with that obtained by 2D FE ABAQUS.

Aldousari (2017) [19] investigated the bending behavior of a function graded s-s beam designed according to Euler-Bernoulli theory and subjected to the distributed load. The stiffness matrix and force vector was formed by using the finite element model and then the problem was solved numerically. The effect of power exponent and modulus ratios on bending of FG beams. The results showed that the symmetric power function can significantly decrease the stress than the other functions. Though, the highest stress was represented by sigmoid function distribution. The static behavior of a layered cantilever beam structure by using the finite element formulation was studied by Othman et al. (2017) [20]. The element of the beam was modeled depending on the first shear beam theory deformation and it has consisted of three layers while the middle layer was FG material with the properties variable along thickness direction according to the exponential or power-law as shown in Fig. 7. The effect of mechanical loadings on variations of the stresses across the depth and displacements along the beam are studied. The results showed that the FG material layer leads to smoothened the stress distribution along with the thickness of the beam, though there was a hard jump in the distribution of stress in the two-layers beam, also the choosing a suitable value for the index value can minimize stresses and deflections.

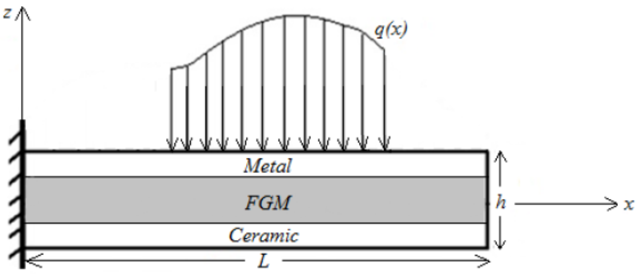

Fig. 7 Layered cantilever beam.

Noha et al. (2017) [21] studied a modified porosity to analyze the static bending of a porous FG beam analytically and numerically. The material variation in beam thickness is due to the non-linear power function. The effect of power-law and porosity index were studied. The results obtained that the static deflection increased by increasing both the porosity and the power-law index.

Hyder et al. (2018) [22] derived the analytical approach depending on the theory of compound beam to calculate the static deflection and to determine the equivalent cross-section area of the new beam. Analytically this approach can be attained when the power index is equal to one while numerically can be achieved for any value of the power index. In this work, the Finite Element Method by using the ANSYS program is used depended on laminated theory to produce three different models.

Atteshamuddin and Yuwaraj (2018) [23] presented a simple modified exponential shear deformation theory (ESDT) for the bending analyses of FG beams. The effect of aspect ratio, boundary conditions, and the power-law index was examined analytically. The result showed that the increase in the power-law index lead to an increase in the transverse displacement because the increase in power index makes this beam flexible. Also, as the value of the aspect ratio increases, the transverse displacement increases.

You-Ming et al. (2019) [24] presented the relations between the solutions of the static bending of the FG beam and those of the homogenous Euler beams HEB. The general solutions of displacement and bending for FG materials were derived in terms of static deflection corresponding to HEBB with similar geometry and external force. Vu et al. (2019) [25] studied a new model of FG beam depending on modified FSDT to analyze the mechanical static bending behavior of the variable thickness FG beams as shown in Fig. 8. The effect of aspect ratio, power index, and degree of non-uniformity on the mechanical behavior of non-uniform thickness FG beam was measured. The benefit of this model was simple in preparation. The result showed that this kind can be used widely to analyze the bending behavior of the FG beam.

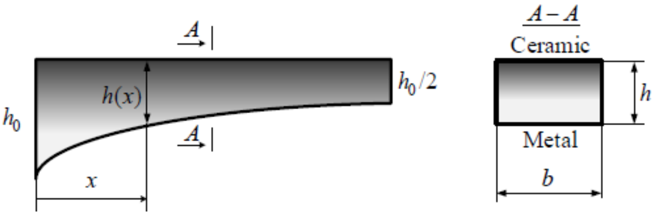

Fig. 8 FG beam with variable thickness.

Sayyad et al. (2019) [26] used high order theory to presented the static analysis of FG beam curved in elevation with both thickness stretching and shear deformation effects numerically and analytically. Sandwich beams with FG material in the skins or core under distributed load are investigated. The effect of power index, skin-core-skin thickness ratios, and boundary conditions was studied.

Yuewu et al. (2019) [27] proposed a novel FG beam reinforced with graphene nano plate by using a new distributed law, that is built by gradient the index value by using the error function. The bending behaviors of the novel nano composite beams under uniform distributed load were studied numerically. The authors from the results observed that the offered distributions of these nano fillers can improve the properties of these composites.

Souhir et al. (2020) [28] studied the effect of porosity on static analysis of FG beams by using the finite element model. Two different types of porosity namely uneven and even distributions were used to estimate the mechanical properties of FG porous beams. The effects of porosity coefficient and power-law index deflections have been studied. The results showed that when the porosity coefficient increase and the power index decrease the deflection increase.

Farshad et al. (2020) [29] studied the static bending of the FG beam with different beam theories. To illustrate the performance of the FG beam, the effect of aspect ratio and porosity index have been studied. FG beam was designed by ANSYS (solid 186). The results showed the deflection is increased as the aspect ratio increase. The first three modes of the FG beam are illustrated in Fig. 9.

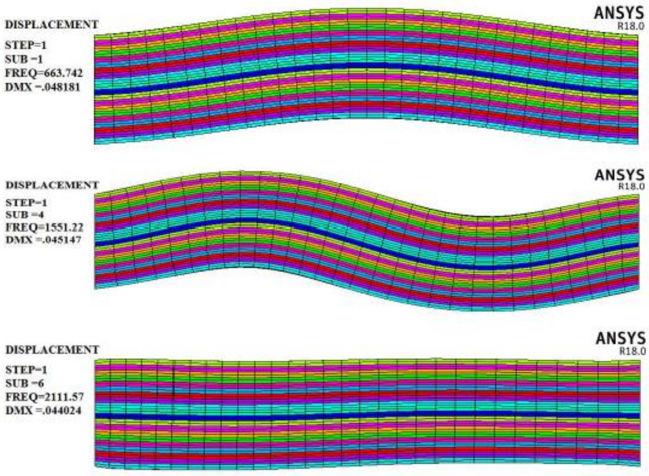

Fig. 9 First three modes l-h 5, n 1, α 0.

Summary of bending in FG beam

Table 1. summary of literature review for bending in FG beam.

Authors	Year	Type	Field	Compute
[9]	2009	Numerical	Comparison between FG and isotropic beams.	Static analysis
[10]	2009	Analytical	Used a generalized nonlocal beam theory	Bending behavior
[11]	2012	Numerical	Effect of power law index	Deflection
[12]	2013	Analytical	Refined shear deformation theory	Static bending
[13]	2014	Numerical	The effect of material distribution and the stresses	Dimensionless axial displacement
[14]	2014	Analytical	The effect of parameter of material length scale	Non dimension deflection
[15]	2015	Numerical	Effect of power law index	Maximum deflection.
[16]	2016	Numerical	Different beam theory	Transverse deflection
[17]	2016	Analytical & numerical	Comparison between isotropic, orthotropic and FG beams	Static deflection
[18]	2016	Numerical	Comparison between 1D and 2D finite element model	static response of FG beam
[19]	2017	Numerical	The influence of power index and elasticity ratio	Static bending
[20]	2017	Numerical	Variations of the stresses across the depth of beam	Static behavior of a layered beam
[21]	2017	Analytical	The effect of power law and porosity index	Deflection
[22]	2018	Analytical & numerical	The three different ANSYS models	static deflection of s-s and cantilever FG beam
[23]	2018	Numerical	The effect of power index and aspect ratio	Dimensionless bending
[24]	2019	Numerical	Comparison between of FG Reddy beam and HEBB.	Static deflection
[25]	2019	Numerical	Effect of power index, degree of non-uniformity, and L-H	the mechanical static bending behavior
[26]	2019	Numerical & analytical	Used high order theory	Bending analysis of beam curved
[27]	2019	Numerical	Distributions GPL nano fillers	Bending behavior
[28]	2020	Numerical	Effect of porosity	Static bending
[29]	2020	Numerical	Effect of aspect ratio, material distribution, and porosity index	Deflection

3.2. Buckling analysis of functionally graded beam without crack

Metin (2009) [10] studied the buckling of nano beams by using the theory of non-locality. Different beam theories (Euler-Bernoulli, Timoshenko, Reddy) were used in this formulation. The effect of beam length and non-locality was studied. The results showed that with increasing nonlocal parameters, the non-dimension buckling load decrease.

Houari et al. (2014) [30] presented the refined theory of first-order shear deformation for the critical buckling load of FG nano beam. The effect of non-locality was examined numerically. The results showed that as the nonlocal parameter increase, the critical buckling decrease.

Saljooghi et al. (2014) [31] used the Reproducing Kernal Practical Method (RKPM) to analyze the buckling load for the FG beam as shown in Fig. 10. The effect of material distribution and different boundary conditions were studied. The RKPM method is shown to be a power full tool to simulate the buckling of the FG beam in which the neutral axis can be displaced drastically.

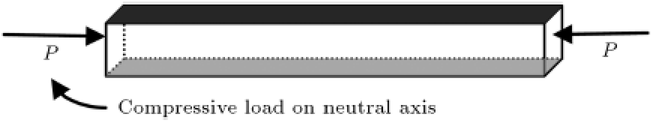

Fig. 10 beam under axial load.

Trung-Kien et al. (2015) [32] advanced a new high order shear deformation theory to analyze the buckling of isotropic and FG sandwich beams. The analytical solution was derived for three types of FG beam with different boundary conditions. The effect of power index, aspect ratio, and skin-core-skin thickness ratio on the critical load of buckling was studied. The results showed that the presented theory is efficient in explaining the buckling performance of FG- sandwich beams and isotropic beams.

Uysal and Kremzer (2015) [33] studied the buckling behavior of short functionally graded cylinders manufactured from polymeric material by centrifugal casting as shown in Fig. 11. Epoxy resin is used as polymeric matrix reinforced with two types of graphite powder materials, PV60/65 and PAM96/98. The powders of graphite were added in quantities of 3, 6, 9, and 12 % of volume respectively. The results presented that the higher buckling load was achieved in the sample containing 12 vol. % of PV60/65 graphite.

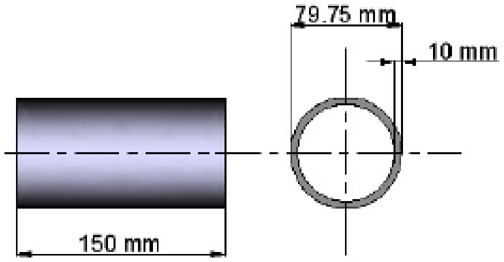

Fig. 11 cylindrical specimen manufactured by centrifugal casting

Jie et al. (2016) [34] investigated the behaviors of buckling and post-buckling of multilayer FG nano beam reinforced with a low content of graphene plates (GPLs). Assuming the distribution of GPLs were uniformly and randomly oriented in each GPL-reinforced composite layer (GPLRC) with weight fraction varying layer-wise along the thickness direction. The effect of pattern distribution, weight fraction, size, and geometry of the GPL nano filler was studied. The results showed that the GPLs have an important effect on the buckling and the post-buckling of nano composite beams.

Mohammad et al. (2016) [35] analyzed the buckling of FG Euler-Bernoulli nano-beam depending on nonlocal elasticity. The effect of the parameter of material length scale and the constant of inhomogeneity were studied. Numerical results showed that the increase in inhomogeneity lead to an increase in the critical buckling load.

Noha et al. (2017) [21] studied an adapted porosity to analyze the buckling behavior of a porous FG beam. The material variation was assumed to be distributed through the beam thickness according to the non-linear power function. The effect of power and porosity index were studied. The results obtained that the critical buckling decreases by increasing both the power and porosity index.

Atteshamuddin and Yuwaraj (2018) [23] presented a simple modified exponential shear deformation theory to analyze the buckling of FG beams with various boundary conditions. The effect of aspect ratio, power-law index, and boundary condition are examined. The result showed that the value of dimensionless buckling decreases with the increase of power-law index and with decreasing of aspect ratio. Also, the clamped beam predicts the highest dimensionless buckling whereas the cantilever beam predicts lower.

Vu et al. (2019) [25] studied a new model of the beam to analyze the buckling behavior of the variable thickness FG beams based on a modified first-order shear theory of deformation. The effect of aspect ratio, degree of non-uniformity, and power index on the mechanical behavior of non-uniform thickness FG beam was studied. The result showed that this model can be extensively applied to analyze the mechanical behavior of the FG beam.

Farshad et al. (2020) [29] used various beam theories to analyze the buckling of the FG beam. The effect of material distribution and porosity index were studied. FG beams are modeled by ANSYS (solid 186). The results showed that the critical buckling load decreased with the increasing porosity and power index value.

Raghad et al. (2021) [36] analyzed the buckling behavior of the FG beam analytically and numerically to calculate the dimensionless critical buckling load. New FG beam models are derived based on Timoshenko and Euler's theories. For simply supported FG beam, the Navier solution is used to calculate the analytical equation for critical buckling load. The material properties are assumed to vary in the thickness direction according to the power-law form. SHELL 281 in ANSYS program is used to analyze the buckling behavior numerically. The effect of several parameters such as aspect ratio, modulus ratio, and power index value on dimensionless buckling are studied numerically and analytically.

Summary of buckling in FG beam:

Table 2. a summary of different methods used to analyze the buckling of FG beam.

Authors	Year	Type	Field
[10]	2009	Analytically	Used a generalized nonlocal beam theory.
[30]	2014	Numerical & Analytical	Effect of nonlocal parameter.
[31]	2014	Numerical & Analytical	Effect of material distribution and different boundary condition.
[32]	2015	Numerical & Analytical	Effect of power index, boundary condition, and aspect ratio.
[33]	2015	Numerically	Comparison between two types of graphite powder.
[34]	2016	Numerical & Analytical	Effect of pattern distribution, weight fraction, and size and geometry of GPL nano filler.
[35]	2016	Numerically	Effect of length scale parameter and inhomogeneity constant.
[21]	2017	Analytically	Effect of power index and porosity index.
[23]	2018	Numerical & Analytical	Effect of aspect ratio, power index, and boundary condition.
[25]	2019	Numerically	The effect of power index, degree of non-uniformity, and aspect ratio.
[29]	2020	Numerically	The effect of material distribution and porosity index.
[36]	2021	Numerical & Analytical	Non dimension critical buckling load.

3.3. Bending and buckling analysis of FG beam with Crack

Yang and Chen (2008) [37] presented a theoretical investigation to elastic buckling of cracked FG beam based on Euler beam theory. The crack was designed by using the model of massless rotational spring. The analytical solution is used for buckling with different boundary conditions. The effect of material properties, location, and cracks number was studied. The results showed that all parameters have a great effect on natural frequency and buckling.

Liao-Liang et al. (2009) [38] studied the buckling behavior of FG beam having open edge crack depending on Timoshenko beam theory. The critical buckling load got analytically for cracked FG beam with various boundary conditions. The effect of modulus ratio, aspect ratio, boundary condition, and crack and location on the buckling of cracked FG beam were studied. The results showed that the increase in crack depth lead to a decrease in the buckling load. Also, the FG beam with a smaller slenderness ratio and low modulus ratio is more effective to edge crack.

Seref (2013) [39] studied numerically the static analysis of cracked Timoshenko FG beam exposed to transverse point load at the free edge of FG beam. The cracked FG beam was modeled by a massless elastic rotational spring. The effect of crack depth and location and the effect of material distribution on static response were investigated. The results showed that the depth and location of crack have an excessive effect on the nonlinear static response FG beam also materials distribution is important for minimizing the negative influence of crack.

Sherafatnia et al. (2014) [40] investigated analytically the buckling analysis of cantilever FG beam having an open edge crack depending on four beam theories, Rayleigh, Euler-Bernoulli, Timoshenko, and shear beam theory. The crack is designed by two massless linear and rotational springs. The effect of crack depth, crack position, and power index on buckling was investigated. The results showed that the existence of crack lead to reducing the buckling load. Also, when the crack is found near the support with a lower modulus ratio lead it reduces the critical buckling load.

Haider et al. (2015) [41] studied the fracture analysis of FG material with different crack locations by an extended element-free Galerkin method (XEFGM). The results showed that the accuracy of the solution is increased by developing this method for isotropic and anisotropic FG material crack analysis along with the use of the sub-triangle technique for appropriate support domain, numerical integration, and enrichment functions in the crack location.

Şeref (2018) [42] investigated the static bending of an edge cracked cantilever nano FG beam exposed to transversal point load at the free end of the beam depending on modified couple stress theory. The cracked nano FG beam was designed by two sub-nano beams connected by a massless elastic rotational spring. The effect of material distribution, depth, and position of crack on static deflection was studied. The numerical results showed that the location and depth of the crack and the material distribution have an excessive effect on the static behavior of the FG nano beams.

Meifung et al. (2019) [43] the finite element method is used to study the compressive buckling of cracked FG graphene nano plates reinforced composite (GPLRC) beam. The weight fraction of grapheme nano plates remains constant in each layer but varies in a thickness direction. The effect of boundary conditions and weight fraction on the critical buckling load was investigated. The results showed that the cracked GPLRS beam with GPLs on the bottom and top surface of this beam lead to enhance the buckling performance. Also, the buckling load is affected by the dimension and geometry of the GPLs nano filler.

Haider et al. (2019) [44] used the digital image correlation technique to compute the fracture parameters, such as intensity factors of a stepwise FG material numerically and experimentally. Five layers of 60 % epoxy 40 % glass are investigated. The effect of crack length and crack tip on non-singular terms are analyzed. The results showed this numerical model can be used to evaluate the fracture parameters for the presented FG material.

Wafaa et al. (2019) [45] investigated the fracture analysis and mechanical properties of FG materials numerically and experimentally. In this study, FG material is made from pure to 40 % epoxy resin and sphere glass manufactured by using the hand lay-up technique. The digital image correlation (DIC) method is used to determine the stress intensity factor for compact tension specimens. This experiment method is compared with the numerical meshless extended element-free Galerkin method and showed good agreement.

Haider and Nathera (2019) [46] used a computational method for crack propagation analysis of FG materials under non-proportional loading and mixed-mode based on the extended element-free Galerkin method. In the crack location for numerical integration, enrichment functions, and appropriate support domain, the sub-triangulation technique are applied. The stress intensity factor is evaluated by using the incompatible interaction integral method. For crack propagation, the maximum hoop stress criterion is assumed for crack propagation criterion.

Summary for crack in FG beam

Table 4. a summary of the above researchers for different methods used to study the effect of crack of FG beam.

Authors	Year	Type	Field
[37]	2008	Analytical	Effect of material properties, location and number of cracks are studied.
[38]	2009	Numerical	Effect of modulus ratio, aspect ratio, boundary condition, crack depth location.
[39]	2013	Numerical	Effect of material distribution, crack depth and location.
[40]	2014	Analytically	The effect of crack depth, crack position, and material properties.
[41]	2015	Numerically	Crack location.
[42]	2018	Numerical	Material distribution, depth and position of crack.
[43]	2019	Numerically	GPLs weight fraction, and boundary condition.
[44]	2019	Numerically and experimentally	Calculate the intensity factor.
[45]	2019	Numerically and experimentally	Analysis the fracture of FG material.
[46]	2019	Numerically	The stress intensity factor.

4. Conclusions

In this review paper, the nature and properties of the FG materials are existing in addition to the analysis of the behavior of buckling and bending of the graded beam with and without cracks.

In this summary, the following research gaps can be filled:

1. In all the above research, the influence of the power index, aspect ratio, modulus ratio, and crack depth and location for the graded beam were studied numerically and analytical because of the high cost of manufacturing such materials to study it experimentally.
2. There is limited work to gradient properties in a longitudinal direction, in two directions, or three directions, as there are applications that require gradation of properties in more than one direction because this gradation has a significant effect on the buckling load and the amount of deflection that occurs in this type of beam.
3. There are also few studies in the experimental fields and areas that depend on the thermal effect. The large gradients in temperature, high pressures, and extreme humidity made engineering structures more prone to failure. Therefore, there is a great need for research efforts in this field.
4. It can be seen that the effect of changing the properties of the material in thickness direction has been studied much more than changing them in the length direction.
5. There are many papers available on the analysis of the static deflection and buckling behavior of the gradient beam, where the material properties are varied in thickness direction due to the mixing rule.
6. The above research showed that the crack has a great effect on the functionally graded beam behavior.

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