

# Numerical Study of Direct Contact Evaporation of Liquid Drops in an Immiscible Liquid

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## Abstract

The present work is a theoretical investigation of the direct contact evaporation process of drops in an immiscible liquid. A theoretical model based on continuity, momentum and energy equations in addition to the energy balance equation is developed, by using cellular model. This model aims to obtain characteristics of the direct contact evaporation of a single droplet bounded by a spherical cell of continuous liquid and these characteristics are used in order to develop a theoretical model to obtain the characteristics of direct contact evaporation of multidroplets in an immiscible liquid. The developed equations were derived and then solved simultaneously applying a numerical method. The energy equations with the potential flow velocity, taken into account the effect of the interaction between the adjacent drops, have been numerically solved using finite-difference techniques. The calculations are performed for n-pentane drops evaporating in distilled water. The theoretical results presented in terms of the average volumetric heat transfer coefficient, and the total time and height for complete evaporation. The influence of several important parameters on the evaporation characteristics were presented. The results indicated that the main parameters affecting the evaporation process were the initial size of droplets and the degree of superheat. The results were compared with the existing theoretical and the experimental results; the agreement between the results was good.

## الخلاصة

تهدف الحالية دراسة نظرية لعملية التبخر بالتماس المباشر قطرات مائع في مائع آخر يمتزج معها. تم تطوير نموذج رياضي لتمثيل العملية باستخدام النموذج لخوي (cellular model) اعتمد بناء هذا النموذج على معادلات الاستمرارية. يهدف مميزات عملية التبخر المحيطة بها، واستخدام هذه المميزات لتطوير نموذج رياضي بخلية كروية مميزات عن تبخر مجموعة من قطرات بالتماس المباشر مع مائع اخر غير امتزاجي. تم اشتقاق المعادلات المطورة ومن ثم حلها سوية بطريقة عددية. أيضا تم حل معادلة الطاقة، المتضمنة سرعة الجريان الكامن والتي أخذ بنظر فيها التأثير بين قطرات المتجاورة، حلاً عددياً باستخدام تقنيات الفروقات المحددة. أجريت الحسابات قطرات من الطبيعي. معامل انتقال الحرارة الحجمي، والزمن والارتفاع اللازمين عملية. بينت لهذه الحالة العوامل الرئيسية المؤثرة على عملية التبخر هي القطر الابتدائي للقطرة بين المائعين. نظرية عملية بين جيد.

**KEYWORDS:** Numerical Study, Direct Contact Evaporation, Liquid Drops, an Immiscible Liquid.

## 1. Introduction:

Direct-contact heat transfer between two immiscible liquids has a lot of advantages over traditional heat exchange methods with metallic heat transfer surfaces: lower driving temperature difference (the availability to operate at relatively small temperature driving forces), simple design and scale-up procedure and no surface corrosion and fouling. The practical applications have been found in a number of engineering processes such as spray cooling tower, accumulator, water desalination, emergency core cooling system, solar, geothermal, ocean-thermal energy conversion and thermal storage systems. A recent increase in publications dealing with the direct-contact heat transfer between two immiscible liquids with change of phase evidences a growing interest in this area[1]. The contact between the streams is promoted by bubbling or dropping one fluid, denominated dispersed phase, through the other, which constitutes the continuous phase. In this type of direct contact evaporation the heat transfer takes place between the continuous liquid and the droplets of a second liquid, dispersed phase, which rise due to buoyancy or other means, through the continuous liquid.

## Nomenclature

$A$	Cross-sectional area of the column, $m^2$	$U$	Instantaneous velocity of rising droplet, m/s
$A_d$	Droplet surface area, $m^2$	$U_r$	Instantaneous relative velocity of droplet, m/s
$B$	Dimensionless radius, $R/R_0$	$U_d$	Superficial velocity of dispersed phase, m/s
$C_d$	Drag coefficient	$V$	Volume, $m^3$
$C_p$	Specific heat capacity, J/kg.K	$\dot{V}$	Volumetric flow rate, $m^3/s$
$D$	Instantaneous diameter of drobble, m	$\dot{V}_d$	Volumetric flow rate of dispersed phase, $m^3/s$
$D_{cell}$	Diameter of Spherical cell of continuous liquid, m	$v_r, v_\theta$	Radial and tangential components of $U_r$ , m/s
$d_c$	Column diameter, m	$y$	Transformed Coordinate, $(r-R)/R$
$d_h$	Hole diameter, m	$Z$	Height of evaporation (axial displacement), Column height, m
$DR$	Density ratio between disperse liquid and continuous liquid, $(\rho_d/\rho_c)$	$Z_f$	Value of $Z$ for complete evaporation, m
$DR$	Density ratio between disperse vapor and dispersed liquids, $(\rho_d/\rho_{pd})$	Creek Symbol	
$D$	Non-dimensional parameter, $(1 - \rho_c/\rho_d - \rho_{pd})$	$\alpha$	Thermal diffusivity of continuous liquid, $m^2/s$
$E$	Non-dimensional parameter, $(1 - \rho_c/\rho_d - \rho_{pd})$	$\alpha_c$	Half opening angle of vapor phase in drobble, rad
$Fr_0$	Initial Froude number, $2$	$\beta$	Temperature difference (degree of superheat), $(T_c - T_{ds})$ , K
$G$	Density ratio, $(\rho_d/\rho_c)$	$\Delta T$	Void fraction or holdup (dispersed phase volum fraction)
$g$	Gravitational acceleration, $m/s^2$	$\epsilon$	Vaporization ratio
$H$	Height of liquid layer in drobble, m	$\theta$	Spherical polar coordinate, rad
$h_{fg}$	Latent heat of evaporation, J/kg	$\Phi$	Non-dimensional temperature, $(T - T_c)/(T_{ds} - T_c)$
$h, h_d$	Heat transfer coefficient, $W/m^2.K$	$\psi$	Transformed coordinate, $(-cos(\theta))$
$h_{mv}$	Average volumetric heat transfer coefficient, $W/m^3.K$	$\mu$	Viscosity of continuous liquid, kg.m/s
$h_{mvf}$	Average volumetric heat transfer coefficient for complete evaporation, $W/m^3.K$	$\mu_k$	Kinematics viscosity of continuous liquid, $m^2/s$
$Ja$	System Jacob number, $c C_{pc} T / (d_v h_{fgd})$	$\rho$	Density, $kg/m^3$
$k$	Thermal conductivity of continuous liquid, $W/m.K$	$\tau$	Dimensionless time, $(t - t_0) / (t_f - t_0)$
$M$	Added mass, Kg	$\psi$	Velocity potential, $m^2/s$
$m$	Mass of growing drobble, Kg	Subscript	
$\dot{m}$	Mass flow rate, kg/s	$av$	Average
$N_h$	Number of holes (orifices) in the distributor	$c$	Continuous liquid
$N_d$	Number density of drobbles, $m^{-3}$	$cd$	Continuous/dispersed liquid interface
$Nu$	Nusselt number, $(h D/k)$	$cell$	Spherical cell of continuous liquid
$Pe$	Peclet number, $(2 \dot{m} D / (k_c \rho_c))$	$d$	Droplet
$Pe$	Modified Peclet number, $(2 \dot{m} D / (k_c \rho_c))$	$ds$	Dispersed liquid (saturation)
$Pr$	Prandtl number of continuous liquid	$dl$	Dispersed Liquid phase
$Q$	Rate of heat transfer, W	$dv$	Dispersed vapor phase
$Q_c$	Rate of convective heat transfer, W	$j, k$	General grid point
$q$	Local Heat flux, $W/m^2$	$f$	Final
$R$	Instantaneous radius of drobbles, m	$0$	Initial value
$R_{cell}$	Radius of cell of continuous liquid, m	Superscript	
$r$	Radial coordinate, m	$+$	Dimensionless
$Re$	Reynolds number, $(2 \dot{m} D / (\mu_k \rho_c))$	$i$	Time grid point
$St$	System Stefan number, $(c C_{pc} T / (h_{fgd}))$		
$T$	Temperature, K		
$t$	Time, s		

If the temperature of the continuous liquid is greater than the saturation temperature of the dispersed phase and the nucleation set in the drops by any mean, the drops evaporate. As evaporation progresses, the droplet mass becomes a two-phase entity of liquid and vapor contained within an envelope of the surrounding immiscible second liquid. Such two-phase configurations have been referred to as a “drobbles” [2]. In this case the dispersed phase stores the heat transferred from the surrounding continuous liquid as latent heat. A critical review of the work carried out in this line of investigation is given by Ojimi [1].

Considerable works have been done to study the phenomenon of single drop evaporation in a column of an immiscible liquid. Among the earlier pioneering studies, Sideman and Taitel [3] developed an analytical expression for the Nusselt number by solving the steady state energy equation. They obtained the following relation for the instantaneous Nusselt number

$$Nu = \left( \frac{3 \cos \beta - \cos^3 \beta + 2}{\pi} \right)^{0.5} Pe^{0.5} \quad (1)$$

they studied the single butane and pentane drops evaporating in stagnant water and determined experimental correlations for the heat transfer coefficient.

Simpson et al.[4] obtained an experimental correlation for the instantaneous heat transfer coefficient which is:

$$h = \frac{2.57B^{1/6}}{1 + 0.206B^{5/12}} \quad (2)$$

Mokhtarzadeh and El-Shirbini[5] presented a theoretical analysis of evaporation of single drops of pentane and butane in a stagnant column of distilled water using available relation for the heat transfer coefficient. Battya et al.[6] studied the process theoretically and they carried out a regression analysis using the experimental data of Sideman and Taitel[3] and obtained the following correlation

$$Nu = 0.64Pe^{0.5}Ja^{-0.35} \quad (3)$$

Furthermore, there are numbers of experimental, theoretical investigations dealing with a swarm of droplets evaporation in an immiscible liquid in the literature. Among the important study along this line was the study of Smith et al.[7] they developed an analytical model to calculate the volumetric heat transfer coefficient for direct-contact evaporation. In their model, the single-droplet velocity was assumed to be of the form

$$U = U_0 \left( \frac{D}{D_0} \right)^{C_1} \quad (4)$$

and the fluid dynamics was described by the so-called drift-flux model. Further, the single droplet heat transfer was calculated as

$$Nu = \frac{hD}{k} = C_2 Re^{C_3} Pr^{1/3} \quad (5)$$

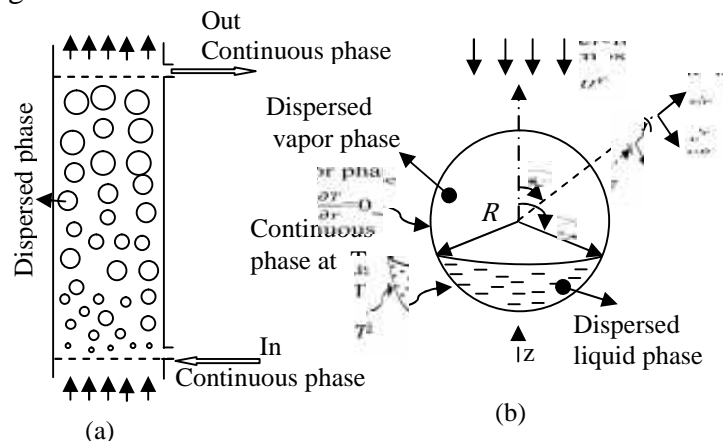
Where  $C_1$ ,  $C_2$  and  $C_3$  were inferred from their experimental data.

Seetharamu and Battya [8] studied the direct contact evaporation of R113 and n-pentane in a stagnant column of distilled water. A modified relation, based on the theoretical analysis of Smith et al.[7], was also developed for predicting the theoretical volumetric heat transfer coefficient. Mori[9] analyzed the heat transfer to drops of a volatile liquid sprayed upward in an immiscible liquid flowing down in a vertical column to enable calculation of the volumetric heat transfer coefficient in the column. For the analysis, a model was contrived that assumes the heat transfer to each of the drops could be approximated by an empirical correlation for heat transfer to an isolated drop evaporating in a quiescent column. Peng et al.[10] studied theoretically and experimentally the heat transfer to dispersed droplets in an immiscible continuous phase for the n-pentane–water system. In the theoretical model, the holdup and relative velocity between continuous and dispersed phase assumed to be constant and the heat transfer to each drobble was described by the same heat transfer correlation as that established for single-drop evaporation. Recently, Al-Jaberi[11] presented experimental investigation deals with the direct contact evaporation of spray n-pentane drops in flowing column of distilled water. In Addition, he developed an analytical model for

calculating volumetric heat transfer coefficients for direct-contact evaporation of multi-drops. Heat transfer was modeled using single drop correlation derived in his theoretical work based on quasi steady state solution of the energy equation, while the holdup and relative velocity between continuous and dispersed phase assumed to be constant. In spite of a large number of experimental and theoretical investigations that can be noted on this line on the review of literature the conclusion noted that, there is no universal theoretical model to study the evaporation of multidrobbles based on using theoretical model for heat transfer and the motion of single drobbles, instead applying an experimental correlations as depicted by nearly all the previous studies. Therefore, the purpose of this research is the attempt to solve, to some extent, this problem.

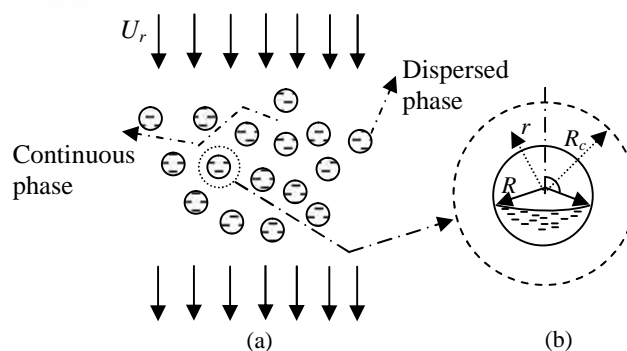
## 2. Analysis

Consider droplets of liquid(dispersed phase) are injected into a column of another immiscible liquid (continuous phase). If the continuous phase has a temperature higher than the boiling point of the dispersed phase, the drops will evaporate and hence extract heat from the continuous phase. The continuous phase is immiscible with either of phases, vapor and unevaporated liquid, of the dispersed phase. As the dispersed droplets complete their evaporation, the vapor exists through the free surface of the continuous liquid and subsequently withdrawn from the head part of column. In the course of evaporation the droplet forms a vapor-liquid two-phase bubble (Drobbles). Figure 1 shows the drobbles column and the most common configuration [1] of drobble.



**Figure 1 Evaporating drobbles (a) Drobbles column (b) Single drobble in a spherical coordinate**

The main problem in modeling of such systems(multi-drobbles systems) is that of drobble-drobble interactions. The present model adapted the cell models[12,13], which replace the difficult of many-drobble problem by a simple and conceptually more attractive one involving only one drobble. Within the framework of the cell model, inter-drobble hydrodynamic interaction are approximated by postulating that each drobble to be surrounded by a hypothetical envelope of continuous fluid of radius  $R_{cell}$  as shown in Figure 2. The size of the envelope is chosen such that the volume fraction of dispersed phase of each cell is equal to the overall mean volume fraction of dispersed phase(or holdup) ( $\epsilon$ )[14].



**Figure 2(a) Schematic representation of flow and (b) cell model idealization.**

Thus the diameter of cell,  $D_{cell}$ , can be related to the mean volume fraction of the dispersed phase in the swarm as follow:

$$D_{cell} = D \varepsilon^{-1/3} \quad (6)$$

Where  $D$  denote the instantaneous diameter. Therefore, one can simulate the swarms of various holdups including the limiting case of a single drobble by setting  $R_{cell} \rightarrow \infty$ , i. e.,  $\varepsilon \rightarrow 0$ . Conversely, one can readily calculate the value of  $R_{cell}$  for known values of the dispersed phase volume fraction and the drobble size.

## 2.1 The Potential Flow Field:

To drive the velocity components of the flow ( $v_r, v_\theta$ ) in  $r$  and  $\theta$  directions, the following assumptions are employed to simplify the actual physical phenomenon:

1. The fluids are Newtonian.
2. The flows are laminar and two dimension in  $r$  and  $\theta$  directions.
3. The physical properties (density and viscosity) are constant.
4. The surface tension is supposed to be large enough to keep the drobbles spherical in shape against any deforming effect of viscous forces. The unvaporized liquid and the vapor inside the drobbles are symmetrical about the vertical axis.
5. The growing drobble is assumed to be rising in a vertical path with an instantaneous relative velocity  $U_r$ . Alternatively, it can be imagined as being at rest, and the continuous medium moving against it with the same velocity.
6. The flow is irrotational and inviscid outside the reference drobble without internal circulation inside it.

The velocity of the flow can be written as the gradient of some scalar function (stream lines function); i.e.

$$v = \nabla \varphi \quad (7)$$

Where  $\varphi$  is conventionally termed a “velocity potential”[15]. According to the above assumptions for incompressible potential flow it can be shown that  $\varphi$  satisfies the Laplace equation (from continuity equation):-

$$\nabla^2 \varphi = 0 \quad (8)$$

Where

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right)$$

Equation (8) can be solved for expanding and translating boundary motion separately and superposition of two solutions gives the solution for the simultaneous motions[5]

$$\varphi = \varphi_1 + \varphi_2 \quad (9)$$

Where ( $\varphi_1, \varphi_2$ ) represent the expanding and translating solution terms.

The conditions for the expanding boundary are:-

$$v_\theta = -\frac{1}{r} \frac{\partial \varphi_1}{\partial \theta} = 0 \text{ at } 0 \leq \theta \leq 2\pi \text{ and } R \leq r \leq R_{cell} \quad (10 a)$$

$$v_r = -\frac{\partial \varphi_1}{\partial r} = \frac{dR}{dt} \text{ at } 0 \leq \theta \leq 2\pi \text{ and } r = R \quad (10 b)$$

And For a translating spherical boundary, the conditions are:

$$v_\theta = -\frac{1}{r} \frac{\partial \varphi_2}{\partial \theta} = U_r \sin \theta \text{ at } 0 \leq \theta \leq 2\pi \text{ and } r = R_{cell} \quad (11 a)$$

$$v_r = -\frac{\partial \varphi_2}{\partial r} = -U_r \cos \theta \text{ at } 0 \leq \theta \leq 2\pi \text{ and } r = R_{cell} \quad (11 b)$$

$$v_r \rightarrow 0 \text{ at } 0 \leq \theta \leq 2\pi \text{ and } r = R \quad (11 c)$$

Using these conditions, the solution for an expanding spherical and the translation motion boundary can be obtained as:

$$\varphi_1(r, t) = \frac{R^2 dR}{r dt} + C \quad (12)$$

$$\varphi_2 = \left( \frac{R_{cell}^3}{(R_{cell}^3 - R^3)} r + \frac{R^3 R_{cell}^3}{2(R_{cell}^3 - R^3)} \frac{1}{r^2} \right) U_r \cos \theta \quad (13)$$

where,  $C$ , arbitrary constant.

Finally, superposition of the expanding and the translating solution gives

$$\varphi(r, \theta, t) = \frac{1}{1 - (R/R_{cell})^3} \left( r + \frac{1}{2} \frac{R^3}{r^2} \right) U_r \cos \theta + \frac{R^2}{r} \frac{dR}{dt} + C \quad (14)$$

The radial and tangential velocity components in the liquid medium can be determined utilizing the above equation to give

$$v_r = -\frac{\partial \varphi}{\partial r} = -\frac{U_r}{(1 - \varepsilon)} \left( 1 - \frac{R^3}{r^3} \right) \cos \theta + \frac{R^2}{r^2} \frac{dR}{dt} \quad (15 a)$$

$$v_\theta = -\frac{1}{r} \frac{\partial \varphi}{\partial \theta} = \frac{U_r}{(1 - \varepsilon)} \left( 1 + \frac{R^3}{2r^3} \right) \sin \theta \quad (15 b)$$

where

$$\varepsilon = \left( \frac{R}{R_{cell}} \right)^3$$

As  $\varepsilon \rightarrow 0$  the above equations reduces to that obtained by sideman and Taitel[3] and other researchers for the velocity components around a single drobble. From the above equation it becomes clear; that the components of the flow velocity around the drobble are affected as a result of the drobble interacting with other drobble around it in the flow field. The means of interaction are coming through the volume fraction (holdup) parameter.

## 2.2 Equation of Motion of the Growing Drobble:

The equation of motion of the growing drobble can be written as follows

$$\frac{d}{dt} [(m_{00} + \bar{m})U] = F_B - F_D \quad (16)$$

For constant mass drobble

$$m_{00} = \frac{4}{3} \pi R^3 \rho_{av} \quad (17 a)$$

where

$$\rho_{av} = \rho_{dl} \left( \frac{R_0}{R} \right)^3 \quad (17 b)$$

For bubbles, the added mass[16]  $\bar{m}$  can be writing as

$$\bar{m} = \frac{1}{2} \frac{4}{3} \pi R^3 \rho_c \quad (17 c)$$

$$F_B = \frac{4}{3} \pi R^3 g (\rho_c - \rho_{av}) \quad (17 d)$$

The drag force over the surface of the growing drobble is given[17] as follows:

$$F_D = \frac{1}{2} \pi R^2 \rho_c U^2 C_d \quad (17 e)$$

Substituting equation (17 a, b, c, d and e) into equation (16), performing the differentiation and rearranging yield the following equation

$$\frac{dU}{dt} = \frac{1}{\frac{4}{3} \pi R^3 \left( \rho_{av} + \frac{\rho_c}{2} \right)} \left[ \frac{4}{3} \pi R^3 g (\rho_c - \rho_{av}) - \frac{\pi \rho_c R^2 U^2}{2} C_d - 2\pi \rho_c R^2 U \frac{dR}{dt} \right] \quad (18)$$

Where  $C_d$  in equation (18) is the drag coefficient of a drobble rising in an immiscible liquid which calculated using the correlation obtained by Haberman and Morton[18] as

$$C_d = 2.6 \quad \text{for} \quad 10^3 \leq Re \leq 10^4 \quad (19 a)$$

While for low Reynolds number the drag coefficient was taken as[19]

$$C_d = \frac{14.9}{Re^{0.78}} \quad \text{for} \quad 1 \leq Re < 10^3 \quad (19 b)$$

the relative velocity between the drobbles and continuous phase can be represented by a swarm model, which was obtained by Marrucci[20]

$$U_r = U \frac{(1 - \varepsilon)^2}{1 - \varepsilon^{5/3}} \quad (20)$$

where  $U$  is the instantaneous single drobble velocity and  $\varepsilon$  dispersed phase holdup.

### 2.3 Equation of Conservation of Mass:

The total volume of drobble  $V$  can be written as

$$V = V_l + V_v \quad (21)$$

The mass balance for a constant drobble mass is

$$\rho_{dl}V_0 = \rho_{dl}V_l + \rho_{dv}V_v \quad (22)$$

From this equation we can obtain relation for liquid volume  $V_l$  in drobble as

$$V_l = V_0(1 - \zeta)$$

Substituting this relation into equation (21) and divided the result by  $V_0$  yields

$$\frac{V}{V_0} = \left(\frac{R}{R_0}\right)^3 = 1 + \zeta \left(\frac{\rho_{dl}}{\rho_{dv}} - 1\right) \quad (23)$$

Where  $\zeta$  (the weight vaporization ratio) is defined as

$$\zeta = \frac{\rho_{dv} V_v}{\rho_{dl} V_0} \quad (24)$$

by simplifying this equation, the relation for vapor volume in drobble  $V_v$  can be obtain as

$$V_v = \frac{\rho_{dl}}{(\rho_{dl} - \rho_{dv})} (V - V_0) = \frac{\rho_{dl}}{(\rho_{dl} - \rho_{dv})} \frac{4\pi}{3} (R^3 - R_0^3) \quad (25)$$

Equation (22) can be written with the geometric relation using the model shown in figure 3 as

$$\frac{4}{3}\pi R_0^3 \rho_{dl} = \frac{\pi}{3} (3RH^2 - H^3) \rho_{dl} + \left[ \frac{4}{3}\pi R^3 - \frac{\pi}{3} (3RH^2 - H^3) \right] \rho_{dv} \quad (26)$$

This equation can be re arranged as

$$H^2(3R - H) = \frac{4 \left( \frac{\rho_{dl}}{\rho_{dv}} R_0^3 - R^3 \right)}{\frac{\rho_{dl}}{\rho_{dv}} - 1} \quad (27)$$

It can be rearranged in terms of the weight vaporization ratio ( $\zeta$ ) as

$$\zeta = \frac{\rho_{dv} \left[ \frac{4}{3}\pi R^3 - \frac{\pi}{3} (3RH^2 - H^3) \right]}{\rho_{dl} \left( \frac{4}{3}\pi R_0^3 \right)} \quad (28)$$

where

$$H = R(1 + \cos \beta) \quad (29)$$

Note that both  $\zeta$ ,  $H$  and  $\beta$  are time dependent. The combination of equations (23), (27) and (29) gives

$$3\cos\beta - \cos^3\beta + 2 = \frac{4(1 - \zeta)}{1 + \zeta \left( \frac{\rho_{dl}}{\rho_{dv}} - 1 \right)} \quad (30)$$

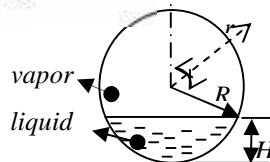


Figure 3 Theoretical model of an evaporating drobble.

### 2.4 Energy Equation:

The heat transfer from the continues phase to dispersed phase is described by the time dependent energy equation which includes conduction and convection. Unlike most of the previous studies by other authors[3, 21,22] etc., the transient term(in the energy equation) is included in this

analysis furthermore the opening angle ( $2\beta$ ) and the non-dimensional bubble growth rates ( $\frac{R}{\alpha} \frac{dR}{dt}$ ) are taken to be changed instantaneously during evaporation process. The additional following assumptions are used to simplify the mathematical model of the problem:

1. The drobble is assumed to be a sphere in which the vapor phase is bounded at the upper part by the opening angle ( $2\beta$ ) and the liquid phase at the lower part is bounded by the angle  $2(\pi - \beta)$ .
2. Heat transfer is assumed to take place at the liquid-liquid interface and the thermal resistance of the dispersed liquid phase is neglected.
3. The interface between the continuous phase and the unvaporized liquid is maintained uniformly at a certain temperature ( $T_{sat}$ ), and uniform temperature of continuous liquid  $T_c$  at infinity (undisturbed water temperature).
4. No heat transfer is assumed at the boundary between cells.
5. The radiation inside and outside drobble is neglected.

The governing equation for the temperature field is as following;

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} = \alpha \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) \right] \quad (31)$$

According to the assumptions the boundary conditions are:

$$\frac{\partial T}{\partial \theta} = 0; \quad \text{at } R \leq r \leq R_{cell} \quad \text{and} \quad \theta = 0, \pi \quad (32 a)$$

$$T = T_c; \quad \text{at } R \leq r \leq R_{cell} \quad \text{and} \quad 0 \leq \theta \leq \beta \quad (32 b)$$

$$T = T_{ds}; \quad \text{at } r = R \quad \text{and} \quad \beta \leq \theta \leq \pi \quad (32 c)$$

$$\frac{\partial T}{\partial r} = 0; \quad \text{at } r = R_{cell} \quad \text{and} \quad 0 \leq \theta \leq \pi \quad (32 d)$$

And the initial conditions are:-

$$\text{at } t = 0; \quad U = U_0; \quad R = R_0; \quad \frac{dR}{dt} = 0 \quad (33 a)$$

$$\begin{aligned} T &= T_c & \text{when } r > R_0 \\ T &= T_{ds} & \text{when } r = R_0 \quad \text{and} \quad 0 \leq \theta \leq \pi \end{aligned} \quad (33 b)$$

where  $R_0$  denotes the initial droplet radius at  $t=0$ . This condition can be used to obtain a steady temperature profile around a drobble (or drop) at this instant. It is obtained as the solution for steady heat transfer to a bubble of constant radius  $R_0$  and used as initial condition in our model.

The evaporation is completed when

$$R_f = R_0 \left( \frac{\rho_{dl}}{\rho_{dv}} \right)^{\frac{1}{3}} \quad (34)$$

## 2.5 The Growth Rate of Growing Drobble:

The growth of a drobble is predicted based on the energy balance on the drobble as

$$\frac{d}{dt} (h_{fgd} \rho_{dv} V_v) = 4\pi R^2 h \Delta T \quad (35)$$

The average outside heat transfer coefficient  $h$  related to the total surface area of drobble is calculated as [21]

$$h = \frac{1}{4\pi R^2} \int_{\beta}^{\pi} \frac{q_\theta}{\Delta T} 2\pi R^2 \sin \theta \, d\theta \quad (36)$$

where the local heat flux  $q_\theta$  through the continuous phase is given by,

$$q_\theta = k \left. \frac{\partial T}{\partial r} \right|_{r=R} \quad (37)$$

Differentiations equation (35) after substitute equation (25), and after rearrangement the resulted equation becomes

$$\frac{dR}{dt} = \frac{\alpha}{2R} \left( 1 - \frac{\rho_{dv}}{\rho_{dl}} \right) \left( \rho_c C_{pc} \frac{\Delta T}{\rho_{dv} h_{fgd}} \right) \left( \frac{2Rh}{k} \right) \quad (38)$$



## 2.6 Method of Solution:

For the purpose of dimensionless formulation, we use the following transformations of the variables[6,23]

$$\begin{aligned}
 U^+ &= \frac{U}{U_0}; \quad B = \frac{R}{R_0}; \quad \tau = \frac{\alpha t}{R_0^2}; \quad Re = \frac{2\rho_c UR}{\mu_c}; \quad Re_0 = \frac{2\rho_c U_0 R_0}{\mu_c} \\
 Pe &= \frac{2UR}{\alpha}; \quad Pe^* = \frac{2R_0 U}{\alpha}; \quad Pe_0 = \frac{2U_0 R_0}{\alpha} \\
 Fr_0 &= \frac{U_0^2}{2R_0 g}; \quad Nu = \frac{2hR}{k}; \quad St = \frac{C_{pc} \Delta T}{h_{fgd}} \\
 DRC &= \frac{\rho_{dl}}{\rho_c}; \quad DRD = \frac{\rho_{dv}}{\rho_{dl}}; \quad Ja = \rho_c C_{pc} \frac{\Delta T}{\rho_{dv} h_{fgd}} = \frac{St}{(DRD \cdot DRC)} \\
 T &= T_c - T_{ds}; \quad G = \frac{\rho_{av}}{\rho_c} = \frac{DRC}{B^3}; \quad E = \frac{(1-\varepsilon)}{(1-\varepsilon^{5/3})} \\
 &= \frac{T - T_c}{T_{ds} - T_c}; \quad y = \frac{r - R}{R}; \quad \mu = -\cos \theta
 \end{aligned} \tag{39}$$

The transformation of  $(r, \theta)$  coordinates to  $(y, \mu)$  coordinates is used in the energy equation in order to transform the geometry into a suitably spaced rectangular grid so that the finite-difference method can be easily applied [24].

Equations (18), (23), (30), (31), (38) and the boundary and initial conditions; equations (32), (33), and the condition for complete evaporation equation (34) can be non-dimensionalized using the above transformations, the resulting dimensionless equations are:-

$$\frac{dU^+}{d\tau} = \frac{Pe_0}{4Fr_0} \frac{1}{(G + 0.5)} \left[ (1 - G) - \left( \frac{1}{B} \right) \left[ 0.75C_d Fr_0 U^{+2} + 6U^+ \frac{Fr_0 dB}{Pe_0 d\tau} \right] \right] \tag{40}$$

$$B^3 = 1 + \zeta \left( \frac{1}{DRD} - 1 \right) \tag{41}$$

$$3\cos\beta - \cos\beta^3 + 2 = \frac{4(1 - \zeta)}{1 + \zeta \left( \frac{1}{DRD} - 1 \right)} \tag{42}$$

$$\begin{aligned}
 \frac{\partial \Phi}{\partial \tau} + \left[ \frac{Pe^* E}{2} \left( 1 - \frac{1}{(1+y)^3} \right) \mu + \frac{1}{(1+y)^2} \frac{dB}{d\tau} - \frac{2}{(1+y)B} \right] \frac{1}{B} \frac{\partial \Phi}{\partial y} \\
 + \left[ \frac{Pe^* E}{2} \left( 1 + \frac{1}{2(1+y)^3} \right) \frac{(1-\mu^2)}{(1+y)} + \frac{2\mu}{(1+y)^2 B} \right] \frac{1}{B} \frac{\partial \Phi}{\partial \mu} - \frac{1}{B^2} \frac{\partial^2 \Phi}{\partial y^2} \\
 - \frac{(1-\mu^2)}{B^2 (1+y)^2} \frac{\partial^2 \Phi}{\partial \mu^2} = 0
 \end{aligned} \tag{43}$$

$$\frac{dB}{d\tau} = \frac{(1 - DRD)}{2B} Ja Nu \tag{44}$$

Where,

$$Nu = \frac{kD}{k} = - \int_{-\cos\beta}^{+1} \frac{\partial \Phi}{\partial y} \Big|_{y=0} d\mu \tag{45}$$

The initial and boundary conditions become:

$$\tau = 0; \quad U^+ = 1; \quad B = 1; \quad \frac{dB}{d\tau} = 0 \tag{46 a}$$

$$\Phi = 0 \quad \text{when} \quad y > 0 \quad \text{and} \quad -1 \leq \mu \leq +1 \tag{46 b}$$

$$\frac{\partial \Phi}{\partial \mu} = 0; \quad \text{at} \quad y_{cell} \geq y \geq 0 \quad \text{and} \quad \mu = -1, 1 \tag{47 a}$$

$$\Phi = 0; \quad \text{at} \quad y_{cell}; \quad y \geq 0 \quad \text{and} \quad -1 \leq \mu \leq -\cos\beta \tag{47 b}$$

$$= 1; \quad \text{at} \quad y = 0 \quad \text{and} \quad -\cos\beta \leq \mu \leq +1 \tag{47 c}$$

$$\frac{\partial \Phi}{\partial y} = 0; \quad \text{at } y = y_{cell} \text{ and } -1 \leq \mu \leq +1 \quad (47 d)$$

The complete evaporation condition becomes

$$B = DRD^{-1/3} \quad (48)$$

The equations (40, 41, 42, and 44) are solved simultaneously by using the variable step Runge-Kutta method for the quantities  $U, \zeta, j$  and  $R$ . The heat transfer coefficient, employed in this solution can be obtained by solving equation (43) numerically using finite difference scheme and utilizing equation (45).

Equation (43) represents the time dependent energy equation written in  $(y, \mu)$  coordinates with heat conduction and convection in the  $y$  and  $\mu$  directions. The geometry of the drobble is changing continuously as it grows during the evaporation process, and the solution points of the problem in each time step is depending on the solution points of the previous step, therefore, uniform discretization step of  $(y$  and  $\mu)$ , but different number of node in  $y$  direction, is used for all the time domain in order to handle this problem. Applying the central difference approximation for diffusion terms and the upwind differencing approximation for convective terms, equation (43) can obtain as

$$\begin{aligned} \Phi_{j,k}^{i+1} = \Phi_{j,k}^i - \tau \left[ a_1^+ \left( \frac{\Phi_{j,k}^i - \Phi_{j-1,k}^i}{\Delta y} \right) + a_1^- \left( \frac{\Phi_{j+1,k}^i - \Phi_{j,k}^i}{\Delta y} \right) + a_2^+ \left( \frac{\Phi_{j,k}^i - \Phi_{j,k-1}^i}{\mu} \right) \right. \\ \left. + a_2^- \left( \frac{\Phi_{j,k+1}^i - \Phi_{j,k}^i}{\Delta \mu} \right) + a_3 \left( \frac{\Phi_{j+1,k}^i + \Phi_{j-1,k}^i - 2\Phi_{j,k}^i}{\Delta y^2} \right) \right. \\ \left. + a_4 \left( \frac{\Phi_{j,k+1}^i + \Phi_{j,k-1}^i - 2\Phi_{j,k}^i}{\Delta \mu^2} \right) \right] \quad (49) \end{aligned}$$

The explicit equation (49) requires a four-point scheme for each point  $(j, k)$  in the  $y, \mu$  directions as shown in figure 5.

Where

$$a_1 = \frac{1}{B} \left[ \frac{Pe^* E}{2} \left( 1 - \frac{1}{(1+y)^3} \right) \mu + \frac{1}{(1+y)^2} \frac{dB}{d\tau} - \frac{2}{B(1+y)} \right]$$

$$a_2 = \frac{1}{B} \left[ \frac{Pe^* E}{2} \left( 1 + \frac{1}{2(1+y)^3} \right) \frac{(1-\mu^2)}{(1+y)} + \frac{2\mu}{(1+y)^2 B} \right]$$

$$a_3 = \frac{-1}{B^2}; \quad a_4 = \frac{-1}{B^2} \frac{(1-\mu^2)}{(1+y)^2}$$

$$a_1^+ = \max(a_1, 0); \quad a_1^- = \min(a_1, 0)$$

$$a_2^+ = \max(a_2, 0); \quad a_2^- = \min(a_2, 0)$$

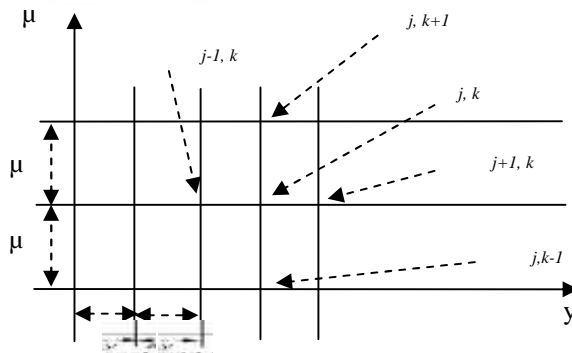


Figure 5 General grid layout for numerical solution for a time step.

The time step was variable and changed from the start of the computation to the final stage. The upwind scheme is stable if the following Courant-Friedrichs-Lewy condition (CFL) is satisfied [24], which is

$$|a \Delta \tau| \leq 1 \quad (50)$$

Hence

$$\Delta\tau = C_1 \frac{1}{a} \quad (51)$$

Where

$$a = \left( \frac{a_1^+}{\Delta y} - \frac{a_1^-}{\Delta y} + \frac{a_2^+}{\Delta \mu} - \frac{a_2^-}{\Delta \mu} - \frac{2a_3}{\Delta y^2} - \frac{2a_4}{\mu^2} \right)$$

The constant ( $C_1$ ) in equation (51) was determined by 'trial and error' of different values (0.01-1.2), after more trails and taking the best value in order to insure the optimum accepted convergences be less than or equal (1.0) when  $\beta$  varying from 0 to  $\pi$ .

The full details of the solution methods was described in [1].

The temperature gradient at the drobble surface is evaluated using the temperatures at the drobble surface and at the nearest node outside the surface with the aid of the second-order upwind scheme that is:

$$\left. \frac{\partial \Phi}{\partial y} \right|_{y=0} = \frac{1}{2} \frac{1}{y} [-3 \Phi_{j-1,k} + 4 \Phi_{j+1,k} - \Phi_{j+2,k}] \quad (52)$$

This equation can be applied in the Nusselt relation, equation (45), and by the integrating numerically; using trapezoidal rule one can obtained the instantaneous average value of Nusselt number which is necessary in the growth rate equation.

## 2.7 The Volumetric Heat Transfer Coefficient:

The volumetric heat-transfer coefficient and other important characteristic quantities, such as the height for complete evaporation, were calculated previously by applying single drops empirical correlations, which are modified and extended for the case of an evaporating multidrobbles. There is a growing demand for an analytical model for situations when empirical correlations do not exist. The purpose of this part is to develop such a model.

In addition to the assumptions mentioned in the previous sections, the following assumptions are employed to simplify the actual physical phenomena:

1. The fundamental assumption in the present model is that multi-droplets direct-contact evaporation can be described using specific heat-transfer rate of individual droplets inside cell of continuous liquid, derived in previous section.
2. The dispersed phase injected in the form of saturated liquid drops of a uniform size, is resulting in an immediate start of evaporation.
3. No significant axial temperature gradient along the column and the temperature of the continuous liquid is spatially uniform and quasi-steady with respect to the evaporation time of drobbles.
4. The fragmentation and coalescence do not occur thus, the number density of drobbles remain constant as the two-phase drobble rising and their diameters increase.

The average volumetric heat-transfer coefficient can be written as[7] (see figure 6)

$$h_{mv}(z) = \frac{1}{z} \int_{z_0}^z A_d(z) N_d h_d(z) dz \quad (53)$$

It can be expressed in terms of  $D$  as

$$h_{mv}(D) = \frac{1}{z} \int_{D_0}^D A_d(D) N_d h_d(D) \frac{dz}{dD} dD \quad (54)$$

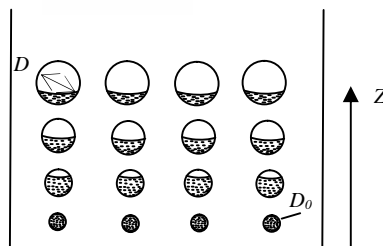


Figure 6 Illustration of drobbles size dependence on height.

Where,  $Z$ , is the axial coordinate,  $A_d$ , the drobble surface area,  $N_d$ , the droplet number density,  $h_d$  is the droplet heat-transfer coefficient (denoted  $h$  in previous section), and  $D$  is the instantaneous equivalent spherical diameter.

The number density of drobbles is assumed to be constant during the evaporating process [10] that is ( $N_d = N_{d0}$ ) as:

$$N_{d0} = \frac{U_d}{(1/6) \pi D_0^3 U_0} \quad (55)$$

where

$$U_d = \frac{\dot{V}_d}{A} \quad (56)$$

Where  $\dot{V}_d$  denote the volumetric flow rate of the dispersed phase;  $U_0$  is the initial velocity of the dispersed phase (drobbles) and  $A$  is the cross section area of the column. Because the mass of the dispersed phase entering the distributor is equal to that leaving the distributor,  $U_0$  can be calculated by using

$$U_0 = \frac{4 \dot{V}_d}{\pi N_h d_h^2} \quad (57)$$

Substituting equations (56) and (57) in to equation (55) yield

$$N_d = \frac{6 N_h d_h^2}{4 D_0^3 A} \quad (58)$$

During the evaporation process, the subsequent calculations is to define a dispersed phase volume fraction (holdup) by[7]:

$$\varepsilon(D) = \frac{\pi}{6} D^3 N_d \quad (59)$$

The net increment of mass of vapor per unit time in the differential element (figure 7)

$$d\dot{m} = \dot{m}_{out} - \dot{m}_{in} = \rho_{dv} A N_d d(U_r V_v) \quad (60)$$

where  $U_r, V_v$  the relative velocity between the drobbles and continuous phase, and volume of dispersed vapor phase of drobble, respectively.

Substituting the  $V_v$  equation (25) into equation (60) yields

$$d\dot{m} = \frac{\pi}{6} C_{m1} A N_d d(U_r (D^3 - D_0^3)) \quad (61)$$

where

$$C_{m1} = \frac{\rho_{dl} \rho_{dv}}{(\rho_{dl} - \rho_{dv})}$$

So the rate of heat transfer from the continuous phase to the dispersed phase is given by

$$dQ = d\dot{m} h_{fg} = \frac{\pi}{6} C_{m1} h_{fg} d A N_d d(U_r (D^3 - D_0^3)) \quad (62)$$

In addition, according to the Newton law of cooling, the rate of heat transfer,  $dQ_c$  can be written as

$$dQ_c = h_d \pi D^2 N_d \Delta T A dz \quad (63)$$

Equating equation (63) to equation (62), the following relation can be obtained;

$$\frac{d(U_r (D^3 - D_0^3))}{dz} = \frac{6 h_d D^2 \Delta T}{h_{fg} C_{m1}} \quad (64)$$

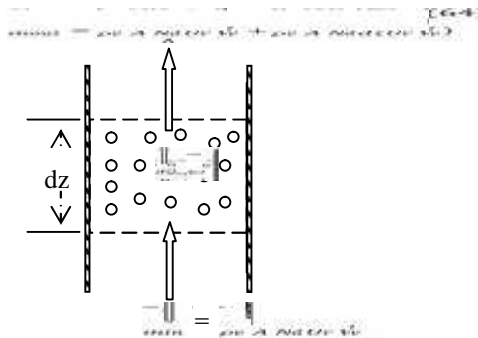


Figure 7 Mass balances in a differential element.

Since  $Z$  and  $U_r$  are time dependent then the left side of equation (64) becomes

$$\frac{d(U_r (D^3 - D_0^3))}{dz} = \frac{1}{dz/dt} \left( 3D^2 U_r \frac{dD}{dt} + (D^3 - D_0^3) \frac{dU_r}{dt} \right) \quad (65)$$

Combining equation (64) with equation (65), the following equation can be obtained:

$$dz = \left\{ \frac{C_{m1} h_{fg}}{6 h_d D^2 \Delta T} \left( 3D^2 U_r \frac{dD}{dt} + (D^3 - D_0^3) \frac{dU_r}{dt} \right) \right\} dt \quad (66)$$

Integrating equation (66) between  $Z=0$ , at  $t=0$  and  $Z=Z$ , at  $t=t$  gives

$$Z(t) = \frac{C_{m1} h_{fg}}{6 \Delta T} \int_0^t \frac{1}{h_d D^2} \left( 3D^2 U_r \frac{dD}{dt} + (D^3 - D_0^3) \frac{dU_r}{dt} \right) dt \quad (67)$$

Where the derivatives  $\left( \frac{dD}{dt}, \frac{dU_r}{dt} \right)$  is calculated numerically using three point formula method.

The average volumetric heat transfer coefficient from the location of drobble formation to the height  $Z$  can be obtained by substitution of equations (55),(66) and (67) into equation (53), followed by a rearrangement, the average volumetric heat transfer coefficient is obtained as:

$$h_{mv}(t) = \frac{\int_0^t \left( \pi N_d \left( 3D^2 U_r \frac{dD}{dt} + (D^3 - D_0^3) \frac{dU_r}{dt} \right) \right) dt}{\int_0^t \left( \frac{1}{h_d(t) D^2} \left( 3D^2 U_r \frac{dD}{dt} + (D^3 - D_0^3) \frac{dU_r}{dt} \right) \right) dt} \quad (68)$$

By solving equation (68) numerically (using trapezoidal rule for the integration) the average volumetric heat transfer coefficient is obtained for each time; and hence for each  $Z$

### 3 Results and Discussion

In the present discussion considered the multi-drobbles evaporation process, where the interactions between drobbles is a significant parameter. The average volumetric heat transfer coefficient, and the total time and height for complete evaporation are evaluated. Since the evaluation of these characteristics are essential for designing and constructing direct contact evaporation equipments. The n-pentane/distilled water system is chosen for calculations in this study since this pair of fluid system is satisfying the assumption and most common configuration of drobbles. The physical properties of fluids forming the drobbles and the continuous medium were obtained with physical properties (using EES (Engineering Equation Solver) software program[25]) relevant to n-pentane drobbles evaporating in a water under the appropriate temperatures and the atmospheric pressure. A comparison between the present theoretical work and the published theoretical and experimental works of other researchers are also performed.

Before starting in obtaining the results , the accuracy of the numerical scheme that used in the solution is verified by comparing the results of the present work with the analytical solutions of the steady state evaporation by Sideman and Taitel [3]. The two physical systems become similar when the unsteady term in the energy equation (43) of the present analysis is neglected.

Figure 8 shows the comparison of the value of  $Nu/Pe^{0.5}$  , the good agreement obtained is an indication of the accuracy of the present numerical scheme.

Based on these results this numerical scheme can be used to investigate the evaporation process of growing drobbles.

The calculations of solving the mathematical model of multi-drobbles evaporation are carried out by numerical method [1]. In order to present a wide scope for the multi-drobbles evaporation, the influences of more parameters, likely to be affective on this process, are studied. These parameters are the initial diameter of drobble, the degree of superheat, the column diameter, and the holes diameters. The values of these parameters were chosen according to the more common cases of the multi-drobble evaporation process which exist in the literature.

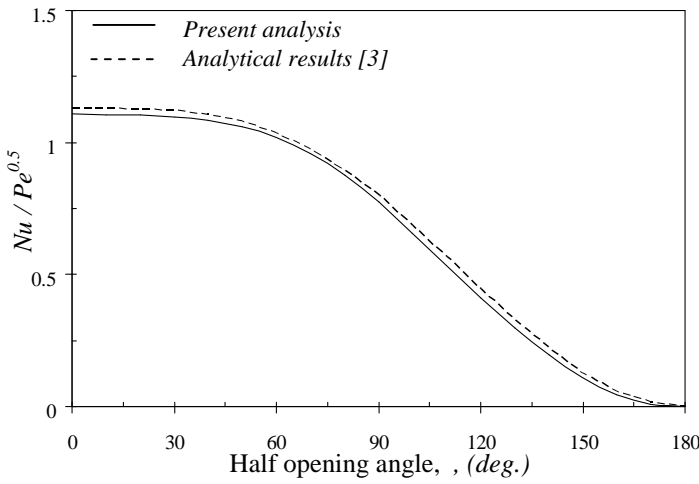


Figure 8 Comparison between the present predictions and other analytical results.

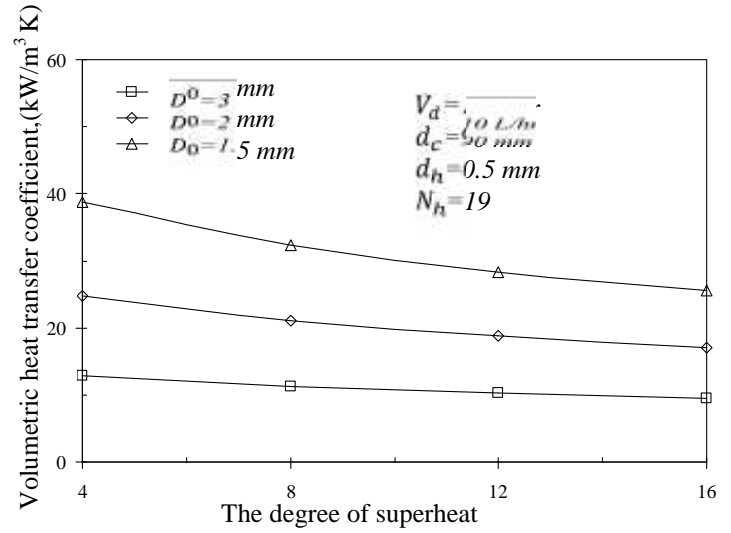


Figure 9 Effect of  $D_0$  on the average volumetric heat transfer coefficient for complete evaporation.

### 3.1 The Average Volumetric Heat Transfer Coefficient:

The variations of the average volumetric heat transfer coefficient for complete evaporation  $h_{mvf}$ , with the the degree of superheat ( $T$ ), are shown in figures 9 to 11. The general trend in these figures is the reduction of the average volumetric heat transfer coefficient for complete evaporation ( $h_{mvf}$ ) with  $\Delta T$ . The effect of initial drobble diameter on  $h_{mvf}$  is shown in figure 9. From this figure it can be seen that  $h_{mvf}$  is increasing with decreasing of  $D_0$  for a fixed value of the degree of superheat  $T$ . The effect the column diameter on the variation of  $h_{mvf}$  with  $T$  is shown in figure 10. As the value of  $d_c$  increases, decrease of the volumetric heat transfer coefficient for a fixed  $T$  is obtained. The reason may be due to, increasing  $d_c$  leads to decreasing the number density of drobbles that in turn reduce the rate of heat absorbed from continuous phase to evaporate the drobbles. The variation of  $h_{mvf}$  with  $\Delta T$  for different values of  $d_h$  is shown in figure 11. Increasing the value of  $d_h$  leads to increase in  $h_{mvf}$ , at a fixed value of  $T$ .

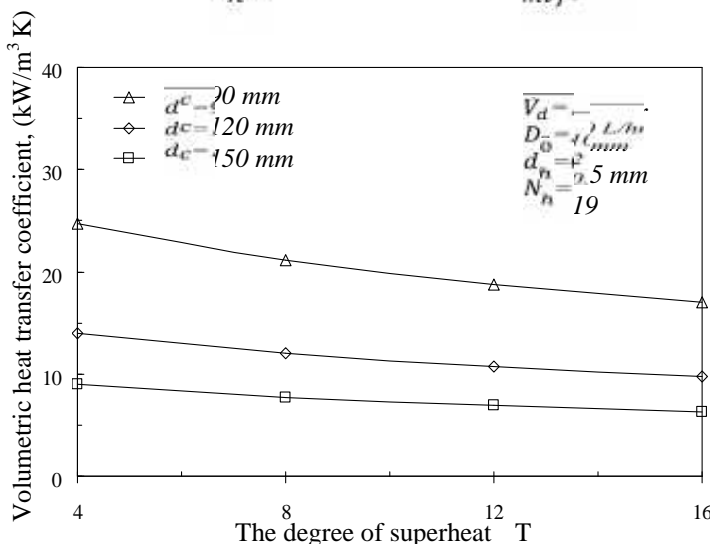


Figure 10 Effect of  $d_c$  on the average volumetric heat transfer coefficient for complete evaporation.

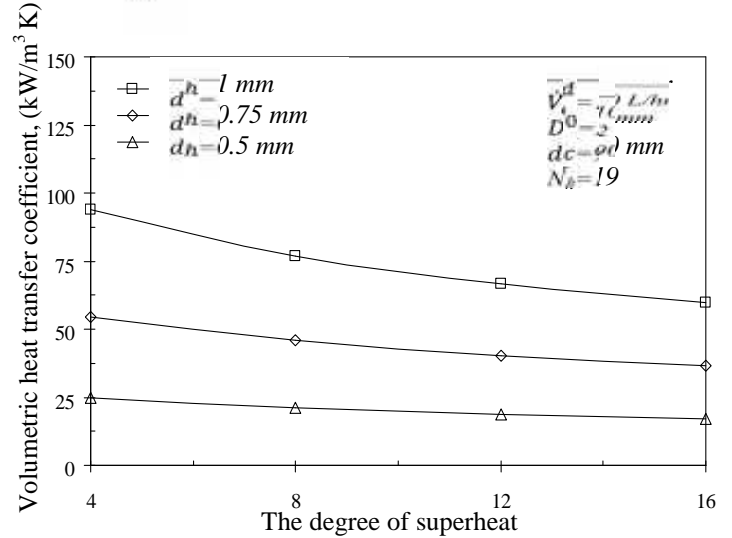


Figure 11 Effect of  $d_h$  on the average volumetric heat transfer coefficient for complete evaporation.

### 3.2 The Total Time and Height for Complete the Evaporation:

Variation of the time ( $t_f$ ) and height ( $Z_f$ ) for complete evaporation with  $T$  is shown in figures (12 to 14) and figures (15 to 17), respectively. The general trend in figures 12 to 14, is reduction of  $t_f$  with increasing  $\Delta T$ . The general trend in figures 15 to 17, is reduction of  $Z_f$  with increasing  $\Delta T$ . The reduction in  $t_f$  and  $Z_f$  with  $T$  may be due to, the increasing in the degree of superheat  $T$  lead to rapid evaporation process of dispersed phase and then the time for complete evaporation is reduced, in turn, reduced the height required for complete evaporation of all

drobbles. The effect of the initial diameter of drobble ( $D_0$ ) is shown in figure 12 and 15, respectively. Increasing the value of  $D_0$  leads to increase in  $t_f$  and  $Z_f$ , at a fixed  $T$  value. Figure 13 and 16 shown the effect the column diameter, respectively. As it is obvious from figure 13, no significant effect of  $d_c$  on  $t_f$  is observed. The effect the holes diameter ( $d_h$ ) on the variation of  $t_f$  and  $Z_f$  with  $\Delta T$ , is shown in figures 14 and 17, respectively. From figure 14 it is noticed that, increasing the value of  $d_h$  lead to small increasing in  $t_f$ , and from figure 17 it is noticed that, the increasing in  $d_h$  lead to marked decrement in  $Z_f$ .

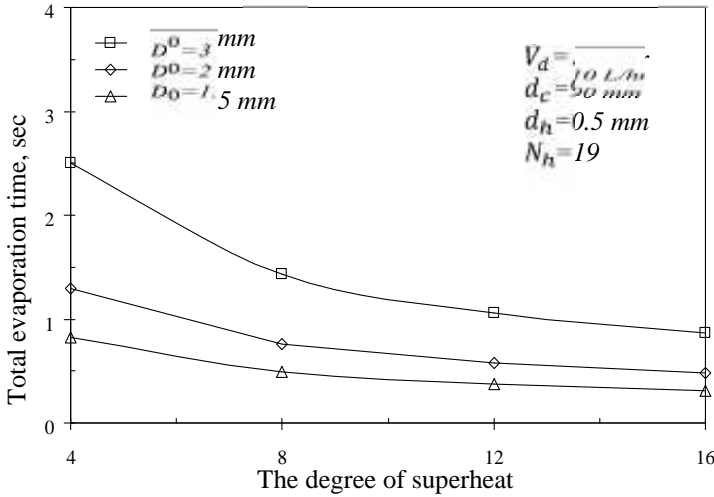


Figure 12 Effect of  $D_0$  on the total evaporation time.

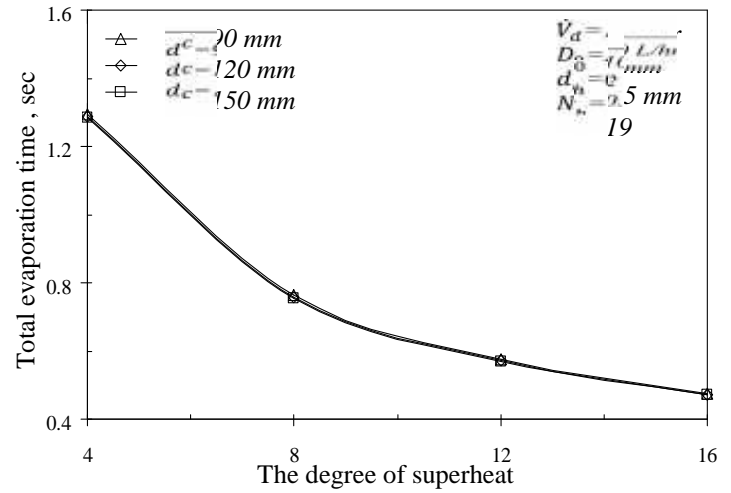


Figure 13 Effect of  $d_c$  on the total evaporation time.

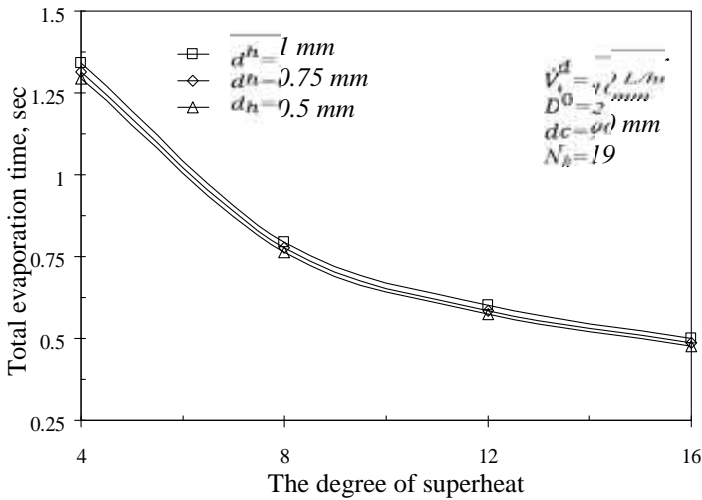


Figure 14 Effect of  $d_h$  on the total evaporation time.

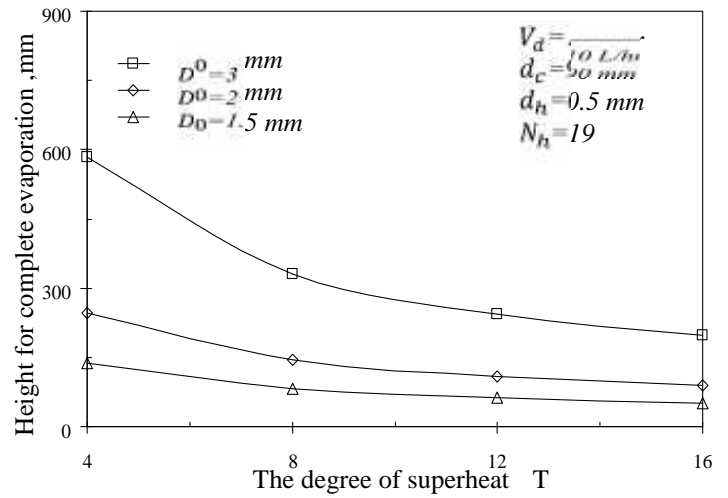


Figure 15 Effect of  $D_0$  on the height for complete evaporation.

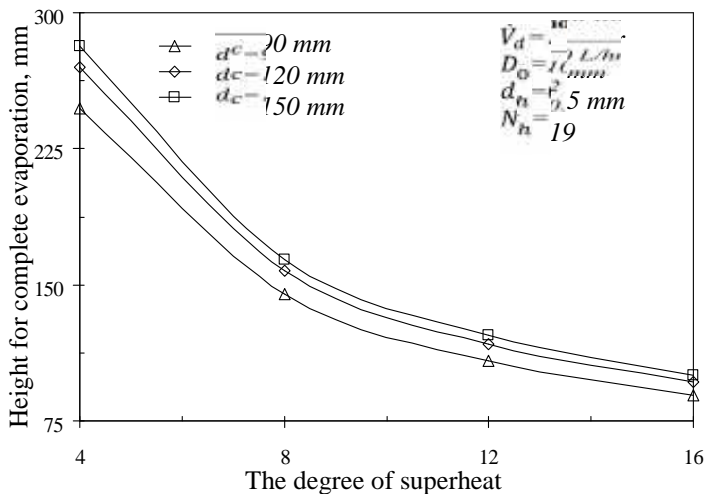


Figure 16 Effect of  $d_c$  on the height for complete evaporation.

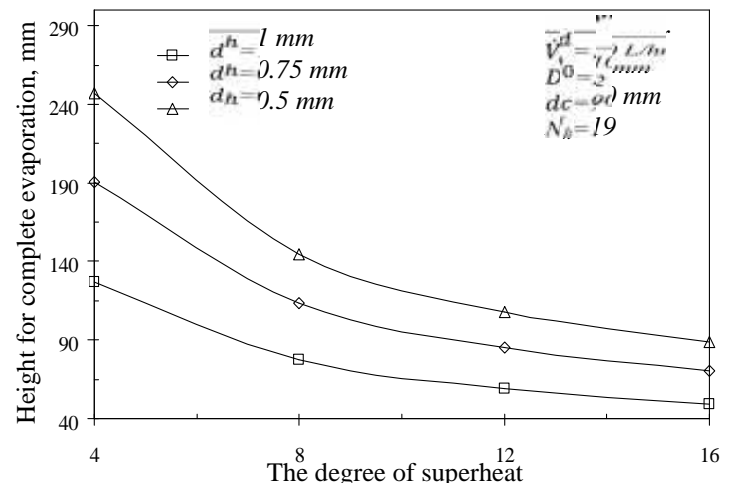
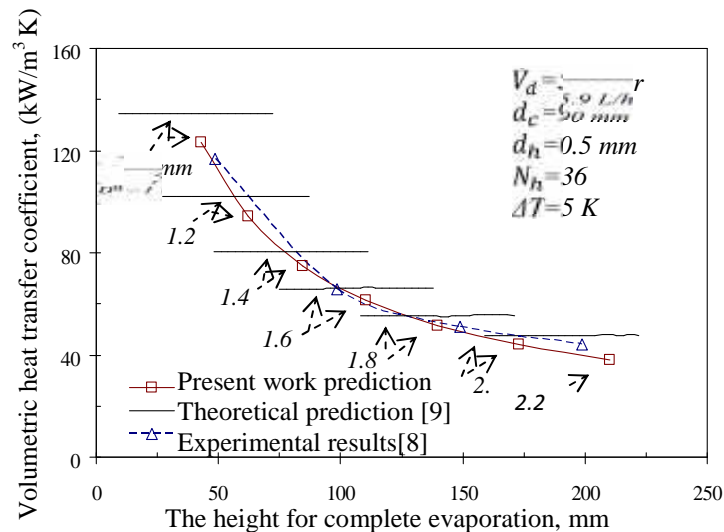


Figure 17 Effect of  $d_h$  on the height for complete evaporation.

### 3.4 Comparison the Present Results with the Other Experimental and Theoretical Results:

In order to prove the validity of the present model, the obtained results are compared with the experimental results of Seetheramu and Battya[8] and the theoretical results of Mori[9] in figure 18, the agreement with both results are good.



**Figure 18 Comparison between the present predictions and other experimental and theoretical results.**

### 4 Conclusions:

Direct contact evaporation of saturated droplets (n-pentane) in a column of an immiscible liquid (distilled water) is theoretically investigated. A theoretical model for direct contact evaporation of drops in an immiscible liquid using cellular model has been developed. The analysis indicated that the evaporation process is controlled by certain parameters, the effects of these parameter are:

- Increasing the degree of superheat leads to decrease the average volumetric heat transfer coefficient, decrease the time and height required for complete evaporation.
- Increasing the initial droplets diameter leads to decrease the average volumetric heat transfer coefficient, increase the time and height required for complete evaporation.
- Increasing the column diameter leads to decrease the average volumetric heat transfer coefficient, no considerable effect on the time required for complete evaporation, increase slightly, the height required for complete evaporation.
- Increasing the holes diameters leads to increase the average volumetric heat transfer coefficient, small effect on the time required for complete evaporation, decrease the height required for complete evaporation.

The present analysis can be extended to include another shape of drobble such as ellipsoid or other shapes of drobbles instead of spherical drobbles. Furthermore the direct contact condensation of multidrops can be investigated.

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# Numerical Study of Direct Contact Evaporation of Liquid Drops in an Immiscible Liquid

## Abstract

The present work is a theoretical investigation of the direct contact evaporation process of drops in an immiscible liquid. A theoretical model based on continuity, momentum and energy equations in addition to the energy balance equation is developed, by using cellular model. This model aims to obtain characteristics of the direct contact evaporation of a single droplet bounded by a spherical cell of continuous liquid and these characteristics are used in order to develop a theoretical model to obtain the characteristics of direct contact evaporation of multidroplets in an immiscible liquid. The developed equations were derived and then solved simultaneously applying a numerical method. The energy equations with the potential flow velocity, taken into account the effect of the interaction between the adjacent drops, have been numerically solved using finite-difference techniques. The calculations are performed for n-pentane drops evaporating in distilled water. The theoretical results presented in terms of the average volumetric heat transfer coefficient, and the total time and height for complete evaporation. The influence of several important parameters on the evaporation characteristics were presented. The results indicated that the main parameters affecting the evaporation process were the initial size of droplets and the degree of superheat. The results were compared with the existing theoretical and the experimental results; the agreement between the results was good.

## الخلاصة

تهدف الحالية دراسة نظرية لعملية التبخر بالتماس المباشر قطرات مائع في مائع آخر يتمزج معها. تم تطوير نموذج رياضي لتمثيل العملية باستخدام النموذج الخلوي (cellular model) اعتمد بناء هذا النموذج على معادلات الاستمرارية. يهدف . مميزات عملية التبخر المحيط بها، واستخدام هذه المميزات لتطوير نموذج رياضي بخلية كروية مميزات عن تبخر مجموعة من لقطرات بالتماس المباشر مع مائع اخر غير امتزاجي. تم اشتقاق المعادلات المطورة ومن ثم حلها سوية بطريقة عددية. أيضا تم حل معادلة الطاقة، المتضمنة سرعة الجريان الكامن والتي أخذ بنظر فيها التأثير بين لقطرات المتجاورة، حلاً عددياً باستخدام تقنيات الفروقات المحددة. أجريت الحسابات قطرات من الطبيعي . بينت لهذه الحالة العوامل الرئيسية المؤثرة على عملية التبخر هي القطر الابتدائي للقطرة عملية بين المائعين. نظرية عملية بين جيد.

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