Improvement of RC4 Security Algorithm

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Abstract

Data confidentiality is the most important security service at present, and to ensure access to this service, efficient encryption algorithms are used to overcome unauthorized access to data, as attacks often exploit weaknesses in these algorithms. Encryption algorithms are used to protect the data, aimed to have a fast rate of execution, low level of complexity, and high level of security. To overcome these challenges, symmetric encryption algorithms are often used. The Rivest Cipher 4 (RC4) is the most common encryption algorithm for meeting the conditions for effective encryption. However, this algorithm has been proven to be vulnerable to a variety of assaults. This is accomplished by exploiting flaws in the critical phases of the generation process, as none of these algorithms are adequately utilized in randomization. Therefore, this paper aimed to improve the RC4 algorithm by overcoming its weaknesses. The proposed method, called Improvement RC4 (IRC4), improves the RC4 key generation based on multiple chaos maps. In addition, IRC4 is stronger against most attacks. This makes the proposed algorithm more secure.

Keywords: Key Scheduling, Random Number, RC4, Security Stream Cipher

1. INTRODUCTION

Currently, data and information protection are critical in order to prevent data from being compromised by others; Attacks should be avoided as many individuals are not responsible for the misuse of technologies especially data theft and eavesdropping[1]. Thus, the encryption methods can be used to data protection. With goals of data integrity, confidentiality, non-repudiation, and authentication, the cryptography is used to protect data by converting it to another language that cannot be identified[1]-[2].

Traditional encryption algorithms are complex and energy-intensive [REF]. As a result, symmetric key ciphering should be used in security systems. One of the most widely used algorithms is the Rivest Cipher 4 (RC4) symmetric stream cipher. The RC4 stream cipher algorithm provides fast encryption and decryption, low resource usage, easy to understand and implement, and low time and space complexity compared to other algorithms [3]. It consists of two phases: KSA, and PRGA. Both phases generate a keystream which is then used to encrypt the data [4]. The size of the encryption file generated by the RC4 algorithm is equal to the size of the original text file and for this reason; it does not reduce the storage space and does not require a long time to implement the encryption process. Therefore, the simple design and non-random behavior between the key, the plaintext, and the ciphertext, as well as statistical failures are some of the weak points of the algorithm [5]. In this paper, we introduced the improving of the RC4 algorithm, which called IRC4. IRC4 will enhance the key that generated by the PRGA algorithm, taking into account the preservation of the original structure of the algorithm. The proposed method leads to an increase the randomness in the resulting key and then to enhance the security of the resulting encryption when compared to the original algorithm variables.

The rest content of this paper is as follows: the related work is presented in Section 2. The RC4 is presented in Section 3. The improvement RC4 Algorithm is explained in Section 4. Evaluation performance is explained in detail in Section 5.

2. The Related Work

RC4 algorithm is one of the most commonly used stream ciphers in a variety of security protocols. It used to generate pseudo-random code; it accepts a secret key as input and uses a deterministic method to generate a stream of random bits. For that, an intruder's aims, in attack the RC4, is to look for non-random behavior in the internal state or the output keystream [REF].

Several researchers tried to improve the safety of RC4 by proposing many types of improvements. Zoltak [6] proposed the (VMPC) which is designed to be effective in program applications To overcome the KSA weakness which Fluhrer and et al. were defined in [7], In comparison to RC4, the structure of the (PRGA) in VMPC was more complicated, increasing the algorithm strength against assaults. Mironov [8]Introduced a new RC4 model and studied it using the random change principle, As a result of this analysis, recommends eliminating the 512 bytes found at the beginning to prevent the weakness that has resulted in a longer execution period. Preneel and Paul [9] proposed an enhancement over RC4 called (RC4A). after discovering a new weakness of statistical in the RC4 keystream generator's first two output bytes They said that the number of outputs necessary to differentiate the output of an RC4 random sequence with bias is 128 and that 256 should be used to overcome this bias. RC4A is thought to be resistant to most of RC4's flaws, notably the distribution flaw in the first two output bytes. However, Maximov [10] developed a differentiating attack on both VMPC and RC4A after a year that can differentiate the cipher output and random values. Hamad and Mousa[11] investigated the impact of several RC4 algorithm parameters by analyzed, such as execution time and file size, and found that the file size and the length of the encryption key had an impact on encryption and decryption speed. Pateriya and Pardeep [12]proposed the (PC-RC4) method as an enhancement to the RC4 to improve the work of both PRGA and KSA algorithms in the randomness, yet there is increases the time of execution. Hammod and et al proposed the RRC4 method, which enhanced the RC4's randomness Furthermore, an RC4 with two state tables (RC4-2S) developed the key obstetrics time while also surpassing the randomness of the keys produced[13].

3. RC4 ALGORITHM

RC4 it is a widely accepted and popular stream cipher devised in 1987 by Ron Rivest. The RC4 algorithm is one of the fastest encryption algorithms used for encryption within a lightweight, robust cipher in terms of memory footprint, power consumption, the flexible main size and CPU and is utilized in email in many popular protocols, such as WEP and TLS /SSL[14]. The security of the algorithm is based on a pseudorandom key scheduling procedure with a configurable key length from 1 to 256 bytes (8 bits to 2048 bits). This is used by initializing the initial vector (S) is completely independent of the plaintext[15]. RC4 algorithm includes two stages called KSA and PRGA algorithms. In the RC4 algorithm two variables, i and j are used. The variable i is a pointer that is increased by 1 at each step, while for the variable j it is a pseudorandom pointer whose content is updated based on the key K and the state vector S.

3.1 KSA Algorithm:

in this part of the RC4 Algorithm, it takes the key stored in K as input and is 1 bytes long, and used K to rearrange the values in the vector S [16]. The KSA sets i and j to zero, and S to change the identity. It then steps i across S looping N times(N=256), and updating j by adding the i-th entries of S and K.Each iteration ends with a two-byte operation in the vector S, indicating the current values of the variables i and j[17], the KSA steps is depicted in Algorithm 1.

```
Algorithm1. Key Schedule Algorithm(KSA)[16]

1: Input: Secret key K

2: K: key length

3: Output: Internal state S

4: j \leftarrow 0

5: for i \leftarrow 0 to N - 1 Do

6: S[i] \leftarrow i

7: end for

8: for i \leftarrow 0 to N - 1 Do

9: j \leftarrow (j + S[i] + K [i \mod k]) \mod 256

10: Swap S [i] with S [j]

11: end for

12: Return (S)
```

3.2 PRGA Algorithm:

in this stage of the RC4 algorithm, setting both i and j to zero, and then does four actions in order: it increases i as a counter, adds S[i] to j, swap the two entries of S indicated by the present values of i and j, and outputs the value of S at index S[i] + S[j] as the value of z [18] the PRGA is shown in algorithm 2.

Algorithm2.Pseudo Random Generation Algorithm (PRGA)[16]

1: Input: Internal state S, generated by KSA 2: Output: keystream Z 3: $i \leftarrow 0$ 4: $j \leftarrow 0$ 5: for each new message byte Do 6: $i \leftarrow (i+1) \mod N$ 7: $j = (j + S[i]) \mod N$ 8: Swap S [i] with S [j] 9: Z = S (S [i] + S [j]) mod N 10: end for 11: Return (Z)

The XOR-ed operation will be performed between the n bit represented by the z value with the n bit of the original message and the output will be a ciphertext of length n bit, on the other hand, to restore the original text from the ciphertext the XOR-ed operation is used between the z value and the ciphertext and the text length is n bits.

4. IMPROVED THE RC4 ALGORITHM

The proposed improvement aims to provide a high level of randomness and complexity to bypass RC4 vulnerabilities by introducing improved RC4 key generation as shown in the diagram below. The original key and id number and the output use as a key in the RC4 algorithm for both encryption and decryption, the schematics below are shown in Figure1.(a) and Figure1.(b) respectively for the optimization process. Note that both algorithms. Note that both the above algorithms are 1 and 2, which include the encryption code and decryption code respectively. Note that both the above algorithms are 1 and 2, which include the KSA code and PRGA code respectively.

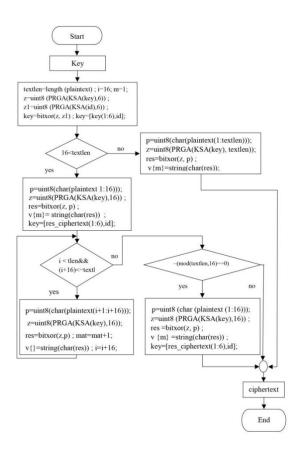


Figure 1.(a). IRC4 Encrypt Algorithm

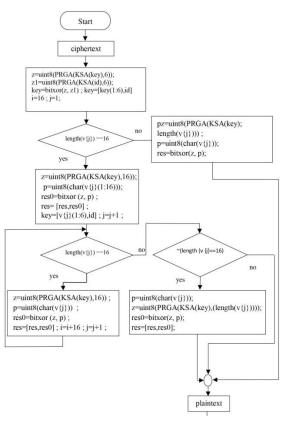


Figure1.(b). IRC4 Decrypt Algorithm

Evaluation Performance and Analyzation

The CRYPTX'98 mathematical analysis program[19] was used to examine the main streams. The frequency test, change point, binary derivative, sequence complexity and sub-block tests, and are all carried out. The suggested algorithm main streams examined own the characteristics of randomness bit streams according to CRYPTX 98. As for the other experiences of CRYPTX'98 aims to a collection of statistic characteristics that attackers can be used it, when it is used the same input plaintext and key directories, their effects compared to the encryption performance in the suggested development. If the algorithm creates a random stream, the p-values produced from a CRYPTX'98 test indicate the possibility of getting an outcome that differs from the test statistic. For the given metric, small p-values would allow non-randomness.

4.1 Frequency Test

A test for the bit stream is checked for an equal number of ones and zeros. In a long random sequence, the number of ones is roughly regularly distributed. That is, a sequence's number of ones and zeroes should be roughly equal. The frequency test estimates the sample stream's tail end probability for the number of ones[20]. The (Fig .5 and table 1) shows that the results of IRC4 the technique is superior to RC4.



Fig.5 (a), (b) RC4 and IRC4 FREQUENCY TEST RESPECTIVELY.

Table 1:	The Results of	Comparing of the	Frequency Test

	Test Parameter									
Algorit	Total	Amount of the	(mean) = foreseeab	ratio	1	is Satiaf				
hm	bits			-	value)	Satisf				
		ones =	le ones	ones		У				
		(x)								
RC		67	64.0	0.52	0.5959	yes				
	128			34		-				
IRC		65	64.0	0.50	0.8597	yes				
	128			78						

4.2 The Binary Derivative

The second test is the binary derivative that is used in the measure of randomness of a string of binary created by a pseudorandom numbering generator used in the system of the cipher [21]. The results of the IRC4 technique are superior to RC4 which shows in Figs. (6 and 7), and Table s (2, and 3).

4.2.1 This sample represents 1st Binary Derivative test (D1) result for RC4 and IRC4 algorithms.



(a)

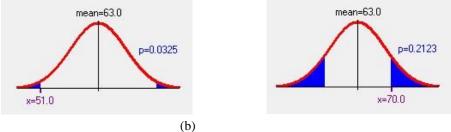
Fig.6 (a), (b) RC4 and IRC4 1st BINARY DERIVATIVE TEST RESPECTIVELY.

(b)

Table 2: The Results of Comparing of the First Binary Derivative Test (D1)

	Test Parameter								
Algori	Tot	Number	Amount	(mean) =	ratio of	(p-value)	Is		
thm	al	of bits	of the	foreseeable	ones		Satisfy		
	bits		ones =	ones					
			(x)						
RC		127	53	63.5	0.4173	0.0624	yes		
	128						2		
IRC		127	68	63.5	0.5354	0.4245	yes		
	128								

4.2.2 This sample represents 2nd Binary Derivative test (D2) result for RC4 and IRC4 algorithms.



(a)

Fig.7 (a), (b) RC4 and IRC4 2nd BINARY DERIVATIVE TEST RESPECTIVLY.

Table 3: The Results of Comparing of the Second Binary Derivative Test (D2).

	Test Parameter									
Algorit	Total	Number	Amount	(mean) =	ratio of	(p-	Is			
hm	bits	of bits	of the	foreseeab	ones	value)	Satisf			
			ones =	le ones			У			
			(x)							
RC		126	51	63	0.4048	0.108	yes			
	128					8	,			
IRC		126	70	63	0.5556	0.032	yes			
	128					5				

4.3 Change Point Test

This test looks for a significant change in the ratio of ones in the bit stream. In bit position in the bit sequence, the proportion of ones to that point is compared to the ratio of ones in the residual stream. The 'change point' is the area where the most change occurs. The test evaluates the significance of the 'change[22].' (Table 4 and Fig .8) shows that the results of IRC4 the technique is superior to RC4.

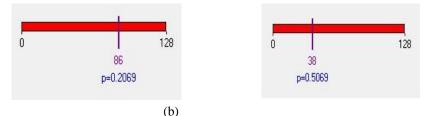


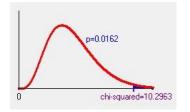
Fig.8 (a), (b) RC4 and IRC4 CHANGE POINT TEST RESPECTIVLY.

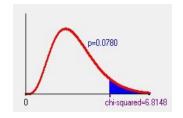
	Test Parameter										
Algorit hm	Total bits	Amount of the	The Change	Amount of the	Ratio of ones	Ratio of	(p- value)	is Satisf			
		ones = (x)	point	ones before	before	ones after	· · ·	У			
RC	128	86	18	11	0.6111	0.409 1	0.137 3	yes			
IRC	128	65	38	16	0.4211	0.544 4	0.506 9	yes			

4.4 Sub-block Test

(a)

Tests of non-overlapping homogeneity sub-blocks of a given length, For sub-block sizes up to 16, the 'uniformity test' requires a sample of at least 5 * b * 2(b) bits, where b is the size of the sub-block. For sizes of sub-block bigger than 16, the 'repetition test' is applied. This test requires a specimen of b * 2(b/2+3) bits [23]. (Fig .9 and table 5) shows that the results of IRC4 the technique is superior to RC4.





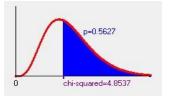
(a)

(b) Fig.9 (a), (b) RC4 and IRC4 SUB-BLOCK TEST RESPECTIVELY.

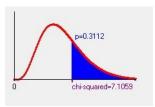
	Test Parameter									
Algorit hm	Total bits	Sub- block	value of Chi-	Degrees freedom	(p- value)	is Satisf				
		size	squared			у				
RC	128	2	10.2963	3	0.0162	yes				
IRC	128	2	6.8148	3	0.0780	yes				

4.5 Runs Test

The purpose of this test is to see if the numeral of runs of ones and zeros of matches what is anticipated from a random series. In particular, this test evaluates if the oscillation between such zeros and ones is too fast or too sluggish[24]. (Fig .10 and table 6) shows that the results of RC4 the technique is superior to IRC4. But the values are close, as it is clear, and therefore doing not affect the strength of the encryption.



(b)



(a)

Fig.10 (a), (b) RC4 and IRC4 RUNS TEST RESPECTIVELY.

	Test Parameter									
Algorit	Total	Number	Number	The	The	Freed	(p-	is		
hm	bits	of	of	Number	value of	om	value)	Satisf		
		runs	blocks	of gaps	Chi-	Degre		у		
					squared	e				
RC		54	27	27	4.8537	6	0.562	yes		
	128						7	5		
IRC		69	35	34	7.1059	6	0.311	yes		
	128						2			

Table 6: The Results of Comparing of the Runs Test.

4.6 Sequence Complexity Test

This check ensures that the stream has a sufficient number of new patterns. A stream is deemed non-random if the sequence complexity metric falls below a certain 'threshold' number, also, the value of An average of the complexity of the sequence for a stream in this length is counted[25]. (Fig .11 and table 7) shows that the results of RC4 the technique is superior to IRC4. But the values are close, as it is clear, and therefore doing not affect the strength of the encryption.





(a)

(b) Fig.11 (a), (b) RC4 and IRC4 SEQUENCE COMPLEXITY TEST RESPECTIVELY.

	1110 1100 01100		8	1						
	Test Parameter									
Algorit hm	Total bits	complexi ty of Sequence	Threshol	Value of Mean	is Satisfy					
RC		19	18	21	ye					
	128				s					
IRC		20	18	21	ye					
	128				s					

Table 7: The Results of Comparing of the Sequence Complexity Test

4.7 Linear Complexity Test

The goal of this test is to see if the sequence is complicated enough to be deemed random or not. A larger (LFSR) 'longer linear feedback shift register' is used to delineated random series[26]. (table 8 and Fig .12) shows that the 'Linear Complexity Profile' test results of IRC4 the technique is superior to RC4, (Fig .13 and table 9) shows that the 'linear complexity-number of jumps' test results of IRC4 technique and RC4 is the same. And 'linear complexity-jump size' test results of IRC4 technique and RC4 is the same results.

4.7.1 This sample represents Linear Complexity Profile test result for RC4 and IRC4 algorithms.

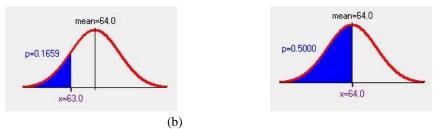
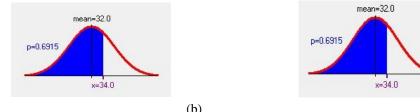


Fig.12 (a), (b) RC4 and IRC4 LINEAR COMPLEXITY TEST RESPECTIVELY.

	Test Parameter								
Algorit	Total	Linear	Expected	(p-value)	is				
hm	bits	Complex	Linear	_	Satisfy				
		ity	Complex						
			ity						
RC		63	64	0.1659	ye				
	128				s				
IRC		64	64	0.5000	ye				
	128				s				

 Table 8: The Results of Comparing of the Linear Complexity Test

4.7.2 This sample represents the Number of Jumps Linear Complexity test result for RC4 and IRC4 algorithms.

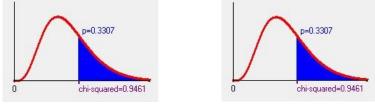


(a) (b) Fig.13 (a), (b) RC4 and IRC4 NUMBER OF JUMPS OF LINEAR COMPLEXITY TEST RESPECTIVELY.

	Test Parameter									
Algorit	Total	Number	Expecte	(p-value)	is					
hm	bits	of	d		Satisfy					
		Jumps	Number							
			of							
			Jumps							
RC	128	34	32	0.6915						
					ye					
					S					
IRC	128	34	32	0.6915						
					ye					
					S					

Table 9: The Results of Comparing of Number of Jumps of Linear Complexity Test

4.7.3 This sample represents Linear Complexity Jumps Size test result for RC4 and IRC4 algorithms.



(a) (b) Fig.14 (a), (b) RC4 and IRC4 LINEAR COMPLEXITY JUMPS SIZE TEST RESPECTIVELY.

(a)

Table 10: The Results of Comparing of the Linear Complexity Jump Size Test

Test Parameter					
Algorit	Total	Number	Expecte	(p-value)	is
hm	bits	of	d		Satisfy
		Jumps	Number		
			of		
			Jumps		
RC	128	0.9461	1	0.3307	
					ye
					S
IRC	128	0.9461	1	0.3307	
					ye
					S

5. **R**ESULTS **AND D**ISCUSSION

To compare the work of both the original and the improved algorithms, the same original script was used for both RC4 and IRC4 algorithms and we found that the time taken to execute both the proposed and the original algorithms is almost the same, when comparing the ciphertext of the RC4 algorithm is less random and complex than the developed algorithm. Statistical randomness test was carried out using CRYPTX'98, after checking the values, the resulting P-value is matched, If the value is less than 0.01, the series is rejected and the series is considered non-random, so the obtained sequences are accepted and described as random and uniformly distributed Through the series, as shown in the above tables, we noticed that most of the tests of the proposed method gave better results than the original algorithm.

6. CONCLUSIONS

RC4 stream encryption is a well-known encryption technology and one of the most popular encryption schemes for maintaining data security. Its implementation is simple and fast, but has flaws in keystream bytes, RC4 biases are now extracted for effective attacks. To provide a solution to bypass RC4 vulnerabilities by offering improved RC4 key generation. In this work, a new algorithm is proposed, Increases the randomness of the generated key by adding an ID number to the keystream before performing an XORed operation with the plaintext to generate a ciphertext as a slight modification has greatly enhanced the RC algorithm.

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