

Mean Square Solutions of Second-Order Random Differential Equations by Using Adomian Decomposition Method

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Abstract

In this paper, the Adomian decomposition method (ADM) is successfully applied for analytic (approximate) mean square solutions of the second-order random differential equations, homogeneous or inhomogeneous. Expectation and variance of the approximate solutions are computed. Several numerical examples are presented to show the ability and efficiency of this method .

Keywords: Random differential equations, Stochastic differential equation and Adomian decomposition method

1. Introduction

A random ordinary differential equations are an ordinary differential equations which contains random constants or random variables. Most scientific problems,

biology, engineering and physical phenomena occur in the form of random differential equations [1-3]. Recently, several first-order random differential models are solved using mean square calculus [4-11]. Many scientific models can be described as a second-order random differential equation in the following form

$$L[X(t)] + N[X(t), A] = g(t), \quad X(0) = Y_0, \quad \left. \frac{dX(t)}{dt} \right|_{t=0} = Y_1, \quad (1)$$

where $L[X(t)] = \frac{d^2 X(t)}{dt^2}$, $N[X(t), A]$ is a nonlinear operator and $g(t)$ is the source inhomogeneous term, as well as A, Y_0 and Y_1 are random variables. Within recent years, a special class of the initial value problem (1) has been treated under appropriate hypotheses on the data to evaluate the main statistical functions, such as the mean and the variance, of the approximate solution stochastic process generated by truncation of the exact power series solution [12-13].

In this paper, the ADM is used to find the mean square solutions for second-order random initial value problems. Several numerical examples are implemented to show the efficiency of this method.

2. Adomian Decomposition Method

The Adomian decomposition method (ADM) was first proposed by the American mathematician, G. Adomian [14,15]. The ADM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations. To illustrate the basic ideas of ADM, let us consider the second-order random differential equation (1)

The inverse operator of L_t , noted, L_t^{-1} is defined by

$$L_t^{-1} = \int_0^t \int_0^t (\cdot) ds$$

Thus, applying the inverse operator L_t^{-1} to (1) yields

$$X(t) = X(0) + X'(0)t + L_t^{-1}g(t) - L_t^{-1}N[X(t), A], \quad (2)$$

The ADM [14,15] assumes an infinite series solutions for unknown function $X(t)$ given by

$$X(t) = \sum_{k=0}^{\infty} x_k(t) \tag{3}$$

The nonlinear operator $N[X(t), A]$ is decomposed as

$$N[X(t), A] = \sum_{k=0}^{\infty} A_k(x_0(t, A), x_1(t, A), \dots, x_k(t, A)) \tag{4}$$

where $A_k(x_0(t, A), x_1(t, A), \dots, x_k(t, A))$ is an appropriate Adomian's polynomial which can be calculated for all forms of nonlinearity according to specific algorithms constructed by Adomian [14,15]. For nonlinearity operator $N[X(t), A]$, these polynomials can be calculated using the basic formula:

$$A_k(x_0(t, A), x_1(t, A), \dots, x_k(t, A)) = \frac{1}{k!} \left[\frac{d^k}{d\lambda^k} N \left(\sum_{n=1}^{\infty} \lambda^n x_n(t, A) \right) \right], \quad k \geq 0 \tag{5}$$

Operating with the twofold integral operator L_t^{-1} and representing $u(t)$ by the decomposition series of components one can obtain

$$\sum_{k=0}^{\infty} X_k(t) = X(0) + X'(0)t - \int_0^t \int_0^t \sum_{k=0}^{\infty} A_k(x_0(s, A), x_1(s, A), \dots, x_k(s, A)) ds + \int_0^t g(s) ds \tag{6}$$

It follows that

$$x_0(t) = X(0) + X'(0)t + \int_0^t \int_0^t g(s) ds, \tag{7}$$

$$x_k(t) = - \int_0^t \int_0^t A_{k-1}(x_0(s, A), x_1(s, A), \dots, x_{k-1}(s, A)) ds ds, \quad k = 1, 2, \dots, N \tag{8}$$

The series solutions follows immediately upon

$$X_N(t) = \sum_{k=0}^N x_k(t). \tag{9}$$

3. Statistical Functions of the Mean Square Random ADM

This section concern with the computation of the main statistical functions of the m.s. solution of (1) given by the iteration formula (7-8).

$$E[X_N(t)] \square \sum_{k=0}^N E[x_k(t)] \tag{10}$$

$$E[X_N^2(t)] \square E \left[\left(\sum_{k=0}^N x_k(t) \right)^2 \right] \tag{11}$$

$$V[X_N(t)] \leq E\left[\left(\sum_{k=0}^N x_k(t)\right)^2\right] - \left(\sum_{k=0}^N E[x_k(t)]\right)^2 \quad (12)$$

The following Lemma guarantee the convergent of the sequence $E[X_N(t)]$ to $E[X(t)]$ and the sequence $V[X_N(t)]$ to $V[X(t)]$ if the sequence the $X_N(t)$ converges to $X(t)$.

Lemma[5]: Let $\{X_N\}$ and $\{Y_N\}$ be two sequences of 2-r.v.s X and Y , respectively, i.e., $\lim_{N \rightarrow \infty} X_N = X$ and $\lim_{N \rightarrow \infty} Y_N = Y$ then $\lim_{N \rightarrow \infty} E[X_N Y_N] = E[XY]$

If $X_N = Y_N$, then $\lim_{N \rightarrow \infty} E[X_N^2] = E[X^2]$, $\lim_{N \rightarrow \infty} E[X_N] = E[X]$ and

$$\lim_{N \rightarrow \infty} V[X_N] = V[X].$$

4. Test Examples

In this section, we adopt several examples to illustrate the using of Adomian decomposition method for approximating the mean and the variance. The results are computed by using Maple 14 and compared to exact solution.

Example 1 [13]: Consider random initial value problem $\frac{d^2 X(t)}{dt^2} + A^2 X(t) = 0$,

$X(0) = Y_0$ and $\left.\frac{dX(t)}{dt}\right|_{t=0} = Y_1$ where A^2 is a Beta r.v. with parameters $\alpha = 2$ and $\beta = 1$, i.e. $A^2 \sim Be(\alpha = 2, \beta = 1)$ and independently of the initial conditions Y_0 and Y_1 which satisfy $E[Y_0] = 1$, $E[Y_0^2] = 2$, $E[Y_1] = 1$, $E[Y_1^2] = 3$ and $E[Y_0 Y_1] = 0$.

$$x_0(t) = Y_0 + Y_1 t,$$

$$x_1(t) = -\frac{1}{6} A^2 Y_1 t^3 - \frac{1}{2} A^2 Y_0 t^2$$

$$x_2(t) = \frac{1}{120} A^4 Y_1 t^5 + \frac{1}{24} A^4 Y_0 t^4,$$

$$x_3(t) = -\frac{1}{5040} A^6 Y_1 t^7 - \frac{1}{720} A^6 Y_0 t^6$$

$$x_4(t) = \frac{1}{362880} A^8 Y_1 t^9 + \frac{1}{40320} A^8 Y_0 t^8$$

$$x_5(t) = -\frac{1}{39916800} A^{10} Y_1 t^{11} - \frac{1}{3628800} A^{10} Y_0 t^{10}$$

$$\vdots$$

For N=5 , one can have

$$X_5(t) = Y_0 + Y_1 t - \frac{1}{6} A^2 Y_1 t^3 - \frac{1}{2} A^2 Y_0 t^2 + \frac{1}{120} A^4 Y_1 t^5 + \frac{1}{24} A^4 Y_0 t^4 - \frac{1}{5040} A^6 Y_1 t^7 - \frac{1}{720} A^6 Y_0 t^6$$

$$+ \frac{1}{362880} A^8 Y_1 t^9 + \frac{1}{40320} A^8 Y_0 t^8 - \frac{1}{39916800} A^{10} Y_1 t^{11} - \frac{1}{3628800} A^{10} Y_0 t^{10}$$

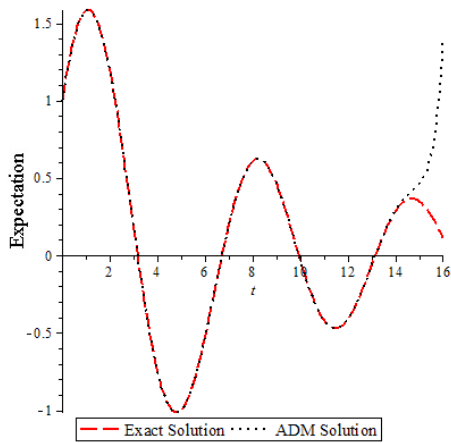


Fig.(1) : Comparison between the exact expectation and its approximation obtained from the ADM with N=18

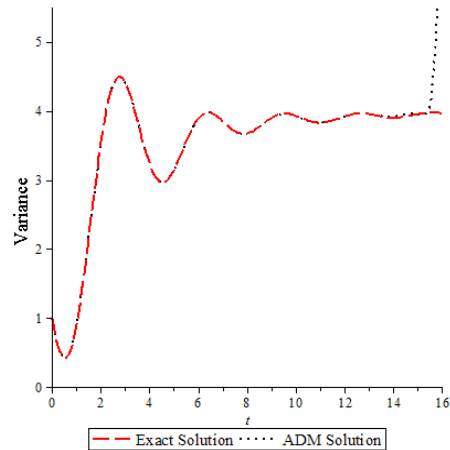


Fig.(2) : Comparison between the exact variance and its approximation obtained from the ADM with N=18

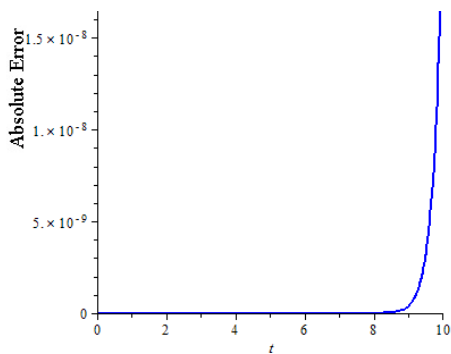


Fig.(3) : Absolute Error of expectation with N=18

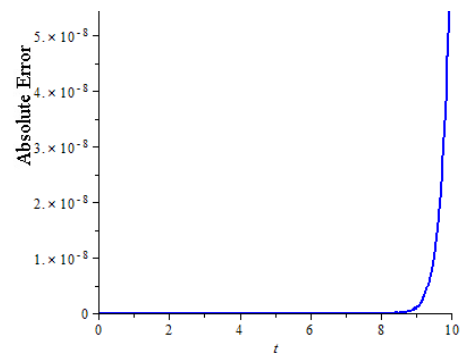


Fig.(4) : Absolute Error of variance with N=18

Example 2 [12] : Consider random initial value problem $\frac{d^2X(t)}{dt^2} + A t X(t) = 0$

$X(0) = Y_0$ and $\left. \frac{dX(t)}{dt} \right|_{t=0} = Y_1$ where A is a Beta r.v. with parameters $\alpha = 2$ and $\beta = 3$, i.e. $A \sim Be(\alpha = 2, \beta = 3)$ and independently of the initial conditions Y_0 and Y_1 which are independent r.v.'s such as $E[Y_0] = 1$, $E[Y_0^2] = 2$, $E[Y_1] = 2$, $E[Y_1^2] = 5$.

$$x_0(t) = Y_0 + Y_1 t,$$

$$x_1(t) = -\frac{1}{12} A Y_1 t^4 - \frac{1}{6} A Y_0 t^3,$$

$$x_2(t) = \frac{1}{504} A^2 Y_1 t^7 + \frac{1}{180} A^2 Y_0 t^6,$$

$$x_3(t) = -\frac{1}{45360} A^3 Y_1 t^{10} - \frac{1}{12960} A^3 Y_0 t^9,$$

$$x_4(t) = \frac{1}{7076160} A^4 Y_1 t^{13} + \frac{1}{1710720} A^4 Y_0 t^{12}$$

$$x_5(t) = -\frac{1}{1698278400} A^5 Y_1 t^{16} - \frac{1}{359251200} A^5 Y_0 t^{15}$$

⋮

For $N=5$, one can

$$X_5(t) = Y_0 + Y_1 t - \frac{1}{12} A Y_1 t^4 - \frac{1}{6} A Y_0 t^3 + \frac{1}{504} A^2 Y_1 t^7 + \frac{1}{180} A^2 Y_0 t^6 - \frac{1}{45360} A^3 Y_1 t^{10} - \frac{1}{12960} A^3 Y_0 t^9$$

$$+ \frac{1}{7076160} A^4 Y_1 t^{13} + \frac{1}{1710720} A^4 Y_0 t^{12} - \frac{1}{1698278400} A^5 Y_1 t^{16} - \frac{1}{359251200} A^5 Y_0 t^{15}$$

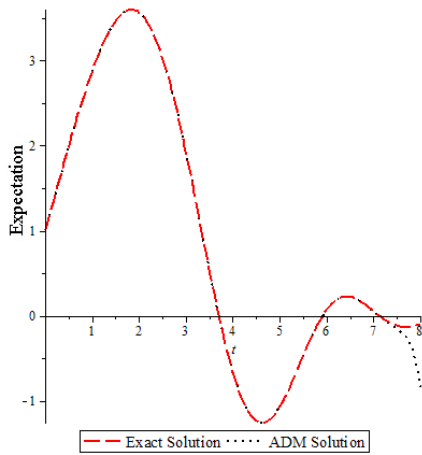


Fig.(5) : Comparison between the exact expectation and its approximation obtained from the ADM with N=15

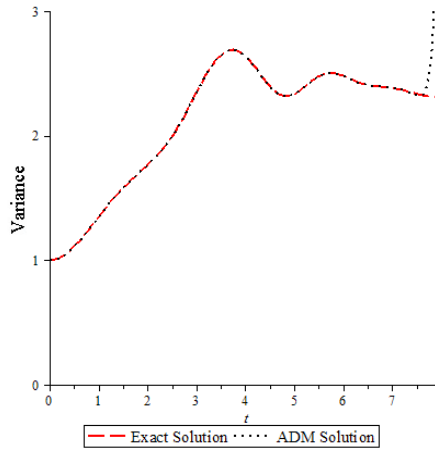


Fig.(6) : Comparison between the exact variance and its approximation obtained from the ADM with N=15

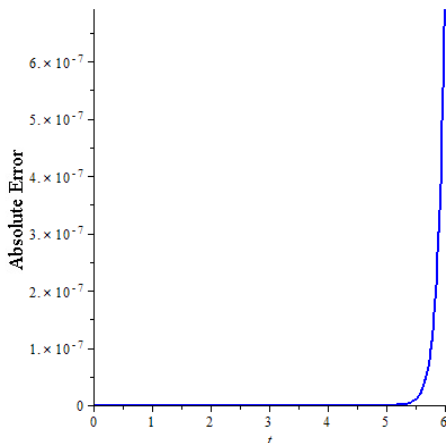


Fig.(7) : Absolute Error of expectation with N=15

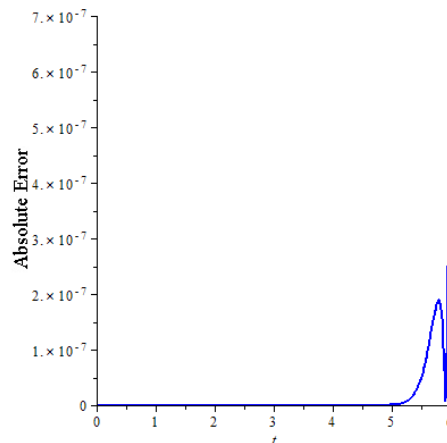


Fig.(8) : Absolute Error of variance with N=15

Example 3: Consider the problem $\frac{d^2 X(t)}{dt^2} + 2A \frac{dX(t)}{dt} + A^2 X(t) = 0$, $X(0) = Y_0$ and $\frac{dX(t)}{dt} \Big|_{t=0} = Y_1$ where A is a Beta r.v. with parameters $\alpha = 2$ and $\beta = 1$, i.e. $A \sim Be(\alpha = 2, \beta = 1)$ and independently of the initial conditions Y_0 and Y_1 which are independent r.v.'s satisfy $E[Y_0] = 1$, $E[Y_0^2] = 2$, $E[Y_1] = 1$, $E[Y_1^2] = 1$.

$$x_0(t) = Y_0 + Y_1 t$$

$$x_1(t) = -\frac{1}{6} A^2 Y_1 t^3 - t^2 A Y_1 - \frac{1}{2} t^2 A^2 Y_0$$

$$x_2(t) = \frac{1}{120} A^4 Y_1 t^5 + \frac{1}{6} t^4 A^3 Y_1 + \frac{1}{24} t^4 A^4 Y_0 + \frac{2}{3} A^2 Y_1 t^3 + \frac{1}{3} A^3 t^3 Y_0$$

$$x_3(t) = -\frac{1}{5040} A^6 Y_1 t^7 - \frac{1}{120} t^6 A^5 Y_1 - \frac{1}{720} t^6 A^6 Y_0 - \frac{1}{10} A^4 Y_1 t^5 - \frac{1}{30} t^5 A^5 Y_0 - \frac{1}{6} t^4 A^4 Y_0 - \frac{1}{3} t^4 A^3 Y_1$$

$$x_4(t) = \frac{1}{362880} A^8 Y_1 t^9 + \frac{1}{5040} t^8 A^7 Y_1 + \frac{1}{40320} t^8 A^8 Y_0 + \frac{1}{210} A^6 Y_1 t^7 + \frac{1}{840} t^7 A^7 Y_0 + \frac{1}{60} t^6 A^6 Y_0$$

$$+ \frac{2}{45} t^6 A^5 Y_1 + \frac{1}{15} t^5 A^5 Y_0 + \frac{2}{15} A^4 Y_1 t^5$$

$$x_5(t) = -\frac{1}{39916800} A^{10} Y_1 t^{11} - \frac{1}{362880} t^{10} A^9 Y_1 - \frac{1}{3628800} t^{10} A^{10} Y_0 - \frac{1}{9072} A^8 Y_1 t^9 - \frac{1}{45360} t^9 A^9 Y_0$$

$$- \frac{1}{1680} t^8 A^8 Y_0 - \frac{1}{504} t^8 A^7 Y_1 - \frac{2}{315} t^7 A^7 Y_0 - \frac{1}{63} A^6 Y_1 t^7 - \frac{1}{45} t^6 A^6 Y_0 - \frac{2}{45} t^6 A^5 Y_1$$

⋮

For $N=5$, one can have

$$X_5(t) = Y_0 + Y_1 t - \frac{1}{6} A^2 Y_1 t^3 - t^2 A Y_1 - \frac{1}{2} t^2 A^2 Y_0 + \frac{1}{120} A^4 Y_1 t^5 + \frac{1}{6} t^4 A^3 Y_1 + \frac{1}{24} t^4 A^4 Y_0 + \frac{2}{3} A^2 Y_1 t^3 + \frac{1}{3} A^3 t^3 Y_0$$

$$- \frac{1}{5040} A^6 Y_1 t^7 - \frac{1}{120} t^6 A^5 Y_1 - \frac{1}{720} t^6 A^6 Y_0 - \frac{1}{10} A^4 Y_1 t^5 - \frac{1}{30} t^5 A^5 Y_0 - \frac{1}{6} t^4 A^4 Y_0 - \frac{1}{3} t^4 A^3 Y_1 + \frac{1}{362880} A^8 Y_1 t^9$$

$$+ \frac{1}{5040} t^8 A^7 Y_1 + \frac{1}{40320} t^8 A^8 Y_0 + \frac{1}{210} A^6 Y_1 t^7 + \frac{1}{840} t^7 A^7 Y_0 + \frac{1}{60} t^6 A^6 Y_0 + \frac{2}{45} t^6 A^5 Y_1 + \frac{1}{15} t^5 A^5 Y_0 + \frac{2}{15} A^4 Y_1 t^5$$

$$- \frac{1}{39916800} A^{10} Y_1 t^{11} - \frac{1}{362880} t^{10} A^9 Y_1 - \frac{1}{3628800} t^{10} A^{10} Y_0 - \frac{1}{9072} A^8 Y_1 t^9 - \frac{1}{45360} t^9 A^9 Y_0 - \frac{1}{1680} t^8 A^8 Y_0$$

$$- \frac{1}{504} t^8 A^7 Y_1 - \frac{2}{315} t^7 A^7 Y_0 - \frac{1}{63} A^6 Y_1 t^7 - \frac{1}{45} t^6 A^6 Y_0 - \frac{2}{45} t^6 A^5 Y_1$$

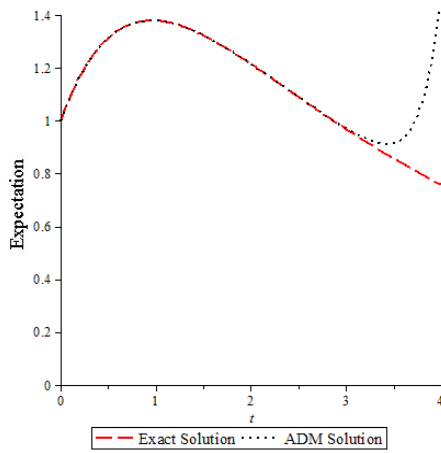


Fig.(9) : Comparison between the exact expectation and its approximation obtained from the ADM with N=16

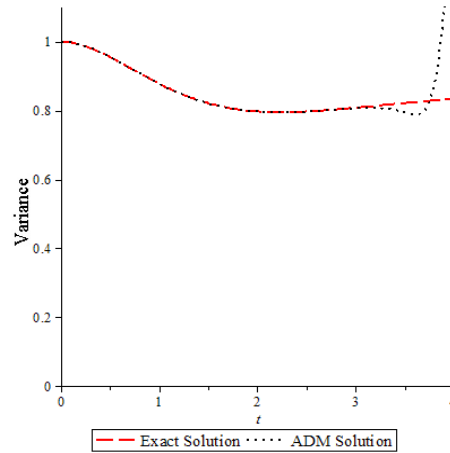


Fig.(10) : Comparison between the exact variance and its approximation obtained from the ADM with N=16

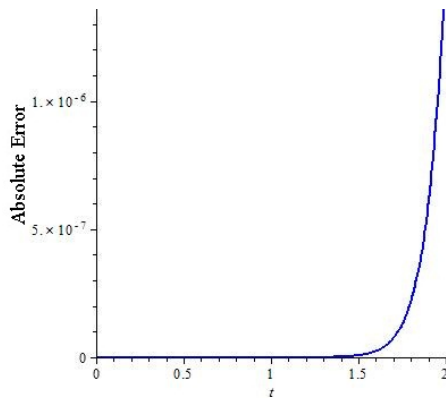


Fig.(11) : Absolute Error of expectation with N=16

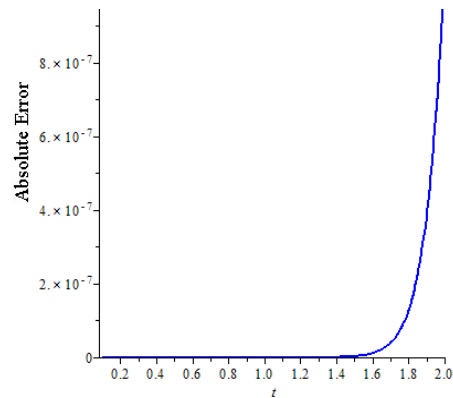


Fig.(12) : Absolute Error of variance with N=16

Example 4 :

Consider the problem $\frac{d^2 X(t)}{dt^2} + At \frac{dX(t)}{dt} = 0$, $X(0) = Y_0$ and $\frac{dX(t)}{dt} \Big|_{t=0} = Y_1$ where A is a Uniform r.v. with parameters $\alpha = 0$ and $\beta = 1$, i.e. $A \square U(\alpha = 0, \beta = 1)$ and independently of the initial conditions Y_0 and Y_1 which are independent r.v.'s satisfy $E[Y_0] = 1$, $E[Y_0^2] = 2$, $E[Y_1] = 1$, $E[Y_1^2] = 1$.
 $x_0(t) = Y_0 + Y_1 t$

$$\begin{aligned}
 x_1(t) &= -\frac{1}{6}AY_1t^3 \\
 x_2(t) &= \frac{1}{40}A^2Y_1t^5 \\
 x_3(t) &= -\frac{1}{336}A^3Y_1t^7 \\
 x_4(t) &= \frac{1}{3456}A^4Y_1t^9 \\
 x_5(t) &= -\frac{1}{42240}A^5Y_1t^{11} \\
 &\vdots
 \end{aligned}$$

For N=5 , one can have

$$X_5(t) = Y_0 + Y_1t - \frac{1}{6}AY_1t^3 + \frac{1}{40}A^2Y_1t^5 - \frac{1}{336}A^3Y_1t^7 + \frac{1}{3456}A^4Y_1t^9 - \frac{1}{42240}A^5Y_1t^{11}$$

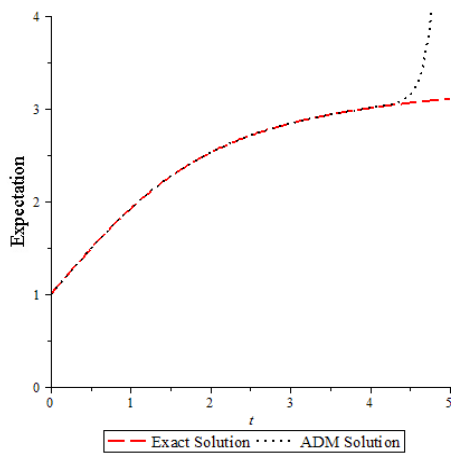


Fig.(13) : Comparison between the exact expectation and its approximation obtained from the ADM with N=20

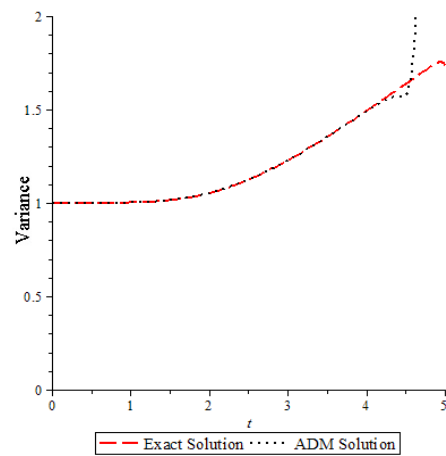


Fig.(14) : Comparison between the exact variance and its approximation obtained from the ADM with N=20

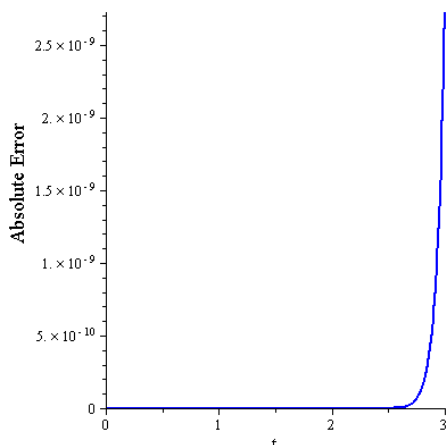


Fig.(15) : Absolute Error of expectation N=20

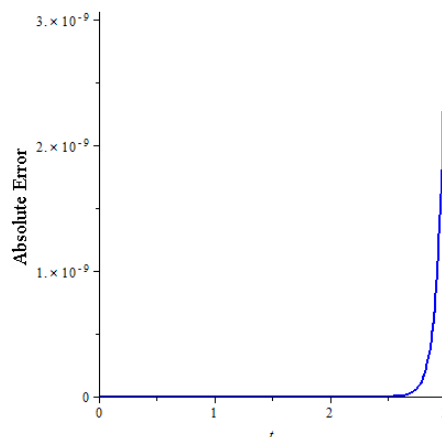


Fig.(16) : Absolute Error of variance with N=20

Example 5 : Consider the problem $\frac{d^2 X(t)}{dt^2} + A X(t) = 0$, $X(0) = Y_0$ and $\frac{dX(t)}{dt} \Big|_{t=0} = Y_1$ where A is a Uniform r.v. with parameters $\alpha = 0$ and $\beta = 2$, i.e. $A \square U(\alpha = 0, \beta = 2)$ and independently of the initial conditions Y_0 and Y_1 which are independent r.v.'s satisfy $E[Y_0] = 1$, $E[Y_0^2] = 4$, $E[Y_1] = 1$, $E[Y_1^2] = 2$.

$$\begin{aligned}
 x_0(t) &= Y_0 + Y_1 t, & x_1(t) &= -\frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2 \\
 x_2(t) &= \frac{1}{120} A^2 Y_1 t^5 + \frac{1}{24} A^2 Y_0 t^4, & x_3(t) &= -\frac{1}{5040} A^3 Y_1 t^7 - \frac{1}{720} A^3 Y_0 t^6 \\
 x_4(t) &= \frac{1}{362880} A^4 Y_1 t^9 + \frac{1}{40320} A^4 Y_0 t^8 \\
 x_5(t) &= -\frac{1}{39916800} A^5 Y_1 t^{11} - \frac{1}{3628800} A^5 Y_0 t^{10} \\
 &\vdots
 \end{aligned}$$

For $N=5$, one can have

$$\begin{aligned}
 X_5(t) &= Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2 + \frac{1}{120} A^2 Y_1 t^5 + \frac{1}{24} A^2 Y_0 t^4 - \frac{1}{5040} A^3 Y_1 t^7 - \frac{1}{720} A^3 Y_0 t^6 \\
 &\quad + \frac{1}{362880} A^4 Y_1 t^9 + \frac{1}{40320} A^4 Y_0 t^8 - \frac{1}{39916800} A^5 Y_1 t^{11} - \frac{1}{3628800} A^5 Y_0 t^{10}
 \end{aligned}$$

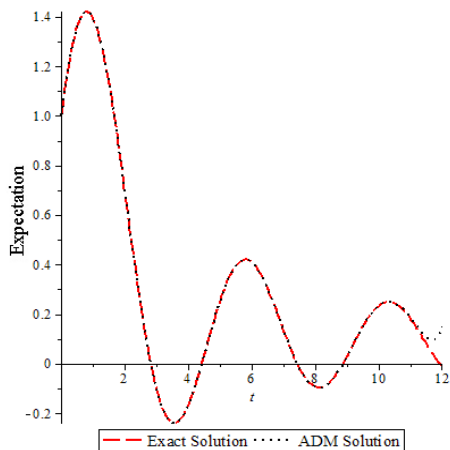


Fig.(17) : Comparison between the exact expectation and its approximation obtained from the ADM with N=20

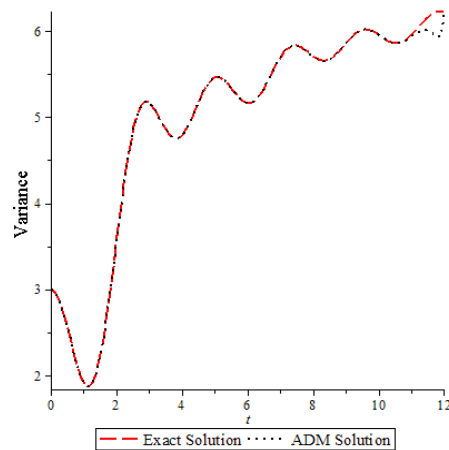


Fig.(18) : Comparison between the exact variance and its approximation obtained from the ADM with N=20

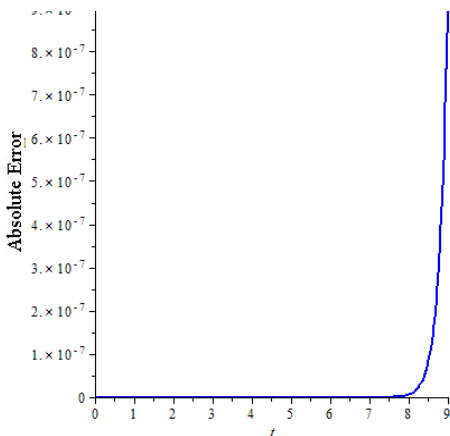


Fig.(19) : Absolute Error of expectation with N=20

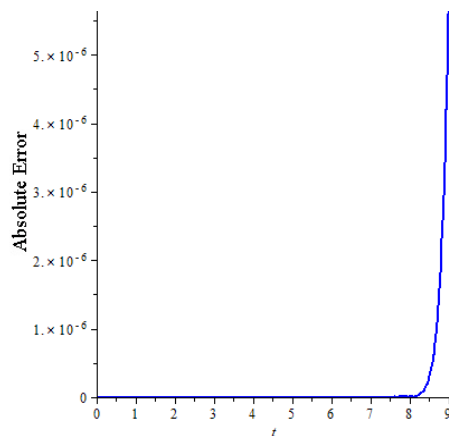


Fig.(20) : Absolute Error of variance with N=20

Example 6: Consider the problem $\frac{d^2 X(t)}{dt^2} + A X(t) = -X(t) + \sin(t)$, $X(0) = Y_0$ and $\frac{dX(t)}{dt} \Big|_{t=0} = Y_1$ where A is a Uniform r.v. with parameters $\alpha = 1$ and $\beta = 2$, i.e. $A \square U(\alpha = 1, \beta = 2)$ and independently of the initial conditions Y_0 and Y_1 which satisfy $E[Y_0] = 1$, $E[Y_0^2] = 2$, $E[Y_1] = 1$, $E[Y_1^2] = 6$ and $E[Y_0 Y_1] = 0$.

$$\begin{aligned}
 x_0(t) &= Y_0 + Y_1 t \\
 x_1(t) &= -\frac{1}{20} A Y_1 t^5 - \frac{1}{12} A Y_0 t^4 \\
 x_2(t) &= \frac{1}{1440} A^2 Y_1 t^9 + \frac{1}{672} A^2 Y_0 t^8 \\
 x_3(t) &= -\frac{1}{224640} A^3 Y_1 t^{13} - \frac{1}{88704} A^3 Y_0 t^{12} \\
 x_4(t) &= \frac{1}{61102080} A^4 Y_1 t^{17} + \frac{1}{21288960} A^4 Y_0 t^{16} \\
 x_5(t) &= -\frac{1}{25662873600} A^5 Y_1 t^{21} - \frac{1}{8089804800} A^5 Y_0 t^{20} \\
 &\vdots
 \end{aligned}$$

For N=5 , one can have

$$\begin{aligned}
 X_5(t) &= Y_0 + Y_1 t - \frac{1}{20} A Y_1 t^5 - \frac{1}{12} A Y_0 t^4 + \frac{1}{1440} A^2 Y_1 t^9 + \frac{1}{672} A^2 Y_0 t^8 - \frac{1}{224640} A^3 Y_1 t^{13} \\
 &- \frac{1}{88704} A^3 Y_0 t^{12} + \frac{1}{61102080} A^4 Y_1 t^{17} + \frac{1}{21288960} A^4 Y_0 t^{16} - \frac{1}{25662873600} A^5 Y_1 t^{21} - \frac{1}{8089804800} A^5 Y_0 t^{20}
 \end{aligned}$$

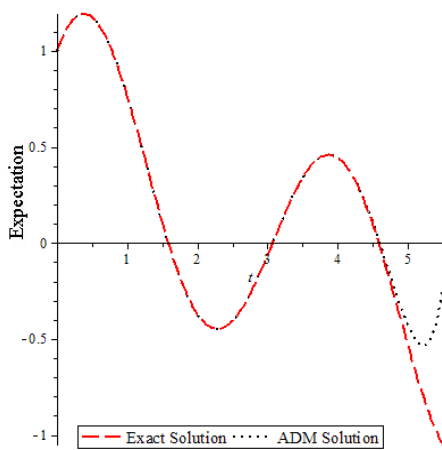


Fig.(21) : Comparison between the exact expectation and its approximation obtained from the ADM with N=10

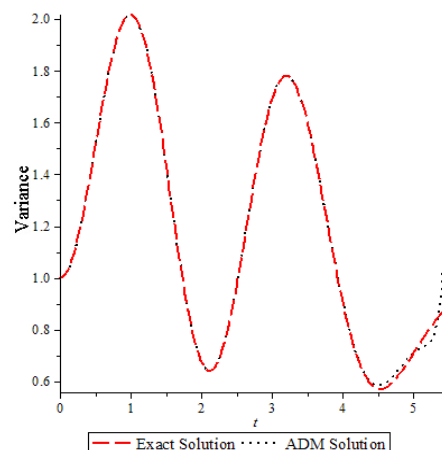


Fig.(22) : Comparison between the exact variance and its approximation obtained from the ADM with N=10

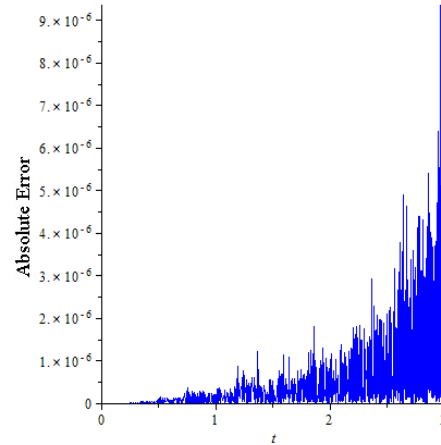
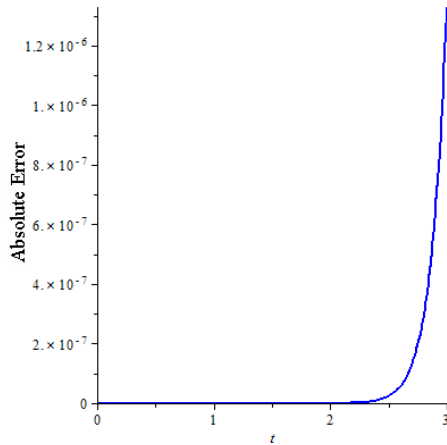


Fig.(23) : Absolute Error of expectation with N=10 Fig.(24) : Absolute Error of variance with N=10

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