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Mean Square Solutions of Second-Order Random Differential Equations by Using Variational Iteration Method

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Abstract

In this paper, the variational iteration method (VIM) is successfully applied for analytic (approximate) mean square solutions of the second-order random differential equations, homogeneous or inhomogeneous. Expectation and variance of the approximate solutions are computed. Several numerical examples are presented to show the ability and efficiency of this method .

Keywords: Random differential equations, Stochastic differential equation and Variational iteration method

1. Introduction

A random ordinary differential equations are an ordinary differential equations which contains random constants or random variables. Most scientific problems, biology, engineering and physical phenomena occur in the form of random differential equations [1-3]. Recently, several first-order random differential models are solved using mean square calculus [4-11]. Many scientific models can be described as a second-order random differential equation in the following form

$$L[X(t)] + N[X(t), A] = g(t), \quad X(0) = Y_0, \quad \frac{dX(t)}{dt}\Big|_{t=0} = Y_{1,} \tag{1}$$

where $L[X(t)] = \dfrac{d^2X(t)}{dt^2}$, $N[X(t), A]$ is a nonlinear operator and $g(t)$ is the source inhomogeneous term, as well as A, Y_0 and Y_1 are random variables. Within recent years, a special class of the initial value problem (1) has been treated under appropriate hypotheses on the data to evaluate the main statistical functions, such as the mean and the variance, of the approximate solution stochastic process generated by truncation of the exact power series solution[12-13].

In this paper, the VIM is used to find the mean square solutions for second-order random initial value problems. Several numerical examples are implemented to show the efficiency of the present method.

2. Variational Iteration Method

The variational iteration method (VIM) was first proposed by He [14,15] and systematically illustrated in 1999 [16]. The VIM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations. In this method, general Lagrange multipliers are introduced to construct correction functional for the problems. The multipliers can be identified optimally via the variational theory.

To illustrate the basic concept of the VIM, we consider the second-order random differential equation (1). According to the VIM, we can construct a correction functional as follows

$$X_{n+1}(t) = X_n(t) + \int_0^t \lambda(t,s)(L[X_n(s)] + N[\tilde{X}_n(s), A] - g(s))ds \tag{2}$$

where $\lambda(t,s) \neq 0$ is a general multiplier, which can be identified optimally via the variational theory, the subscript n denotes the n^{th} approximation, and $\tilde{X}_n$ is considered as a restricted variation i.e., $\delta \tilde{X}_n = 0$. The successive approximation, $X_{n+1},\ n \geq 0$, of the solution $X(t)$ can then be readily obtained by using the Lagrange multiplier determined and any selective function $X_0(t)$ satisfy the given conditions; consequently, the solution is given by $X(t) = \lim_{n\to\infty} X_n(t)$.

Under the restricted condition $(\delta \tilde{X}_n = 0)$, the stationary conditions of the above correction functional (2) can be expressed as follows:

$$\frac{\partial^2 \lambda(t,s)}{\partial s^2} = 0,$$

$$1 - \frac{\partial \lambda(t,s)}{\partial s}\Big|_{t=s} = 0,$$

$$\lambda(t,s)\Big|_{t=s} = 0.$$

The Lagrange multiplier, $\lambda(t,s)$, can be identified as $\lambda(t,s) = s - t$, which leads to the iterative formula

$$X_{n+1}(t) = X_n(t) + \int_0^t (s-t)\frac{d^2 X_n(s)}{ds^2}\, ds + \int_0^t (s-t)(N[X_n(s), A] - g(s))\, ds \qquad (3)$$

By using integration by parts to the first integral in the above equation, one can have

$$\int_0^t (s-t)\left(\frac{d^2 X_n(s)}{ds^2}\right) ds = X_n(0) + \frac{dX_n(s)}{ds}\Big|_{s=0} t - X_n(t)$$

So that (3) reduce to

$$X_{n+1}(t) = X_0(t) + h(t) + \int_0^t (s-t)\, N[X_n(s), A]\, ds \qquad (4)$$

where $X_0(t) = Y_0 + Y_1 t$ and $h(t) = \int_0^t (t-s)\, g(s)\, ds$

3. Statistical Functions of the Mean Square Random VIM

This section concern with the computation of the main statistical functions of the m.s. solution of (1) given by the iteration formula (4).

$$E[X_{n+1}(t)] = E[X_0(t)] + h(t) + \int_0^t (s-t)\ E[N[X_n(s), A]]\, ds, \qquad (5)$$

$$E[X_{n+1}^2(t)] = E[X_0^2(t)] + 2h(t)E[X_0(t)] + 2E\left[X_0(t)\int_0^t (s-t)\, N[X_n(s),A]\, ds \right]$$
$$+(h(t))^2 + 2h(t)E\left[\int_0^t (s-t)\, N[X_n(s),A]\, ds \right]$$
$$+E\left[\left(\int_0^t (s-t)\, N[X_n(s),A]\, ds \right)^2 \right]$$
$$V[X_{n+1}(t)] = V[X_0(t)] + 2Cov\left[X_0(t), \int_0^t (s-t)\, N[X_n(s),A]\, ds \right]$$
$$+V\left[\int_0^t (s-t)\, N[X_n(s),A]\, ds \right]$$

The following Lemma guarantee the convergent of the sequence $E[X_n(t)]$ to $E[X(t)]$ and the sequence $V[X_n(t)]$ to $V[X(t)]$ if the sequence the $X_n(t)$ converges to $X(t)$.

Lemma[5]: Let $\{X_n\}$ and $\{Y_n\}$ be two sequences of 2-r.vs X and Y, respectively, i.e., $\lim_{n\to\infty} X_n = X$ and $\lim_{n\to\infty} Y_n = Y$ then $\lim_{n\to\infty} E[X_n Y_n] = E[XY]$
If $X_n = Y_n$, then $\lim_{n\to\infty} E[X_n^2] = E[X^2]$, $\lim_{n\to\infty} E[X_n] = E[X]$ and $\lim_{n\to\infty} V[X_n] = V[X]$.

4. Test Examples

In this section, we adopt several examples to illustrate the using of variational iteration method for approximating the mean and the variance. The results are computed by using Maple 14 and compared to exact solution .

Example 1 [13] : Consider random initial value problem $\frac{d^2X(t)}{dt^2} + A^2 X(t) = 0$, $X(0) = Y_0$ and $\left.\frac{dX(t)}{dt}\right|_{t=0} = Y_1$ where $A^2 \sim Be(\alpha = 2, \beta = 1)$ and independently of the initial conditions Y_0 and Y_1 which satisfy $E[Y_o] = 1$, $E[Y_o^2] = 2$, $E[Y_1] = 1$, $E[Y_1^2] = 3$ and $E[Y_0 Y_1] = 0$.

$$X_0(t) = Y_0 + Y_1 t$$
$$X_1(t) = Y_0 + Y_1 t - \frac{1}{6}A^2 Y_1 t^3 - \frac{1}{2}A^2 Y_0 t^2$$

$$X_2(t)=Y_0+Y_1t-\frac{1}{6}A^2Y_1t^3-\frac{1}{2}A^2Y_0t^2+\frac{1}{120}A^4Y_1t^5+\frac{1}{24}A^4Y_0t^4$$

$$X_3(t)=Y_0+Y_1t-\frac{1}{6}A^2Y_1t^3-\frac{1}{2}A^2Y_0t^2+\frac{1}{120}A^4Y_1t^5+\frac{1}{24}A^4Y_0t^4-\frac{1}{5040}A^6Y_1t^7-\frac{1}{720}A^6Y_0t^6$$

$$X_4(t)=Y_0+Y_1t-\frac{1}{6}A^2Y_1t^3-\frac{1}{2}A^2Y_0t^2+\frac{1}{120}A^4Y_1t^5+\frac{1}{24}A^4Y_0t^4-\frac{1}{5040}A^6Y_1t^7-\frac{1}{720}A^6Y_0t^6$$
$$+\frac{1}{362880}A^8Y_1t^9+\frac{1}{40320}A^8Y_0t^8$$

$$X_5(t)=Y_0+Y_1t-\frac{1}{6}A^2Y_1t^3-\frac{1}{2}A^2Y_0t^2+\frac{1}{120}A^4Y_1t^5+\frac{1}{24}A^4Y_0t^4-\frac{1}{5040}A^6Y_1t^7-\frac{1}{720}A^6Y_0t^6$$
$$+\frac{1}{362880}A^8Y_1t^9+\frac{1}{40320}A^8Y_0t^8-\frac{1}{39916800}A^{10}Y_1t^{11}-\frac{1}{3628800}A^{10}Y_0t^{10}$$

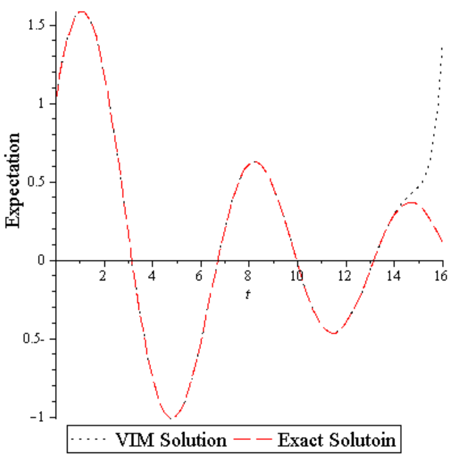

Fig.(1) : Comparison between the exact expectation and its approximation obtained from the VIM with n=18

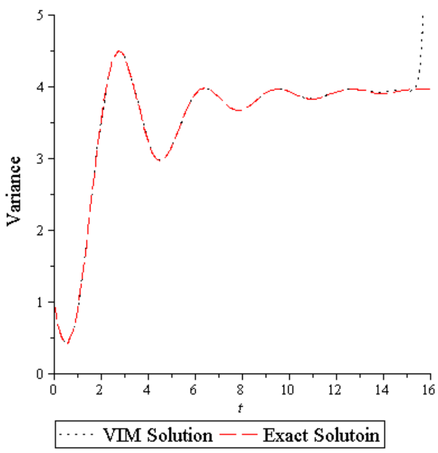

Fig.(2) : Comparison between the exact variance and its approximation obtained from the VIM with n=18

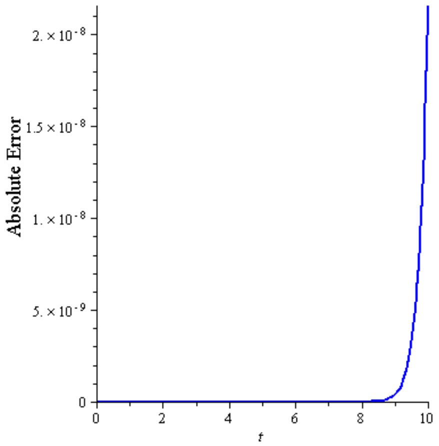

Fig.(3) : Absolute Error of expectation with n=18

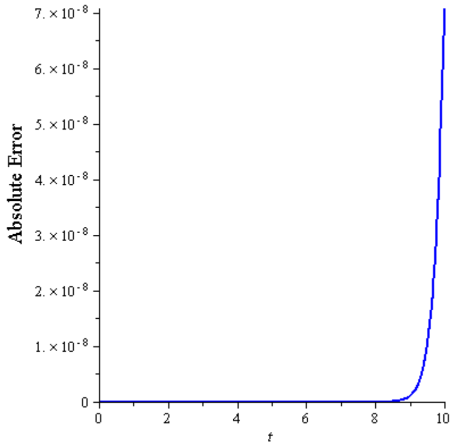

Fig.(4) : Absolute Error of variance with n=18

Example 2 [12] : Consider random initial value problem $\frac{d^2X(t)}{dt^2}+A\,t\,X(t)=0$ $X(0)=Y_0$ and $\frac{dX(t)}{dt}\Big|_{t=0}=Y_1$ where A is a Beta r.v. with parameters $\alpha=2$ and $\beta=3$, i.e. $A \sim Be(\alpha=2,\beta=3)$ and the initial conditions Y_o and Y_1 are independent r.v.'s such as $E[Y_o]=1$, $E[Y_o^2]=2$, $E[Y_1]=2$, $E[Y_1^2]=5$.

$$X_0(t)=Y_0+Y_1t$$

$$X_1(t)=Y_0+Y_1t-\frac{1}{12}AY_1t^4-\frac{1}{6}AY_0t^3$$

$$X_2(t)=Y_0+Y_1t-\frac{1}{12}AY_1t^4-\frac{1}{6}AY_0t^3+\frac{1}{504}A^2Y_1t^7+\frac{1}{180}A^2Y_0t^6$$

$$X_3(t)=Y_0+Y_1t-\frac{1}{12}AY_1t^4-\frac{1}{6}AY_0t^3+\frac{1}{504}A^2Y_1t^7+\frac{1}{180}A^2Y_0t^6-\frac{1}{45360}A^3Y_1t^{10}-\frac{1}{12960}A^3Y_0t^9$$

$$X_4(t)=Y_0+Y_1t-\frac{1}{12}AY_1t^4-\frac{1}{6}AY_0t^3+\frac{1}{504}A^2Y_1t^7+\frac{1}{180}A^2Y_0t^6-\frac{1}{45360}A^3Y_1t^{10}$$
$$-\frac{1}{12960}A^3Y_0t^9+\frac{1}{7076160}A^4Y_1t^{13}+\frac{1}{1710720}A^4Y_0t^{12}$$

$$X_5(t) = Y_0 + Y_1 t - \frac{1}{12}AY_1t^4 - \frac{1}{6}AY_0t^3 + \frac{1}{504}A^2Y_1t^7 + \frac{1}{180}A^2Y_0t^6 - \frac{1}{45360}A^3Y_1t^{10} - \frac{1}{12960}A^3Y_0t^9$$
$$+\frac{1}{7076160}A^4Y_1t^{13} + \frac{1}{1710720}A^4Y_0t^{12} - \frac{1}{1698278400}A^5Y_1t^{16} - \frac{1}{359251200}A^5Y_0t^{15}$$

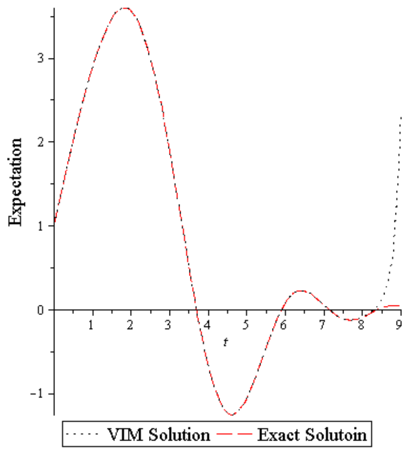

Fig.(5) : Comparison between the exact expectation and its approximation obtained from the VIM with n=18

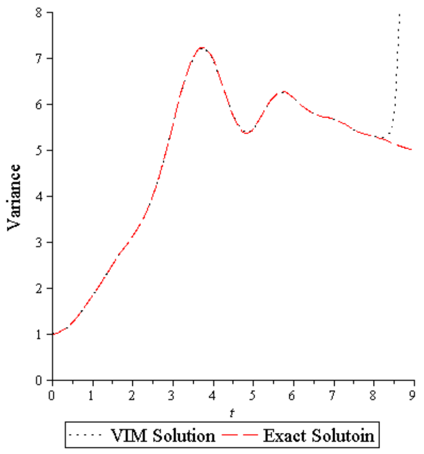

Fig.(6) : Comparison between the exact variance and its approximation obtained from the VIM with n=18

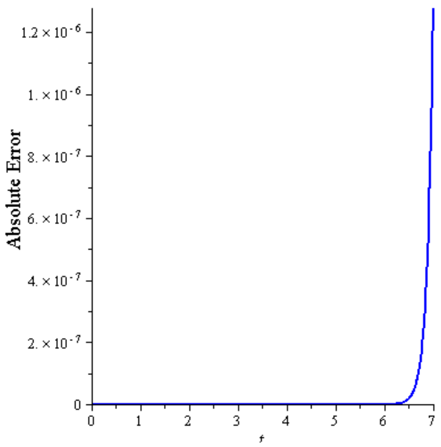

Fig.(7) : Absolute Error of expectation with n=18

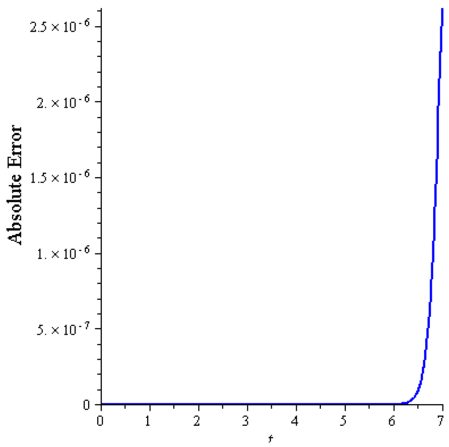

Fig.(8) : Absolute Error of variance with n=18

Example 3: Consider the problem $\frac{d^2X(t)}{dt^2}+2A\frac{dX(t)}{dt}+A^2\,X(t)=0$, $X\,(0)=Y_0$ and $\frac{dX\,(t)}{dt}\Big|_{t=0}=Y_1$ where A is a Beta r.v. with parameters $\alpha=2$ and $\beta=1$, i.e. $A \sim Be(\alpha=2,\beta=1)$ and independently of the initial conditions Y_o and Y_1 which are independent r.v.' satisfy $E[Y_o]=1$, $E[Y_o^2]=2$, $E[Y_1]=1$, $E[Y_1{}^2]=1$.

$$X_0(t)=Y_0+Y_1t$$

$$X_1(t)=Y_0+Y_1t-\frac{1}{6}A^2Y_1t^3-t^2AY_1-\frac{1}{2}t^2A^2Y_0$$

$$X_2(t)=Y_0+Y_1t+\frac{1}{2}A^2Y_1t^3-t^2AY_1-\frac{1}{2}t^2A^2Y_0+\frac{1}{120}A^4Y_1t^5+\frac{1}{6}t^4A^3Y_1+\frac{1}{24}t^4A^4Y_0+\frac{1}{3}A^3t^3Y_0$$

$$X_3(t)=Y_0+Y_1t+\frac{1}{2}A^2Y_1t^3-t^2AY_1-\frac{1}{2}t^2A^2Y_0-\frac{11}{120}A^4Y_1t^5-\frac{1}{6}t^4A^3Y_1-\frac{1}{8}t^4A^4Y_0+\frac{1}{3}A^3t^3Y_0$$
$$-\frac{1}{5040}A^6Y_1t^7-\frac{1}{120}t^6A^5Y_1-\frac{1}{720}t^6A^6Y_0-\frac{1}{30}t^5A^5Y_0$$

$$X_4(t)=Y_1t+Y_0+\frac{1}{2}A^2Y_1t^3-t^2AY_1-\frac{1}{2}t^2A^2Y_0+\frac{1}{24}A^4Y_1t^5-\frac{1}{6}t^4A^3Y_1-\frac{1}{8}t^4A^4Y_0+\frac{1}{3}A^3t^3Y_0$$
$$+\frac{1}{5040}t^8A^7Y_1+\frac{23}{5040}A^6Y_1t^7+\frac{13}{360}t^6A^5Y_1+\frac{11}{720}t^6A^6Y_0+\frac{1}{30}t^5A^5Y_0+\frac{1}{362880}A^8Y_1t^9$$
$$+\frac{1}{40320}t^8A^8Y_0+\frac{1}{840}t^7A^7Y_0$$

$$X_5(t)=Y_1t+Y_0+\frac{1}{2}A^2Y_1t^3-t^2AY_1-\frac{1}{2}t^2A^2Y_0+\frac{1}{24}A^4Y_1t^5-\frac{1}{6}t^4A^3Y_1-\frac{1}{8}t^4A^4Y_0+\frac{1}{3}A^3t^3Y_0$$
$$-\frac{1}{560}t^8A^7Y_1-\frac{19}{1680}A^6Y_1t^7-\frac{1}{120}t^6A^5Y_1-\frac{1}{144}t^6A^6Y_0+\frac{1}{30}t^5A^5Y_0-\frac{13}{120960}A^8Y_1t^9$$
$$-\frac{23}{40320}t^8A^8Y_0-\frac{13}{2520}t^7A^7Y_0-\frac{1}{39916800}A^{10}Y_1t^{11}-\frac{1}{362880}t^{10}A^9Y_1-\frac{1}{3628800}t^{10}A^{10}Y_0$$
$$-\frac{1}{45360}t^9A^9Y_0$$

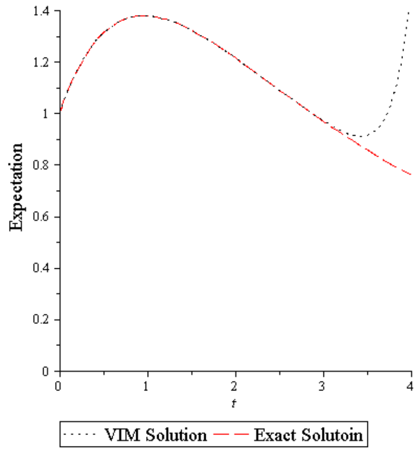

Fig.(9) : Comparison between the exact expectation and its approximation obtained from the VIM with n=16

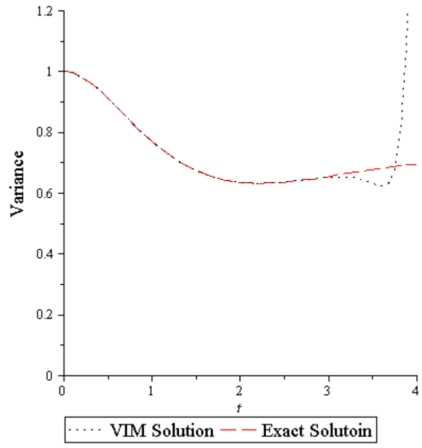

Fig.(10) : Comparison between the exact variance and its approximation obtained from the VIM with n=16

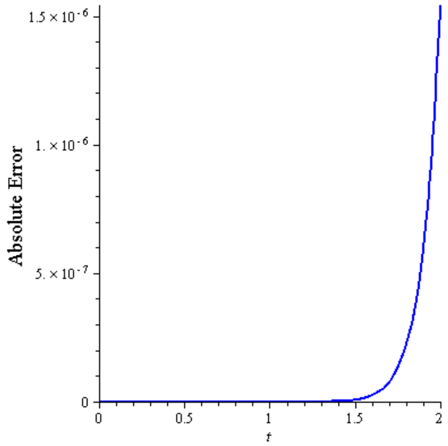

Fig.(11) : Absolute Error of expectation with n=16

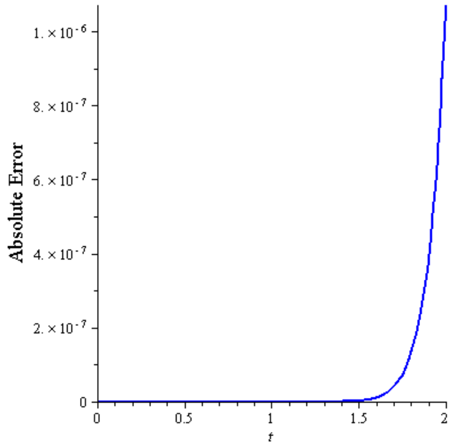

Fig.(12) : Absolute Error of variance with n=16

Example 4 : Consider the problem $\frac{d^2X(t)}{dt^2}+At\frac{dX(t)}{dt}=0$, $X(0)=Y_0$ and $\frac{dX(t)}{dt}\Big|_{t=0}=Y_1$ where A is a Uniform r.v. with parameters $\alpha=0$ and $\beta=1$, i.e. $A \sim U(\alpha=0,\beta=1)$ and independently of the initial conditions Y_o and Y_1

which are independent r.v.'s satisfy $E[Y_o]=1$, $E[Y_o^2]=2$, $E[Y_1]=1$, $E[Y_1^{\,2}]=1$.

$$X_0(t) = Y_0 + Y_1 t$$

$$X_1(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3$$

$$X_2(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 + \frac{1}{40} A^2 Y_1 t^5$$

$$X_3(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 + \frac{1}{40} A^2 Y_1 t^5 - \frac{1}{336} A^3 Y_1 t^7$$

$$X_4(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 + \frac{1}{40} A^2 Y_1 t^5 - \frac{1}{336} A^3 Y_1 t^7 + \frac{1}{3456} A^4 Y_1 t^9$$

$$X_5(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 + \frac{1}{40} A^2 Y_1 t^5 - \frac{1}{336} A^3 Y_1 t^7 + \frac{1}{3456} A^4 Y_1 t^9 - \frac{1}{42240} A^5 Y_1 t^{11}$$

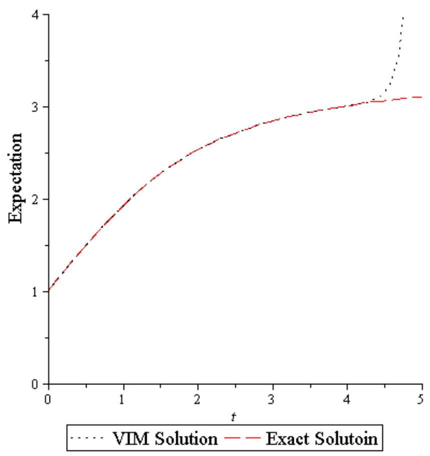

Fig.(13) : Comparison between the exact expectation and its approximation obtained from the VIM with n=20

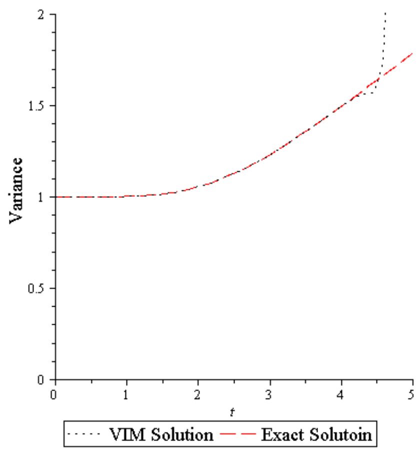

Fig.(14) : Comparison between the exact variance and its approximation obtained from the VIM with n=20

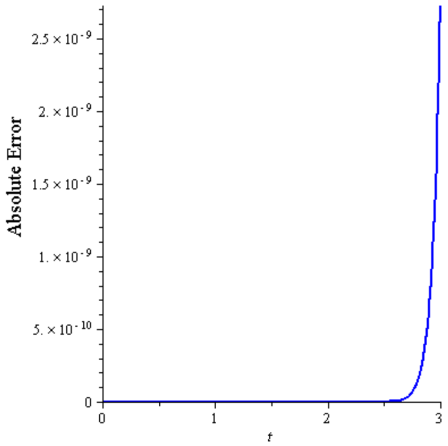

Fig.(15) : Absolute Error of expectation n=20

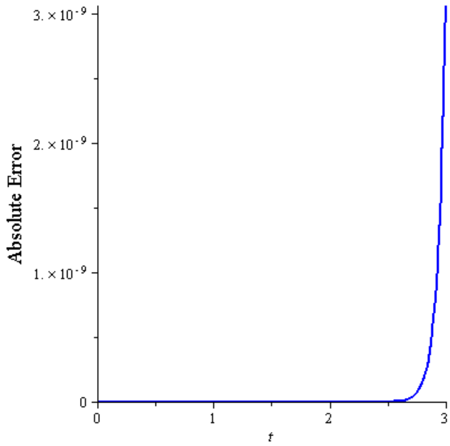

Fig.(16) : Absolute Error of variance with n=20

Example 5 : Consider the problem $\frac{d^2X(t)}{dt^2} + AX(t) = 0$, $X(0) = Y_0$ and $\left.\frac{dX(t)}{dt}\right|_{t=0} = Y_1$ where A is a Uniform r.v. with parameters $\alpha = 0$ and $\beta = 2$, i.e. $A \sim U(\alpha = 0, \beta = 2)$ and independently of the initial conditions Y_o and Y_1 which are independent r.v.'s satisfy $E[Y_o] = 1$, $E[Y_o^{\ 2}] = 4$, $E[Y_1] = 1, E[Y_1^{\ 2}] = 2$.

$$X_0(t) = Y_0 + Y_1 t$$

$$X_1(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2$$

$$X_2(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2 + \frac{1}{120} A^2 Y_1 t^5 + \frac{1}{24} A^2 Y_0 t^4$$

$$X_3(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2 + \frac{1}{120} A^2 Y_1 t^5 + \frac{1}{24} A^2 Y_0 t^4 - \frac{1}{5040} A^3 Y_1 t^7 - \frac{1}{720} A^3 Y_0 t^6$$

$$X_4(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2 + \frac{1}{120} A^2 Y_1 t^5 + \frac{1}{24} A^2 Y_0 t^4 - \frac{1}{5040} A^3 Y_1 t^7 - \frac{1}{720} A^3 Y_0 t^6 + \frac{1}{362880} A^4 Y_1 t^9 + \frac{1}{40320} A^4 Y_0 t^8$$

$$X_5(t) = Y_0 + Y_1 t - \frac{1}{6} A Y_1 t^3 - \frac{1}{2} A Y_0 t^2 + \frac{1}{120} A^2 Y_1 t^5 + \frac{1}{24} A^2 Y_0 t^4 - \frac{1}{5040} A^3 Y_1 t^7 - \frac{1}{720} A^3 Y_0 t^6 + \frac{1}{362880} A^4 Y_1 t^9 + \frac{1}{40320} A^4 Y_0 t^8 - \frac{1}{39916800} A^5 Y_1 t^{11} - \frac{1}{3628800} A^5 Y_0 t^{10}$$

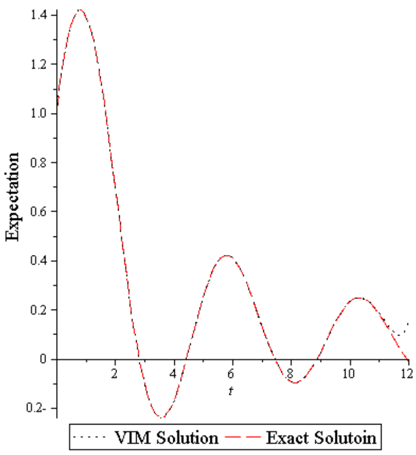

Fig.(17) : Comparison between the exact expectation and its approximation obtained from the VIM with n=20

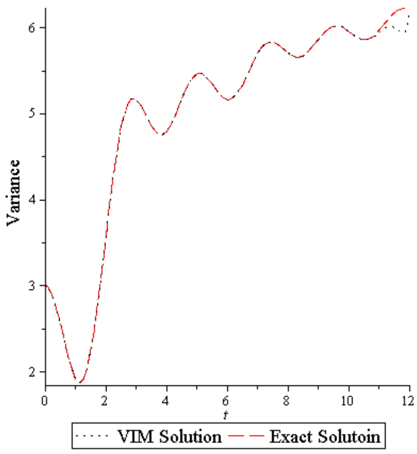

Fig.(18) : Comparison between the exact variance and its approximation obtained from the VIM with n=20

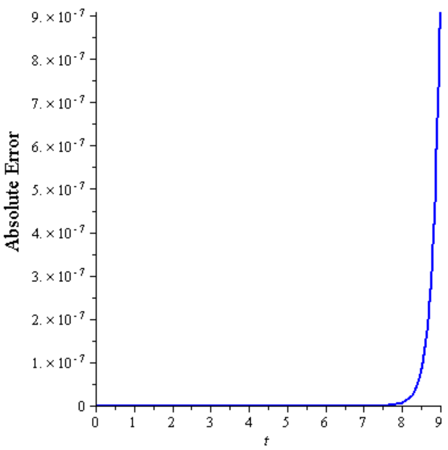

Fig.(19) : Absolute Error of expectation with n=20

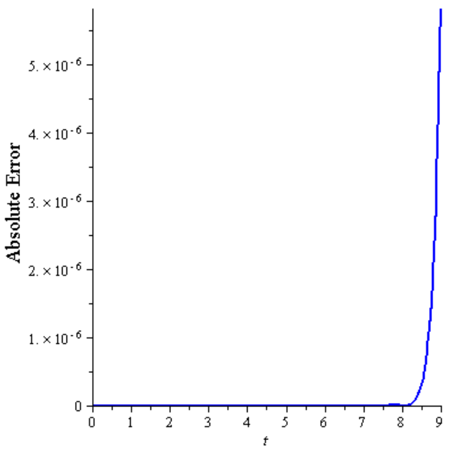

Fig.(20) : Absolute Error of variance with n=20

Example 6: Consider the problem $\frac{d^2X(t)}{dt^2}+A\,X(t)=-X(t)+\sin(t)$, $X(0)=Y_0$ and $\frac{dX(t)}{dt}\Big|_{t=0}=Y_1$ where A is a Uniform r.v. with parameters $\alpha=1$ and $\beta=2$, i.e. $A \sim U(\alpha=1,\beta=2)$ and independently of the initial conditions Y_o and Y_1 which satisfy $E[Y_o]=1$, $E[Y_o^2]=2$, $E[Y_1]=1$, $E[Y_1^2]=6$ and $E[Y_0Y_1]=0$.

$$X_0(t)=Y_0+Y_1t$$

$$X_1(t)=Y_0+Y_1t-\frac{1}{20}AY_1t^5-\frac{1}{12}AY_0t^4$$

$$X_2(t)=Y_0+Y_1t-\frac{1}{20}AY_1t^5-\frac{1}{12}AY_0t^4+\frac{1}{1440}A^2Y_1t^9+\frac{1}{672}A^2Y_0t^8$$

$$X_3(t)=Y_0+Y_1t-\frac{1}{20}AY_1t^5-\frac{1}{12}AY_0t^4+\frac{1}{1440}A^2Y_1t^9+\frac{1}{672}A^2Y_0t^8-\frac{1}{224640}A^3Y_1t^{13}-\frac{1}{88704}A^3Y_0t^{12}$$

$$X_4(t)=Y_0+Y_1t-\frac{1}{20}AY_1t^5-\frac{1}{12}AY_0t^4+\frac{1}{1440}A^2Y_1t^9+\frac{1}{672}A^2Y_0t^8-\frac{1}{224640}A^3Y_1t^{13}-\frac{1}{88704}A^3Y_0t^{12}+\frac{1}{61102080}A^4Y_1t^{17}+\frac{1}{21288960}A^4Y_0t^{16}$$

$$X_5(t)=Y_0+Y_1t-\frac{1}{20}AY_1t^5-\frac{1}{12}AY_0t^4+\frac{1}{1440}A^2Y_1t^9+\frac{1}{672}A^2Y_0t^8-\frac{1}{224640}A^3Y_1t^{13}-\frac{1}{88704}A^3Y_0t^{12}+\frac{1}{61102080}A^4Y_1t^{17}+\frac{1}{21288960}A^4Y_0t^{16}-\frac{1}{25662873600}A^5Y_1t^{21}-\frac{1}{8089804800}A^5Y_0t^{20}$$

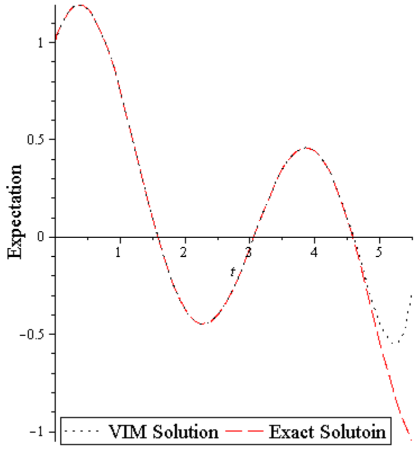

Fig.(21) : Comparison between the exact expectation and its approximation obtained from the VIM with n=10

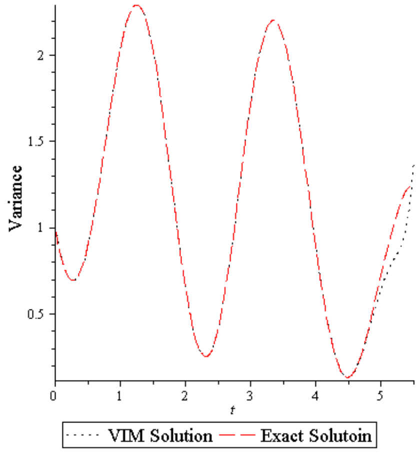

Fig.(22) : Comparison between the exact variance and its approximation obtained from the VIM with n=10

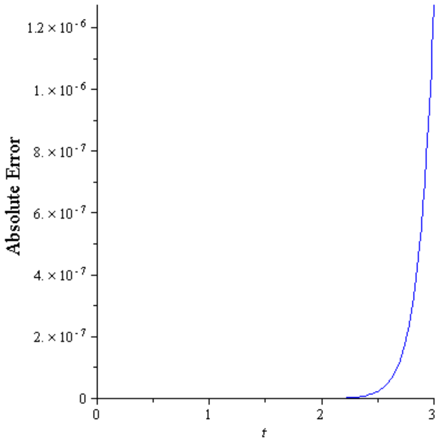

Fig.(23) : Absolute Error of expectation with n=10

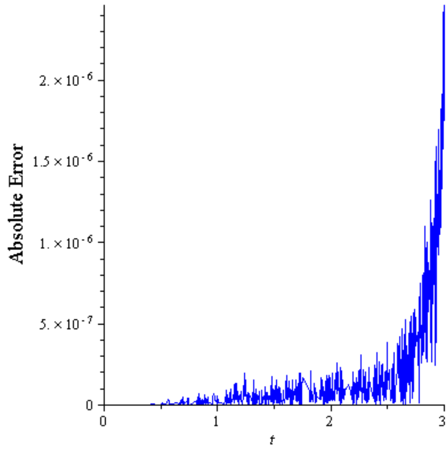

Fig.(24) : Absolute Error of variance with n=10

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