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The Principle IndecomposableProjective Characters of The Symmetric Groups S_{22}Modulo $p = 19$

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ABSTRACT

In this paper we invention all the principle indecomposablespin(projective) characters of the symmetric group S_n ,when $n = 22$ and the characteristic of the field is 19. The principle indecomposable spincharacters of the symmetric group S_{22},also can get it by determine all irreducible spin characters for S_{22} ,$p = 19$, and all irreducible modular spin characters for S_{22} , p = 19 , the sum of multiplication of these characters it represents the principle indecomposable spin(projective) characters of the symmetric group S_{22}.For their more can get on the irreducible spin characters for S_{22} , $p = 19$ by fixing all bar partitions for S_{22} , also we can get on all the irreducible modular spin characters for S_{22} , $p = 19$ by$(r, \bar{r})$-inducing method, finally induce the principal indecomposable characters(P.i.s)fromS_{21} (see creek*) give usprincipal indecomposable characters (P.i.s) or principal characters (P.s) of S_{22}.

MSC : 15C15,15C20,15C25

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1 . Introduction

The principle indecomposablespin(projective) characters is represent the sum of multiplication for irreducible projective characters and irreducible modular projective characters[1].In General characters is known modular or ordinary corresponding to the characteristic of the field is prime or zero, respectively[2] . Every finite group has covering group[3] ,then S_n has as this group. The characters of the covering group which are identical the characters of S_n are called modular or ordinary characters of S_n ,the survival characters are called projective(spin) of S_n [4].The irreducible projective characters are described by the bar partitions of n ,also these characters are recognize double or associate corresponding to the result $n - m$ is even or odd, respectively where$m$ is the number of parts for the bar partition of

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n[3]. A lot of researchersactivate in this field like A. K..Yaseen, S. A.Taban, A. H.Jassimand M. M.Jawad[5],[7].In this paper the principle indecomposablespin(projective) characters of S_{22} modulo $p = 19$ have been calculated by using $(r, \bar{r})$-inducing method, we induce the principal indecomposable characters of S_{21} (see creek*) to have the principal indecomposable characters or principal characters of S_{22}.

2. Rudiments

1-The spin characters of S_n can be written as a linear combination , with non-negative integer coefficients , of the irreducible spin characters[8].
2- Projective characters $\langle\omega\rangle = \langle\omega_1, \dots, \omega_u\rangle$for S_z have degree which is equal $2^{\left[\frac{z-u}{2}\right]} \frac{z!}{\prod_{i=1}^{u}(\omega_i!)} \prod_{1\le i<j\le u} \frac{(\omega_i-\omega_j)}{(\omega_i+\omega_j)}$ where u it represents the number of parts [9].
3-The values of associate characters $\langle\omega\rangle, \langle\omega\rangle'$ are same on the class except on the class corresponding to ω they have values $\pm i^{\frac{z-u+1}{2}} \sqrt{\left(\frac{\omega_1,\dots,\omega_u}{2}\right)}$[9].
4-The inducing from group or restriction from the subgroup of the projective characters are also projective characters [1] .
5- If z is even and $p \nmid (z)$,then $\langle z\rangle$ and $\langle z\rangle'$ are distinctirreducible modular spin charactersI.m.s of grade $2^{\left[\frac{(z-1)}{2}\right]}$ which are denoted by$\omega\langle z\rangle$ and$\omega\langle z\rangle'$[9].
6- Let p be an odd prime and let μ, ω be a bar partition of$z$ which are not p-bar core. Then $\langle\mu\rangle$(and $\langle\mu\rangle'$ if μ is odd) and $\langle\omega\rangle$(and $\langle\omega\rangle'$ if ω is odd) are in the same p-block$\leftrightarrow \widetilde{\langle\mu\rangle} = \widetilde{\langle\omega\rangle}$(where $\widetilde{\langle\mu\rangle}, \widetilde{\langle\omega\rangle}$ are represents$p$-bar core forμ , ωrespectively). If αbe a bar partition of z and $\mu = \widetilde{\langle\mu\rangle}$, then $\langle\mu\rangle$(and $\langle\mu\rangle'$ if σ is odd) forms a p-block of defect 0[4].
7- Let p be an odd prime and $\omega = (\omega_1, \dots, \omega_u)$be a bar partition of z,then allirreducible modular spin characters I.m.s in the block B are double(associate) , if $(z - u - u_0)$ is even(odd), where u_0 the number of parts of ω divisible by p[4] .
8-Each block of defect zero contains exactly one irreducible ordinary characters τ, one irreducible modular charactersϵ and one principal indecomposable character ∂ such that $\tau = \epsilon = \partial$[10].
9- If C is a principal character for an odd prime p and all the entries in C are divisible by positive integer q , then (1/q)C is a principal character[1] .

3-The projective block of S_{22}

The number of the principle indecomposablespin(projective) characters of S_{22}, $p = 19$ is 130 which is equal to the $(19,\alpha)$-regular classes [10].
From preliminaries (6) ,there are 103 blocks of $S_{22}, p = 19$, theses blocks are $j_3, \dots, j_{105}$ of defect zero except the blockj_1, $j_2$of defect one.
The blocks of defect zero $j_3, \dots, j_{105}$ includes
$\langle 1,3,5,6,7\rangle, \langle 1,3,5,6,7\rangle', \langle 2,3,4,6,7\rangle, \langle 2,3,4,6,7\rangle', \langle 4,5,6,7\rangle^*$,
$\langle 1,2,3,4,5,6,7\rangle^*, \langle 1,2,5,6,8\rangle, \langle 1,2,5,6,8\rangle', \langle 3,5,6,8\rangle^*, \langle 2,3,4,5,8\rangle$,
$\langle 2,3,4,5,81\rangle', \langle 1,3,4,6,8\rangle, \langle 1,3,4,6,8\rangle', \langle 2,5,7,8\rangle^*, \langle 3,4,7,8\rangle^*$,
$\langle 1,2,4,7,8\rangle, \langle 1,2,4,7,8\rangle', \langle 1,6,7,8\rangle^*, \langle 1,3,4,5,9\rangle, \langle 1,3,4,5,9\rangle'$,

$\langle 1,2,4,6,9\rangle, \langle 1,2,4,6,9\rangle', \langle 3,4,6,9\rangle^*, \langle 2,5,6,9\rangle^*, \langle 1,2,3,7,9\rangle,$
$\langle 1,2,3,7,9\rangle', \langle 2,4,7,9\rangle^* \langle 1,5,7,9\rangle^* \langle 6,7,9\rangle, \langle 6,7,9\rangle',$
$\langle 2,3,8,9\rangle^*, \langle 1,4,8,9\rangle^*, \langle 5,8,9\rangle, \langle 5,8,9\rangle', \langle 1,2,4,5,10\rangle,$
$\langle 1,2,4,5,10\rangle', \langle 3,4,5,10\rangle^*, \langle 1,2,3,6,10\rangle, \langle 1,2,3,6,10\rangle',$
$\langle 2,4,6,10\rangle^*, \langle 1,5,6,10\rangle^*, \langle 2,3,7,10\rangle^*, \langle 1,4,7,10\rangle^*, \langle 5,7,10\rangle,$
$\langle 5,7,10\rangle', \langle 1,3,8,10\rangle^*, \langle 4,8,10\rangle, \langle 4,8,10\rangle', \langle 1,2,3,5,11\rangle,$
$\langle 1,2,3,5,11\rangle', \langle 2,4,5,11\rangle^*, \langle 2,3,6,11\rangle^*, \langle 1,4,6,11\rangle^*,$
$\langle 5,6,11\rangle, \langle 5,6,11\rangle', \langle 1,3,7,11\rangle^*, \langle 4,7,11\rangle, \langle 4,7,11\rangle'$
$\langle 2,9,11\rangle, \langle 2,9,11\rangle', \langle 1,10,11\rangle, \langle 1,10,11\rangle', \langle 1,2,3,4,12\rangle, \langle 1,2,3,4,12\rangle', \langle 2,3,5,12\rangle^*, \langle 1,4,5,12\rangle^*, \langle 1,3,6,12\rangle^*,$
$\langle 4,6,12\rangle, \langle 4,6,12\rangle', \langle 2,8,12\rangle, \langle 2,8,12\rangle', \langle 1,9,12\rangle, \langle 1,9,12\rangle',$
$\langle 10,12\rangle^*, \langle 2,3,4,13\rangle^*, \langle 1,3,5,13\rangle^*,$
$\langle 4,5,13\rangle, \langle 4,5,13\rangle', \langle 2,7,13\rangle, \langle 2,7,13\rangle', \langle 1,8,13\rangle, \langle 1,8,13\rangle',$
$\langle 9,13\rangle^*, \langle 1,3,4,14\rangle^*, \langle 2,6,14\rangle, \langle 2,6,14\rangle',$
$\langle 1,7,14\rangle, \langle 1,7,14\rangle', \langle 8,14\rangle^*, \langle 2,5,15\rangle, \langle 2,5,15\rangle', \langle 1,6,15\rangle, \langle 1,6,15\rangle', \langle 7,15\rangle^*, \langle 2,4,16\rangle, \langle 2,4,16\rangle',$
$\langle 1,5,16\rangle, \langle 1,5,16\rangle', \langle 6,16\rangle^*, \langle 1,4,17\rangle, \langle 1,4,17\rangle', \langle 5,17\rangle^*, \langle 1,3,18\rangle, \langle 1,3,18\rangle', \langle 4,18\rangle^*$
.respectively ,these characters areprinciple indecomposablespin characters (preliminaries 6).
The block j_2 contains the projective characters$\langle 1,21\rangle^*$, $\langle 2,20\rangle^*$,$\langle 1,2,19\rangle$, $\langle 1,2,19\rangle'$, $\langle 1,2,3,16\rangle^*$, $\langle 1,2,4,15\rangle^*$, $\langle 1,2,5,14\rangle^*$, $\langle 1,2,6,13\rangle^*$,
$\langle 1,2,7,12\rangle^*$, $\langle 1,2,8,11\rangle^*$,$\langle 1,2,9,10\rangle^*$.
The principle block $j_1$contains the remaining projective characters.

4-The principle indecomposable spin(projective) characters for the block j_2 of defect one

From preliminaries (7,3) allirreducible modular spin characters I.m.s. for the block $\boldsymbol{j_2}$ are double and $\langle\omega\rangle = \langle\omega\rangle'$on (19,$\omega$)-regular classes respectively.

Theorem(4.1): The required characters of S_{22}arex_1,x_2, x_3 , x_4 , x_5, x_6,x_7,x_8,x_9.

Proof: Through technique and the method $(r,\bar{r})$-inducing ofprinciple indecomposable spin charactersP.i.s. of $S_{21}, p = 19$(see creek *) to S_{22} we have
$y_1 \uparrow^{(2,18)} S_{22} = x_1$, $y_2 \uparrow^{(1,0)} S_{22} = x_2$, $y_3 \uparrow^{(17,3)} S_{22} = x_3$,
$y_4 \uparrow^{(15,5)} S_{22} = x_4$, $y_5 \uparrow^{(14,6)} S_{22} = x_5$, $y_6 \uparrow^{(12,8)} S_{22} = x_6$,
$y_7 \uparrow^{(12,8)} S_{22} = x_7$, $y_8 \uparrow^{(13,7)} S_{22} = x_8$, $y_9 \uparrow^{(12,8)} S_{22} = x_9$.

Issue(4.1.1):$x_3$is not subtracted from x_1
Suppose x_3 is subtracted from x_1, then x_3-x_1=$\langle 1,21\rangle^*+2\langle 2,20\rangle^*$,and we have $2\langle 1,21\rangle^*-\langle 2,20\rangle^*+\langle 1,2,19\rangle$
Is m.s for S_{22} , but $(2\langle 1,21\rangle^* - \langle 2,20\rangle^* + \langle 1,2,19\rangle)_{\downarrow}S_{21} = \langle 1,2,18\rangle^*$- $\langle 21\rangle^*$ is not m.s for S_{21}, so, x_3 is not subtracted from x_1.

Issue(4.1.2):½ $x_2$is not P.i.sfor S_{22}
Suppose ½ x_2 is P.i.s for S_{22} , then $\langle 1,21\rangle^*$- $\langle 2,20\rangle^*$+ $\langle 1,2,19\rangle$
Is m.s for S_{22} , but $(\langle 1,21\rangle^* - \langle 2,20\rangle^* + \langle 1,2,19\rangle)_{\downarrow}S_{21} = \langle 1,2,18\rangle^*$- $\langle 21\rangle^*$ is not m.s for S_{21}, so, ½ x_2 is not P.i.s for S_{22}.

Issue(4.1.3):x_3 is not subtracted from x_2

Suppose x_3 is subtracted from x_2, then x_3- x_2=2 $\langle 2,20\rangle^*\langle 1,2,19\rangle + \langle 1,2,19\rangle'$and we have 2 $\langle 1,21\rangle^*$- $\langle 2,20\rangle^*$+ $\langle 1,2,19\rangle$
Is m.s for S_{22} , but $(2\,\langle 1,21\rangle^* - \langle 2,20\rangle^* + \langle 1,2,19\rangle)_{\downarrow}S_{21} = \langle 1,2,18\rangle^*$- $\langle 21\rangle^*$ is not m.s for S_{21}, so x_3 is not subtracted from x_2.
So, the characters required for this block are given as:

x_1 =$\langle 1,21\rangle^*$+2$\langle 2,20\rangle^*$+$\langle 1,2,19\rangle + \langle 1,2,19\rangle'$, x_4=$\langle 1,2,4,15\rangle^* + \langle 1,2,5,14\rangle^*$,
x_2 =2 $\langle 2,20\rangle^*$+2$\langle 1,2,19\rangle$ + 2$\langle 1,2,19\rangle'$,x_5=$\langle 1,2,4,15\rangle^* + \langle 1,2,5,14\rangle^*$
x_3 =$\langle 1,2,19\rangle + \langle 1,2,19\rangle'$,x_6=$\langle 1,2,5,14\rangle^*$+$\langle 1,2,6,13\rangle^*$
x_7 =$\langle 1,2,6,13\rangle^*$+$\langle 1,2,7,12\rangle^*$,x_8=$\langle 1,2,7,12\rangle^*$+$\langle 1,2,8,11\rangle^*$,$x_9$ =$\langle 1,2,8,11\rangle^*$+$\langle 1,2,9,10\rangle^*$.

5-The principle indecomposable spin characters for the principle block$\boldsymbol{j_1}$ of defect one

From preliminaries (7,3) allirreducible modular spin characters I.m.s. for the block $\boldsymbol{j_1}$ are associate and $\langle\omega\rangle \neq \langle\omega\rangle'$ on (19,ω)-regular classes respectively.

Theorem(5.1): The required characters for this block are x_{10} , x_{11},…, x_{27}

Proof: Through technique and the method $(r,\bar{r})$-inducing of P.i.s. of $S_{21}, p = 19$ (see creek *) to S_{22}, $\langle\omega\rangle \neq \langle\omega\rangle'$ on (19,ω)-regular classes and rudiments 5 we have:
$y_1 \uparrow^{(2,18)} S_{22}$ =x_{10} , $y_1 \uparrow^{(1,0)} S_{22}$=$x_{11}$, $y_2 \uparrow^{(1,0)} S_{22}$=$x_{12}$,$y_2 \uparrow^{(1,0)} S_{22}$=$x_{13}$,$y_3 \uparrow^{(17,3)} S_{22}$=$x_{14}$
$y_3 \uparrow^{(17,3)} S_{22}$=$x_{15}$, $y_4 \uparrow^{(17,3)} S_{22}$=$x_{16}$, $y_4 \uparrow^{(17,3)} S_{22}$=$x_{17}$, $y_5 \uparrow^{(17,3)} S_{22}$=$x_{18}$, $y_5 \uparrow^{(17,3)} S_{22}$=$x_{19}$
$y_6 \uparrow^{(17,3)} S_{22}$=$x_{20}$, $y_6 \uparrow^{(17,3)} S_{22}$=$x_{21}$, $y_7 \uparrow^{(17,3)} S_{22}$=$x_{22}$, $y_7 \uparrow^{(17,3)} S_{22}$=$x_{23}$, $y_8 \uparrow^{(17,3)} S_{22}$=$x_{24}$
$y_8 \uparrow^{(17,3)} S_{22}$=$x_{25}$, $y_9 \uparrow^{(17,3)} S_{22}$=$x_{26}$, $y_9 \uparrow^{(17,3)} S_{22}$=$x_{27}$.
So, the characters required for this block are :

x_{10} =$\langle 22\rangle + \langle 3,19\rangle^*$, x_{11} =$\langle 22\rangle' + \langle 3,19\rangle^*$, x_{12} =$\langle 3,19\rangle^* + \langle 1,3,18\rangle$, x_{13} =$\langle 3,19\rangle^* + \langle 1,3,18\rangle'$
x_{14} =$\langle 1,3,18\rangle + \langle 2,3,17\rangle$, x_{15} =$\langle 1,3,18\rangle' + \langle 2,3,17\rangle'$, x_{16} =$\langle 2,3,17\rangle + \langle 3,4,15\rangle$, x_{17} =$\langle 2,3,17\rangle' + \langle 3,4,15\rangle'$
x_{18} =$\langle 3,4,15\rangle + \langle 3,5,14\rangle$, x_{19} =$\langle 3,4,15\rangle' + \langle 3,5,14\rangle'$, x_{20} =$\langle 3,4,14\rangle + \langle 3,6,13\rangle$, x_{21} =$\langle 3,4,14\rangle' + \langle 3,6,13\rangle'$
x_{22} =$\langle 3,6,13\rangle + \langle 3,7,12\rangle$, x_{23} =$\langle 3,6,13\rangle' + \langle 3,7,12\rangle'$, x_{24} =$\langle 3,7,12\rangle + \langle 3,8,11\rangle$, x_{25} =$\langle 3,7,12\rangle' + \langle 3,8,11\rangle'$
x_{26} =$\langle 3,8,11\rangle + \langle 3,9,10\rangle$, x_{27} =$\langle 3,8,11\rangle' + \langle 3,9,10\rangle'$

Creek(*)

The grade of the projective characters	The projective characters	$H^{1}_{21,19}$								
1024	$\langle 21\rangle^{*}$	1								
193536	$\langle 19,2\rangle$	1	1							
193536	$\langle 19,2\rangle'$	1	1							
487424	$\langle 18,2,1\rangle^{*}$		1	1						
62899200	$\langle 16,3,2\rangle^{*}$			1	1					
253338624	$\langle 15,4,2\rangle^{*}$				1	1				
684343296	$\langle 14,5,2\rangle^{*}$					1	1			
1316044800	$\langle 13,6,2\rangle^{*}$						1	1		
1809561600	$\langle 12,7,2\rangle^{*}$							1	1	
1663334400	$\langle 11,8,2\rangle^{*}$								1	1
684343296	$\langle 10,9,2\rangle^{*}$									1
		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9

Conclusion and Future work

We characterize allprinciple indecomposable spin characters P.i.s. for S_{22} modulo $p = 19$ by method $(r,\bar{r})$-inducing.Our proof strongly depends on theorems (4.1) and (5.1). In future work we can find the principle indecomposablespin(projective) characters for the symmetric group S_{23} modulo $p = 19$.

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