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The irreducible modular projective characters of the symmetric groups S_{21} modulo $p = 19$

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Abstract In this paper we invention all irreducible modular spin(projective) characters of the symmetric group S_n , when $n = 12$ and the characteristic of the field =19.

Key words spin(projective) characters , modular characters , decomposition matrix , AMS 2010,15C15,15C20,15C25.

1-Introduction

The decomposition matrix for the projective characters is appear the link between the irreducible projective characters and irreducible modular projective characters[1] , then when we find this matrix as amounting to find all irreducible modular projective characters. Characters is known modular or ordinary when the characteristic of the field is prime or zero[2]. Every finite group has covering group[3] ,then S_n has like this group. The characters of the covering group which are identical the characters of S_n are called modular or ordinary characters of S_n ,the rest characters are called projective(spin) of S_n [4]. Finding the decomposition matrix for the projective characters will become more difficult when n increasing ,and there is no general method to find this matrix[4]. Many Mathematicians work in this field like Adul Kareem A.Yaseen, Saeed Abdul-Ameer Taban ,Ahmed Hussein Jassim and Marwa Mohammed Jawad [5],[6],[7],[8]. In this paper decomposition matrices S_{21} modulo $p = 19$ have been calculated by using $(r, \bar{r})$ -inducing method, we induce the principal indecomposable characters(P.i.s)of S_{20} (see creek*) to have (P.i.s) or principal characters (P.s) of S_{21}.

2-Rudiments

1-The spin characters of S_n can be written as a linear combination , with non-negative integer coefficients , of the irreducible spin characters[8].

2- Projective characters $\langle\gamma\rangle = \langle\gamma_1, \ldots, \gamma_l\rangle$ have degree which is equal $2^{\left[\frac{o-l}{2}\right]}\frac{o!}{\prod_{i=1}^{l}(\gamma_i!)}\prod_{1\le i<j\le l}\frac{(\gamma_i-\gamma_j)}{(\gamma_i+\gamma_j)}$ [9].

3-The values of associate characters $\langle\gamma\rangle, \langle\gamma\rangle'$ are same on the class except on the class corresponding to γ they have values $\pm i^{\frac{o-l+1}{2}}\sqrt{\left(\frac{\gamma_1 \ldots \gamma_l}{2}\right)}$ [9].

4-The inducing from group or restriction from the subgroup of the projective characters are also projective characters [1].

5-If o is odd and $p \nmid (o-1)$,then $\langle o-1,1\rangle$ and $\langle o-1,1\rangle'$ are distinct I.m.s of grade $2^{\left[\frac{(o-3)}{2}\right]} \times (o-2)$ which are denoted by $\rho\langle o-1,1\rangle$ and $\rho\langle o-1,1\rangle'$ [9].

6- Let p be an odd prime and let σ, γ be a bar partition of o which are not p-bar core. Then $\langle\sigma\rangle$(and $\langle\sigma\rangle'$ if σ is odd) and $\langle\gamma\rangle$(and $\langle\gamma\rangle'$ if γ is odd) are in the same p-block$\leftrightarrow$ $\widetilde{\langle\sigma\rangle} = \widetilde{\langle\gamma\rangle}$.If α be a bar partition of o and $\langle\sigma\rangle$= $\widetilde{\langle\sigma\rangle}$, then $\langle\sigma\rangle$(and $\langle\sigma\rangle'$ if σ is odd) forms a p-block of defect 0 [4].

7- Let p be an odd prime and $\gamma = (\gamma_1, \dots \gamma_l)$ be a bar partition of o ,then all I.m.s in the block B are double(associate) , if $(o - l - m_0)$ is even(odd), where m_0 the number of parts of γ divisible by p [4].

8- If o is even and $p \nmid (o)$,then $\langle o \rangle$ and $\langle o \rangle'$ are distinct I.m.s of grade $2^{\left[\frac{(o-1)}{2}\right]}$ which are denoted by $\gamma\langle o \rangle$ and$\gamma\langle o \rangle'$ [9].

3-The spin block of S_{21}

The matrix required of the projective(spin) characters of $S_{21}, p = 19$ has 115 irreducible spin characters and 113 (19, α)-regular classes [10].

From preliminaries (6) ,there are 105 blocks of $S_{21}, p = 19$, theses blocks are $M_2, \dots, M_{104}$ of defect zero except the block M_1 of defect one.

The blocks of defect zero $M_2, \dots, M_{104}$ includes
$\langle 20,1\rangle, \langle 20,1\rangle', \langle 18,3\rangle, \langle 18,3\rangle', \langle 17,4\rangle, \langle 17,4\rangle', \langle 17,3,1\rangle^*, \langle 16,5\rangle, \langle 16,5\rangle', \langle 16,4,1\rangle^*,$

$\langle 15,6\rangle, \langle 15,6\rangle', \langle 15,5,1\rangle^*, \langle 15,3,2,1\rangle, \langle 15,3,2,1\rangle', \langle 14,7\rangle, \langle 14,7\rangle', \langle 14,6,1\rangle^*, \langle 14,4,3\rangle^*,$

$\langle 14,4,2,1\rangle, \langle 14,4,2,1\rangle', \langle 13,8\rangle, \langle 13,8\rangle', \langle 13,7,1\rangle^*, \langle 13,6,2\rangle^*, \langle 13,5,3\rangle^*, \langle 13,5,2,1\rangle, \langle 13,5,2,1\rangle',$

$\langle 13,4,3,1\rangle, \langle 13,4,3,1\rangle', \langle 12,9\rangle, \langle 12,9\rangle', \langle 12,8,1\rangle^*, \langle 12,6,3\rangle^*, \langle 12,6,2,1\rangle, \langle 12,6,2,1\rangle', \langle 12,5,4\rangle^*,$

$\langle 12,5,3,1\rangle, \langle 12,5,3,1\rangle', \langle 12,4,3,2\rangle, \langle 12,4,3,2\rangle', \langle 11,10\rangle, \langle 11,10\rangle', \langle 11,9,1\rangle^*, \langle 11,7,3\rangle^*,$

$\langle 11,7,2,1\rangle, \langle 11,7,2,1\rangle', \langle 11,6,4\rangle^*, \langle 11,6,3,1\rangle, \langle 11,6,3,1\rangle', \langle 11,5,4,1\rangle, \langle 11,5,4,1\rangle', \langle 11,5,3,2\rangle, \langle 11,5,3,2\rangle',$

$\langle 11,4,3,2,1\rangle^*, \langle 10,9,2\rangle^*, \langle 10,8,3\rangle^*, \langle 10,8,2,1\rangle, \langle 10,8,2,1\rangle', \langle 10,7,4\rangle^*, \langle 10,7,3,1\rangle, \langle 10,7,3,1\rangle',$

$\langle 10,6,5\rangle^*, \langle 10,6,4,1\rangle, \langle 10,6,4,1\rangle', \langle 10,6,3,2\rangle, \langle 10,6,3,2\rangle', \langle 10,5,4,2\rangle, \langle 10,5,4,2\rangle', \langle 10,5,3,2,1\rangle^*,$

$\langle 9,8,4\rangle^*, \langle 9,8,3,1\rangle, \langle 9,8,3,1\rangle', \langle 9,7,5\rangle^*, \langle 9,7,4,1\rangle, \langle 9,7,4,1\rangle', \langle 9,7,3,2\rangle, \langle 9,7,3,2\rangle',$

$\langle 9,6,5,1\rangle, \langle 9,6,5,1\rangle', \langle 9,6,4,2\rangle, \langle 9,6,4,2\rangle', \langle 9,6,3,2,1\rangle^*, \langle 9,5,4,3\rangle, \langle 9,5,4,3\rangle', \langle 9,5,4,2,1\rangle^*, \langle 8,7,6\rangle^*,$

$\langle 8,7,5,1\rangle, \langle 8,7,5,1\rangle', \langle 8,7,4,2\rangle, \langle 8,7,4,2\rangle', \langle 8,7,3,2,1\rangle^*, \langle 8,6,5,2\rangle, \langle 8,6,5,2\rangle', \langle 8,6,4,3\rangle, \langle 8,6,4,3\rangle',$

$\langle 8,6,4,2,1\rangle^*, \langle 8,5,4,3,1\rangle^*, \langle 7,6,5,3\rangle, \langle 7,6,5,3\rangle', \langle 7,6,5,2,1\rangle^* \ \langle 7,6,4,3,1\rangle, \langle 7,6,4,3,1\rangle', \langle 7,5,4,3,2\rangle^*,$

$\langle 6,5,4,3,2,1\rangle, \langle 6,5,4,3,2,1\rangle'$ .respectively ,these characters are irreducible modular spin character (preliminaries 6).The principle block M_1contains the remaining projective characters.

4-The decomposition matrix for the block M_1 of defect one

From preliminaries (7,3) all I.m.s. for the block B_1 are double and $\langle \alpha \rangle = \langle \alpha \rangle'$ on (19, α)-regular classes respectively.

Theorem(4.1):

The matrix required of the projective(spin) characters of S_{21} is

$H_{19,19} = H_{21,19}^{(1)} \oplus \dots \oplus H_{21,19}^{(104)}$

Proof:

Through technique and the method $(r, \bar{r})$-inducing of P.i.s. of $S_{20}, p = 19$ (see creek *) to S_{21} we have

$z_1 \uparrow^{(1,0)} S_{20} = y_1$, $z_3 \uparrow^{(1,0)} S_{20} = y_2$, $z_5 \uparrow^{(17,3)} S_{20} = y_3$,

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$z_7 \uparrow^{(16,4)} S_{20}=y_4$, $z_9 \uparrow^{(1,0)} S_{20}=y_5$, $z_{11} \uparrow^{(1,0)} S_{20}=y_6$,

$z_{13} \uparrow^{(1,0)} S_{20}=y_7$, $z_{15} \uparrow^{(1,0)} S_{20}=y_8$, $z_{17} \uparrow^{(1,0)} S_{20}=y_9$.

So, the matrix required for this block is as given in creek(1).

Creek(1)

The grade of the projective characters	The projective characters	$H^1_{21,19}$								
1024	$\langle 21\rangle^*$	1								
193536	$\langle 19,2\rangle$	1	1							
193536	$\langle 19,2\rangle'$	1	1							
487424	$\langle 18,2,1\rangle^*$		1	1						
62899200	$\langle 16,3,2\rangle^*$			1	1					
253338624	$\langle 15,4,2\rangle^*$				1	1				
684343296	$\langle 14,5,2\rangle^*$					1	1			
1316044800	$\langle 13,6,2\rangle^*$						1	1		
1809561600	$\langle 12,7,2\rangle^*$							1	1	
1663334400	$\langle 11,8,2\rangle^*$								1	1
684343296	$\langle 10,9,2\rangle^*$									1
		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9

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Creek(*)

The grade of the projective character	The projective characters	$D^{1}_{20,19}$																	
512	$\langle 20\rangle$	1																	
512	$\langle 20\rangle'$		1																
9216	$\langle 19,1\rangle^*$	1	1	1	1														
204800	$\langle 17,2,1\rangle$			1		1													
204800	$\langle 17,2,1\rangle'$				1		1												
1497600	$\langle 16,3,1\rangle$					1		1											
1497600	$\langle 16,3,1\rangle'$						1		1										
6031872	$\langle 15,4,1\rangle$							1		1									
6031872	$\langle 15,4,1\rangle'$								1		1								
16293888	$\langle 14,5,1\rangle$									1		1							
16293888	$\langle 14,5,1\rangle'$										1		1						
31334400	$\langle 13,6,1\rangle$											1		1					
31334400	$\langle 13,6,1\rangle'$												1		1				
43084800	$\langle 12,7,1\rangle$													1		1			
43084800	$\langle 12,7,1\rangle'$														1		1		
39603200	$\langle 11,8,1\rangle$															1		1	
39603200	$\langle 11,8,1\rangle'$																1		1
16293888	$\langle 10,9,1\rangle$																	1	
16293888	$\langle 10,9,1\rangle'$																		1
		z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}	z_{16}	z_{17}	z_{18}

Future work

We can find the irreducible modular projective characters or indecomposable principal characters for the symmetric group S_{21} modulo $p = 19$.

المستخلص

في هذا البحث تم أيجاد المشخصات الاسقاطية للزمر التناظرية S_{21} معيار19

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