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Decomposition numbers of the decomposition matrices for the projective characters of the symmetric groups S_{19}, S_{20} modulo $p = 19$

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Decomposition numbers of the decomposition matrices for the projective characters of the symmetric groups S_{19}, S_{20} modulo $p = 19$

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Abstract In this paper we has determined the decomposition matrices of the symmetric group S_{19}, S_{20} modulo $p = 19$,which is equivalent determined the irreducible modular spin(projective) characters.
Key words spin(projective) characters , modular characters , decomposition matrix , AMS 2010,15C15,15C20,15C25.

1- Introduction

Characters of the finite group is recognized modular when the characteristic of the field is p>0, and is known ordinary when the characteristic of the field is zero[1].Every finite group has a covering group (representation group),the symmetric group S_n has like that group and the number of the elements of this group is $2(n!)$,when $n \geq 4$[2].The characters for the covering group either corresponding the characters for S_n or not , when these characters are not corresponding to the characters for S_n ,these characters are known projective(spin) characters ,also the projective characters is known modular or ordinary corresponding to the characteristic of the filed p [3]. The characters for S_n which is described by bar partition of n are irreducible spin characters [3]. In this paper decomposition matrices S_{19} , S_{20} modulo $p = 19$ have been calculated by using $(r, \bar{r})$-inducing method and technique for finding decomposition matrices , where every spin(projective) characters of S_{18} modulo $p = 19$ are located in blocks of defect zero , that is mean theses blocks contains exactly one irreducible projective(spin)character (I.s.),one irreducible modular projective(spin)character (I.m.s.) and one principal indecomposable character(P.i.s.) all these characters(I.s ,I.m.s, P.i.s) are equals.

2- Rudiments

1-The spin characters of S_n can be written as a linear combination , with non-negative integer coefficients , of the irreducible spin characters[4].

2- Projective characters $\langle\beta\rangle = \langle\beta_1, \dots, \beta_k\rangle$ have degree which is equal $2^{\left[\frac{k-l}{2}\right]} \frac{k!}{\prod_{i=1}^{l}(\alpha_i!)} \prod_{1\leq i<j\leq m} \frac{(\alpha_i-\alpha_j)}{(\alpha_i+\alpha_j)}$ [5].

3-The values of associate characters $\langle\beta\rangle, \langle\beta\rangle'$ are same on the class except on the class corresponding to α they have values $\pm i^{\frac{k-l+1}{2}} \sqrt{\left(\frac{\beta_1 \dots \beta_l}{2}\right)}$ [5].

4-Let H be a subgroup of S_n ,if θ is a projective character of S_n (H) ,then the restriction (inducing)of θ is a spin character of H (S_n) [6].

5-If n is odd and $p \nmid (n-1)$,then $\langle n-1,1\rangle$ and $\langle n-1,1\rangle'$ are distinct I.m.s of grade $2^{\left[\frac{(k-3)}{2}\right]} \times (k-2)$ which are denoted by $\gamma\langle k-1,1\rangle$ and $\gamma\langle k-1,1\rangle'$ [5].

6- Let p be an odd prime and let α, β be a bar partition of n which are not p-bar core. Then $\langle\sigma\rangle$(and $\langle\sigma\rangle'$ if α is odd) and $\langle\beta\rangle$(and $\langle\beta\rangle'$ if β is odd) are in the same p-block$\leftrightarrow$ $\widetilde{\langle\sigma\rangle} = \widetilde{\langle\beta\rangle}$.If α be a bar partition of k and $\langle\sigma\rangle = \widetilde{\langle\sigma\rangle}$, then $\langle\sigma\rangle$(and $\langle\sigma\rangle'$ if α is odd) forms a p-block of defect 0 [3].
7- Let p be an odd prime and $\beta = (\beta_1, \dots \beta_l)$ be a bar partition of n ,then all I.m.s in the block B are double(associate) , if $(k - l - m_0)$ is even(odd), where m_0 the number of parts of β divisible by p [3].
8- If k is even and $p \nmid (k)$,then $\langle k\rangle$ and $\langle k\rangle'$ are distinct I.m.s of grade $2^{\left[\frac{(n-1)}{2}\right]}$ which are denoted by $\gamma\langle k\rangle$ and$\gamma\langle k\rangle'$ [5].

3- The spin block of S_{19}

The matrix required of the projective(spin) characters of $S_{19}, p = 19$ has 80 rows and 79 columns since the group S_{19} has 80 irreducible spin characters and 80 (19, α)-regular classes respectively [8]. From preliminaries (6) ,there are 62 blocks of $S_{19}, p = 19$, theses blocks are $B_1, \dots, B_{62}$ of defect zero except the block B_1 of defect one.
The blocks of defect zero $B_2, \dots, B_{62}$ includes
$\langle 16,2,1\rangle^*, \langle 15,3,1\rangle^* \langle 14,4,1\rangle^*, \langle 14,3,2\rangle^* \langle 13,5,1\rangle^*, \langle 13,4,2\rangle^*, \langle 13,3,2,1\rangle, \langle 13,3,2,1\rangle'$,
$\langle 12,6,1\rangle^*, \langle 12,5,2\rangle^*, \langle 12,4,3\rangle^* \langle 12,4,2,1\rangle, \langle 12,4,2,1\rangle', \langle 11,7,1\rangle^*, \langle 11,6,2\rangle^*, \langle 11,5,3\rangle^*$,
$\langle 11,5,2,1\rangle, \langle 11,5,2,1\rangle', \langle 11,4,3,1\rangle, \langle 11,4,3,1\rangle'$, $\langle 10,8,1\rangle^*, \langle 10,7,2\rangle^*, \langle 10,6,3\rangle^*$,
$\langle 10,6,2,1\rangle, \langle 10,6,2,1\rangle', \langle 10,5,4\rangle^*, \langle 10,5,3,1\rangle, \langle 10,5,3,1\rangle', \langle 10,4,3,2\rangle, \langle 10,4,3,2\rangle'$,
$\langle 9,7,3\rangle^*, \langle 9,7,2,1\rangle, \langle 9,7,2,1\rangle', \langle 9,6,4\rangle^*, \langle 9,6,3,1\rangle, \langle 9,6,3,1\rangle', \langle 9,5,4,1\rangle, \langle 9,5,4,1\rangle', \langle 9,5,3,2\rangle$
, $\langle 9,5,3,2\rangle'$, , $\langle 9,4,3,2,1\rangle^*, \langle 8,7,4\rangle^*, \langle 8,7,3,1\rangle, \langle 8,7,3,1\rangle', \langle 8,6,5\rangle^*, \langle 8,6,4,1\rangle, \langle 8,6,4,1\rangle'$,
$\langle 8,6,3,2\rangle, \langle 8,6,3,2\rangle', \langle 8,5,4,2\rangle', \langle 8,5,4,2\rangle', \langle 8,5,3,2,1\rangle^*, \langle 7,6,5,1\rangle, \langle 7,6,5,1\rangle', \langle 7,6,4,2\rangle$,
$\langle 7,6,4,2\rangle', \langle 7,6,3,2,1\rangle^*, \langle 7,5,4,3\rangle$, , $\langle 7,5,4,3\rangle', \langle 7,5,4,2,1\rangle^*, \langle 6,5,4,3,1\rangle^*$
respectively ,these characters are irreducible modular spin character (preliminaries 6).The principle block B_1contains the remaining projective characters.

4- The decomposition matrix for the block B_1 of defect one

From preliminaries (7,3) all I.m.s. for the block B_1 are associate and $\langle\alpha\rangle \neq \langle\alpha\rangle'$ on $(19, \alpha)$-regular classes respectively.

Theorem(4.1):
The matrix required of the projective(spin) characters of S_{19} is
$D_{19,19} = D_{19,19}^{(1)} \oplus \dots \oplus D_{19,19}^{(62)}$
Proof:
Through technique and the method $(r, \bar{r})$-inducing of P.i.s. of $S_{18}, p = 19$(all blocks of decomposition matrix of $S_{18}, p = 19$ of defect 0) to S_{19} we have
$\langle 18\rangle \uparrow^{(18,2)} S_{19} = t_1$, $\langle 18\rangle' \uparrow^{(18,2)} S_{19} = t_2$, $\langle 17,1\rangle^* \uparrow^{(17,3)} S_{19} = k_3$, $\langle 16,2\rangle^* \uparrow^{(16,4)} S_{19} = k_4$, $\langle 15,3\rangle^* \uparrow^{(15,5)} S_{19} = k_5$, $\langle 14,4\rangle^* \uparrow^{(14,6)} S_{19} = k_6$, $\langle 13,5\rangle^* \uparrow^{(13,7)} S_{19} = k_7$, $\langle 12,6\rangle^* \uparrow^{(12,8)} S_{19} = k_8$, $\langle 11,7\rangle^* \uparrow^{(119)} S_{19} = k_9$, $\langle 10,8\rangle^* \uparrow^{(10,10)} S_{19} = k_{10}$.
From the (preliminaries 5) we have k_3 must be split to t_3 and t_4 .
Now since each $\langle 16,3\rangle \neq \langle 16,3\rangle'$, $\langle 15,4\rangle \neq \langle 15,4\rangle'$, $\langle 14,5\rangle \neq \langle 14,5\rangle'$, $\langle 13,6\rangle \neq \langle 13,6\rangle'$, $\langle 12,7\rangle \neq \langle 12,7\rangle'$, $\langle 11,8\rangle \neq \langle 11,8\rangle'$, $\langle 10,9\rangle \neq \langle 10,9\rangle'$ on $(19, \alpha)$-regular classes ,then k_4 should be divided to t_5 and t_6 ,k_5 must be split to t_7 and t_8 , k_6 must be split to t_9 and t_{10} ,k_7 must be split to t_{11} and t_{12}, k_8 must be split to t_{13} and t_{14},k_9 must be split to t_{15} and t_{16} ,k_{10} must be split to t_{17} and t_{18}, respectively (preliminaries 5).
So, the matrix required for this block is as given in creek(1).

The grade of the projective characters	The projective characters	$D^1_{19,19}$																	
512	$\langle 19\rangle^*$	1	1																
4352	$\langle 18,1\rangle$	1		1															
4352	$\langle 18,1\rangle'$		1		1														
34560	$\langle 17,2\rangle$			1		1													
34560	$\langle 17,2\rangle'$				1		1												
169728	$\langle 16,3\rangle$					1		1											
169728	$\langle 16,3\rangle'$						1		1										
574464	$\langle 15,4\rangle$							1		1									
574464	$\langle 15,4\rangle'$								1		1								
1410048	$\langle 14,5\rangle$									1		1							
1410048	$\langle 14,5\rangle'$										1		1						
2558976	$\langle 13,6\rangle$											1		1					
2558976	$\langle 13,6\rangle'$												1		1				
3394560	$\langle 12,7\rangle$													1		1			
3394560	$\langle 12,7\rangle'$														1		1		
3055104	$\langle 11,8\rangle$															1		1	
3055104	$\langle 11,8\rangle'$																1		1
1244672	$\langle 10,9\rangle$																	1	
1244672	$\langle 10,9\rangle'$																		1
		t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}

Creek(1)
is the matrix for this block

5- The spin block of S_{20}

The matrix required of the projective characters of $S_{20}, p = 19$ has 96 rows and 95 columns since the group S_{20} has 80 irreducible spin characters and 80 (19, α)-regular classes respectively [8]. From preliminaries (6) ,there are 77 blocks of $S_{20}, p = 19$, theses blocks are $B_1, \dots, B_{77}$ of defect zero except the block B_1 of defect one.
The blocks of defect zero $B_2, \dots, B_{77}$ includes
$\langle 18,2\rangle^*, \langle 17,3\rangle^* \langle 16,4\rangle^*, \langle 15,5\rangle^*, \langle 15,3,2\rangle, \langle 15,3,2\rangle', \langle 14,6\rangle^*, \langle 14,4,2\rangle, \langle 14,4,2\rangle', \langle 14,3,2,1\rangle^*, \langle 13,7\rangle^*, \langle 13,5,2\rangle, \langle 13,5,2\rangle', \langle 13,4,3\rangle, \langle 13,4,3\rangle', \langle 13,4,2,1\rangle^*, \langle 12,8\rangle^*, \langle 12,6,2\rangle, \langle 12,6,2\rangle', \langle 12,5,3\rangle, \langle 12,5,3\rangle', \langle 12,5,2,1\rangle^*, \langle 12,4,3,1\rangle^*, \langle 11,9\rangle^*, \langle 11,7,2\rangle', \langle 11,7,2\rangle', \langle 11,6,3\rangle, \langle 11,6,3\rangle', \langle 11,6,2,1\rangle^*, \langle 11,5,4\rangle, \langle 11,5,4\rangle', \langle 11,5,3,1\rangle^*, \langle 11,4,3,2\rangle^*, \langle 10,8,2\rangle', \langle 10,8,2\rangle', \langle 10,7,3\rangle, \langle 10,7,3\rangle', \langle 10,7,2,1\rangle^*, \langle 10,6,4\rangle, \langle 10,6,4\rangle', \langle 10,6,3,1\rangle^*, \langle 10,5,4,1\rangle^*, \langle 10,5,3,2\rangle^*, \langle 10,4,3,2,1\rangle', \langle 10,4,3,2,1\rangle', \langle 9,8,3\rangle, \langle 9,8,3\rangle', \langle 9,8,2,1\rangle^*, \langle 9,7,4\rangle', \langle 9,7,4\rangle', \langle 9,7,3,1\rangle^*, \langle 9,6,5\rangle, \langle 9,6,5\rangle', \langle 9,6,4,1\rangle^*, \langle 9,6,3,2\rangle^*, \langle 9,5,4,2\rangle^*, \langle 9,5,3,2,1\rangle, \langle 9,5,3,2,1\rangle', \langle 8,7,5\rangle, \langle 8,7,5\rangle', \langle 8,7,4,1\rangle^*, \langle 8,7,3,2\rangle^*, \langle 8,6,5,1\rangle^*,, \langle 8,6,4,2\rangle^*, \langle 8,6,3,2,1\rangle', \langle 8,6,3,2,1\rangle' \langle 8,5,4,3\rangle^*, \langle 8,5,4,2,1\rangle', \langle 8,5,4,2,1\rangle', \langle 7,6,5,2\rangle^*, \langle 7,6,4,3\rangle^*, \langle 7,6,4,2,1\rangle', \langle 7,6,4,2,1\rangle', \langle 7,5,4,3,1\rangle, \langle 7,5,4,3,1\rangle', \langle 6,5,4,3,2\rangle', \langle 6,5,4,3,2\rangle'.,$
respectively ,these characters are I.m.s.(preliminaries 6).The principle block B_1contains the remaining projective characters.

6- The decomposition matrix for the block B_1 of defect one

From preliminaries (7,3) all I.m.s. for the block B_1 are associate and $\langle\alpha\rangle \neq \langle\alpha\rangle'$ on (19, α)-regular classes respectively.

Theorem(6.1):

The matrix required of the projective characters of S_{19} is

$D_{20,19} = D_{20,19}^{(1)} \oplus \ldots \oplus D_{20,19}^{(77)}$

Proof:

Through technique and the method $(r,\bar{r})$-inducing of P.i.s. of $S_{18}, p = 19$(all blocks of decomposition matrix of $S_{18}, p = 19$ of defect 0) to S_{19} we have

$t_1 \uparrow^{(0,1)} S_{20} = k_1$, $t_3 \uparrow^{(17,3)} S_{20} = z_3$, $t_4 \uparrow^{(17,3)} S_{20} = z_4$, $t_5 \uparrow^{(16,4)} S_{20} = z_5$, $t_6 \uparrow^{(16,4)} S_{20} = z_6$, $t_7 \uparrow^{(15,5)} S_{20} = z_7$, $t_8 \uparrow^{(15,5)} S_{20} = z_8$ $t_9 \uparrow^{(14,6)} S_{20} = z_9$,$t_{10} \uparrow^{(14,6)} S_{20} = z_{10}$,$t_{11} \uparrow^{(13,7)} S_{20} = z_{11}$, $t_{12} \uparrow^{(13,7)} S_{20} = z_{12}$, $t_{13} \uparrow^{(12,8)} S_{20} = z_{13}$, $t_{14} \uparrow^{(12,8)} S_{20} = z_{14}$, $t_{15} \uparrow^{(11,9)} S_{20} = z_{15}$, $t_{16} \uparrow^{(11,9)} S_{20} = z_{16}$, $t_{17} \uparrow^{(10,10)} S_{20} = z_{17}$, $t_{18} \uparrow^{(10,10)} S_{20} = z_{18}$.

From the (preliminaries 8) we have k_1 must be split to z_1 and z_2.

Highly, the matrix required for this block is as given in creek(2).

The grade of the projective character	The projective characters	$D_{20,19}^{1}$																	
512	⟨20⟩	1																	
512	⟨20⟩′		1																
9216	⟨19,1⟩*	1	1	1	1														
204800	⟨17,2,1⟩			1		1													
204800	⟨17,2,1⟩′				1		1												
1497600	⟨16,3,1⟩					1		1											
1497600	⟨16,3,1⟩′						1		1										
6031872	⟨15,4,1⟩							1		1									
6031872	⟨15,4,1⟩′								1		1								
16293888	⟨14,5,1⟩									1		1							
16293888	⟨14,5,1⟩′										1		1						
31334400	⟨13,6,1⟩											1		1					
31334400	⟨13,6,1⟩′												1		1				
43084800	⟨12,7,1⟩													1		1			
43084800	⟨12,7,1⟩′														1		1		
39603200	⟨11,8,1⟩															1		1	
39603200	⟨11,8,1⟩′																1		1
16293888	⟨10,9,1⟩																	1	
16293888	⟨10,9,1⟩′																		1
		z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}	z_{16}	z_{17}	z_{18}

Creek(2)

is the matrix for this block

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