

## Dimensional Analysis Approach for Coefficient of Transmission Over Stepped Mound Breakwater

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### Abstract

Breakwaters considering one of the most hydraulic structures that covered by the history. At the first, the waves action hits the stones that formed breakwater and by the time it's eroded and scattered in the sea, after that, the enhancement comes to made a breakwaters that have an equilibrium shape by filling the voids with more stones. Although, another type of breakwaters were built from rock fill or blocks to protect the shoreline, in this study, stepped mound breakwaters used to dissipate sea wave energy by suggesting a new dimensional group to predict the transmission coefficient. After Buckingham theory. Results show that the depth of water (d), height of incidence wave ( $H_i$ ), height of transmission wave ( $H_t$ ), length of wave (L), Width of the crest (W), Step width (Ls), Step height (hs), Number of steps (N) and Inclination angle of breakwater ( $\alpha$ ) are controlled factors to represent breaking wave behavior.

**Key words:** Dimensional Analysis, Breakwater, Sea Waves, Transmission Coefficient.

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### Introduction

In some countries, where huge rocks cannot be assemblage and accumulate without any problems, such as Japan and Korea the breakwaters become widespread. This type of breakwater has changed step by step from a simple rock fill breakwater throughout an intermediate stages such as concrete, cellular, and cyclopean blocks to reach the concrete breakwater [1].

Obtained theoretical solution for the transmission of waves over a fixed horizontal plate based upon linear wave theory, equation (1) represent the proposed transmission coefficient equation [16].

$$K_t = \frac{H_t}{H_i} = \frac{1}{\left[1 + \left(\frac{\pi B}{L}\right)^2\right]^{0.5}} \quad \text{Eq.1}$$

Where:  $K_t$  represent the transmission coefficient,  $H_t$  and  $H_i$  represent the transmitted wave height and the incident wave height, respectively, B represent the length of breakwater and L represent the wave length.

Experiments for submerged breakwater with water depth 2 feet and the range vary from 0.1 to 0.6 feet in depth of submergence [2]. When the transmission coefficient was plotted it was found that the  $K_t$  increase with increase in depth of submergence and its increase with increase in (h/d) ratio for a given value of (L/B) ratio.

Modified Stokers theory by inserting a non-dimensional constant as a correction factor as shown in equation (2) [3].

$$K_t = \frac{H_t}{H_i} = \alpha \left[ 1 + \left( \frac{\pi B}{L} \right)^2 \right]^{-0.5} \quad \text{Eq.2}$$

Based on experiments found that the constant  $\alpha$  is a function of  $H_i/L$  and  $h/d$ . [3]

U.S. Army Corps of Engineers, 1984, presented many empirical graphs and equations that derived from wave tank data, the experiments that included submerged breakwaters tests showed an indication about wave energy, where the less wave energy caused by the greater of submergence height. Transmission coefficients can be calculated from the proposed equation (3).

$$K_t = -0.4 \frac{R_c}{H_i} + 0.8 \left( \frac{B}{H_i} \right)^{-0.31} (1 - e^{-0.5 \epsilon}) \quad \text{Eq.3}$$

The behavior of low crested breakwaters by studying the experimental tests of U.S. Army Corps of Engineers, 1984 [4]. The researcher aimed to identify the stability of mound type breakwaters and the effect of this type on wave transmission coefficient, therefore an equation (4 and 5) proposed for wave transmission coefficient.

$$K_t = \frac{1}{1 + \left( \frac{H_i A_t}{L d_{50}^2} \right)^{0.592}} \quad \text{for } \frac{R_c}{H_i} < 1 \quad \text{Eq.4}$$

$$K_t = \frac{1}{1 + \left( \frac{h_c}{d} \right)^{c1} \left( \frac{A_t}{h L p} \right)^{c2} \exp \left[ c3 \left( \frac{R_c}{H_i} \right) + c4 \left( \frac{A_t^{3/2}}{L d_{50}^2} \right) \right]} \quad \text{for } \frac{R_c}{H_i} > 1 \quad \text{Eq.5}$$

Where:  $A_t$  represent cross section area of breakwater,  $d_{50}$  represent sieve dimension of stone and the constants  $C1=1.188$ ,  $C2=0.261$ ,  $C3=-0.592$  and  $C4=0.00551$ .

A different equation (6) with dimensionless groups including the permeability and freeboard characteristics [5], by reanalyzed the data of [4] experiments that belong to US Army Engineers. The hydraulic behavior of this equation is different usual breakwaters equations. This equation is applicable when the limits satisfied:

$$0.075 < K_t < 0.75, 1 < \frac{H_i}{d_{50}} < 6 \text{ and } 0.01 < \frac{2\pi H_i}{g T^2} < 0.05$$

$$K_t = D_1 \left( \frac{R_c}{d_{50}} \right) + D_2 \quad \text{Eq.6}$$

$$\text{Where: } D_1 = 0.031 \left( \frac{H_s}{d_{50}} \right) - 0.24 \text{ and}$$

$$D_2 = -5.24 \left( \frac{2\pi H_i}{g T^2} \right) + 0.0323 \left( \frac{H_i}{d_{50}} \right) - 0.0017 \left( \frac{B}{d_{50}} \right)^{1.84} + 0.51$$

[6], introduced modifications on the [5] equation with the same limits by reanalyzed the laboratory results from previous studies on wave transmission coefficient for low crested breakwater. Based on these results, the effect of mean rock materials diameter ( $d_{50}$ ) examined in different dimensionless groups as  $(R_c/d_{50})$ ,  $(H_i/d_{50})$ ,  $(B/d_{50})$ . The same linear relationship that proposed in equation (6) between transmission coefficient and  $R_c/d_{50}$ , is valid, this equation applied for permeable breakwaters with specified the size of its material.

submerged breakwater with permeable and impermeable stats, by considering the effects of crest width and side slope an on the wave transmission coefficient [7]. New modification sited up on equation of [5], for this purpose, equation (7) produced when the limit of  $K_t$  satisfied as:  $0.075 < K_t < 0.75$

$$K_t = a \left( \frac{R_c}{H_i} \right) + b \left( \frac{B}{H_i} \right)^c (1 - e^{-0.5 \xi}) \quad \text{Eq.7}$$

Where: the values of variables a, b and c are -0.4, 0.8 and -0.31 for impermeable breakwater, respectively, and values of variables a, b and c are -0.4, 0.64 and -0.31 for permeable breakwater, respectively.

[8], focused the experimental work on the submerged longitudinal breakwater, the submerged breakwater arranged with the longitudinal direction normal to the shore. The incident wave height, period and angle, with submergence depth, lateral spacing, crest width and length have been investigated with respect to the transmission coefficient and wave dissipation energy. The results show that the height of wave depends on the submerged depth of the crest and less effective by the crest length and width. Also, it's discovered that ratios of relative submergence  $\frac{R_c}{H_i}$  and relative lateral obstruction  $\frac{c_e}{B}$  were the controlled variables.

The equations (8 and 9) proposed by [8] with regression coefficient  $R^2$  equal to 0.728 and 0.77 for the transmission coefficient and wave dissipation energy, respectively. These equations showed a practical statistical concord for the tested data.

$$K_t = 0.51 + 0.21 \frac{R_c}{H_i} + 0.015 \frac{c_e}{B} \quad \text{Eq.8}$$

$$E = 80 - 33 \frac{R_c}{H_i} - 2.2 \frac{c_e}{B} \quad \text{Eq.9}$$

Where:  $0 < \frac{R_c}{H_i} < 2.7$  ,  $1.2 < \frac{c_e}{B} < 9.9$  and  $c_e$  represent the effective lateral spacing between breakwater units.

[9] thought that the equation presented by [7] can be applied in wide range of data set; therefore, the researchers suggested an adaptation on [7] equation with some enhancements. These enhancements came after merge the [7] data set with the data of laboratory studies, the equations (10 and 11) proposed for a wide range of data and applied only for impermeable submerged breakwater with taking into account the limits of Iribarren number and the limit of [7] equation:

$$K_t = -0.3 \left( \frac{R_c}{H_i} \right) + 0.75 (1 - e^{-0.5\xi}) \quad \xi < 3 \quad \text{Eq.10}$$

$$K_t = -0.3 \left( \frac{R_c}{H_i} \right) + 0.75 \left( \frac{B}{H_i} \right)^{-0.31} (1 - e^{-0.5\xi}) \quad \xi \geq 3 \quad \text{Eq.11}$$

Cleared that equation of (U.S. Army Corps of Engineers, 1984) can be improved with specifying the upper and lower limits of transmission coefficient as shown in equations (12 and 13) [10]. The upper limit equation of transmission coefficient stated that 0.9, 0.081 and 0.06 in regression coloration, root mean square of error and standard deviation, respectively. The lower limit equal to 0.08

$$K_t = -0.4 \frac{R_c}{H_i} + 0.51 \left( \frac{B}{H_i} \right)^{-0.65} (1 - e^{-0.41 \varepsilon}) \quad \text{Eq.12}$$

$$K_t \text{ upper limit} = 0.93 - 0.006(B/H_i) \quad \text{Eq.13}$$

[11], reconsidered in the application of equations that introduced by [7] equation and [9]. The results of reconsiderations showed that these equations can be applied for the relative crest width  $\left( \frac{B}{H_i} < 8 \right)$ , therefore, the researchers proposed an improvement for these equations to get the equations (14 and 15) as shown:

$$K_t = -0.4 \left( \frac{R_c}{H_i} \right) + 0.64 \left( \frac{B}{H_i} \right)^{-0.31} (1 - e^{-0.5\xi}) \quad \frac{B}{H_i} < 10 \quad \text{Eq.14}$$

$$K_t = -0.35 \left( \frac{R_c}{H_i} \right) + 0.51 \left( \frac{B}{H_i} \right)^{-0.65} (1 - e^{-0.41\xi}) \quad \frac{B}{H_i} \geq 12 \quad \text{Eq.15}$$

[12] proposed an artificial mangrove root system using numerical simulation model, the variables, wave height, length and depth with structure height, porosity and width, were examined to show their influence on the wave transmission, reflection and energy dissipation coefficients. The results of numerical model compared with experimental model to give a sensible conformity.

[13] demonstrate different systems for the protection of the shoreline by using a CGWAVE as numerical model, this study belong to the Ras El-Bar beach in Egypt; many structures have been proposed to protect the beach by dissipation the energy of sea currents. The researchers concluded many points such as the groin structure has no any notable effect on wave energy dissipation, otherwise the detached and seawall breakwater shows a significant effect on energy dissipation rate, also the submerged breakwater could be used to minimize the cost of construction and to reduce the energy action of incident waves. Equation (16) shows an acceptable statistical  $R^2$  equal to (0.83) with exponential relationship, this equation represent the suggested relationship between the relative energy dissipated and the distance to the shoreline for the submerged breakwater.

$$R_{Ed} = 0.4 e^{-0.09 x} \quad \text{Eq.16}$$

Where:  $R_{Ed}$  represent the relative energy and  $x$  represent the distance to the offshore.

[14] presented a special type of shoreline protection, pile breakwater were experimentally investigated to identify the ability of this breakwater type to dissipate wave energy, the effective parameters such as wave height, period, steepness and pile diameter, arrangement were tested. From the physical model observations, it's clear that the pile breakwater has the ability to dissipate wave energy by the range (15 to 55) % which is considering a significant amount of dissipation energy in shoreline protections.

Investigate, in the lab, the energy dissipation rate of submerged breakwater with three rectangular shapes, and these shapes have different 0.3 ,0.6 and 0.9m in width with 0.25m in height, breakwaters were tested in three water depth, 0.30, 0.35 and 0.40 m, the wave period of 1.1 to 1.9 second and wave height of 4.0 to 10.0 cm this study show that any increasing in breakwater width will cause a decrease in the transmission coefficient and decreasing when the water depth threshold to increase near the breakwater [15].

### Dimensional Analysis

To study the performance of wave breaking over steed breakwater, dimensional analysis approach adopted to find out the confident behavior of physical parameters that has mainly effect on breaking waves phenomenon. This approach depended on a mathematical form to yield a relationship, expression or equation, for combining the effect of all physical values by dimensionless groups. The most useful method to explain the dimensional analyses approach is Buckingham theory, in this study the dimensionless groups produced to represent the behavior of breaking waves over mound stepped breakwater. The hydraulic parameters witch described the physical study, breakwater geometry and wave features shown in Tables 1, 2 and 3, respectively.

Table 1. Parameters describe physical study

No.	Sample	Define Parameters	Units	Dimensions
1	$K_T$	Coefficient of Transmission	-	-

Table 2. Parameters describe breakwater geometry

No.	Sample	Define Parameters	Units	Dimensions
1	B	Width of the crest	m	L
2	$L_s$	Step width	m	L
3	$h_s$	Step height	m	L

4	N	Number of steps	-	-
5	$\alpha$	Inclination angle of breakwater	-	-

Table 3. Parameters describe wave features

No.	Sample	Define Parameters	Units	Dimensions
1	$\rho$	Density of the fluid	kg/m <sup>3</sup>	ML-3
2	$\mu$	Dynamic viscosity of the fluid	Kg/(m*s)	M/(L*T)
3	g	Gravitational acceleration	m/s <sup>2</sup>	LT-2
4	L	Length of wave	m	L
5	H <sub>i</sub>	Incident height of wave	m	L
6	H <sub>t</sub>	Transmit height of wave	m	L
7	d	Depth of water	m	L
8	V	Velocity of water	m/s	LT-1

The hydraulic parameters that affect the breaking waves behavior can be presented as follow:

$$f(\rho, \mu, g, L, H_i, H_t, d, V, B, h_s, L_s, N, \alpha) = 0 \quad \text{Eq.17}$$

The dimensionless groups establish by the benefits of Buckingham  $\pi$ -theorem to identifying the repeated variables of theory that should indicate wave behavior and fluid properties as  $\rho, V, H_i$ , the outcomes of equation (17) present (10 dimensionless groups) as:

No.	Group	Results	No.	Group	Results
1	$f_1(\rho, V, H_i, g)$	$\frac{V^2}{g H_i}$	6	$f_6(\rho, V, H_i, L_s)$	$\frac{H_i}{L_s}$
2	$f_2(\rho, V, H_i, \mu)$	$\frac{\rho V H_i}{\mu}$	7	$f_7(\rho, V, H_i, h_s)$	$\frac{H_i}{h_s}$
3	$f_3(\rho, V, H_i, \alpha)$	$\alpha$	8	$f_8(\rho, V, H_i, N)$	$N$
4	$f_4(\rho, V, H_i, L)$	$\frac{H_i}{L}$	9	$f_9(\rho, V, H_i, B)$	$\frac{H_i}{B}$
5	$f_5(\rho, V, H_i, d)$	$\frac{H_i}{d}$	10	$f_{10}(\rho, V, H_i, H_t)$	$\frac{H_i}{H_t}$ or $K_t$

The resulted dimensionless groups can arranged to be:

$$\left( \frac{V^2}{g H_i}, \frac{\rho V H_i}{\mu}, \alpha, \frac{H_i}{L}, \frac{H_i}{d}, \frac{H_i}{L_s}, \frac{H_i}{h_s}, N, \frac{H_i}{B}, \frac{H_i}{H_t} \right)$$

The terms  $\left( \frac{\rho V H_i}{\mu}, \frac{V^2}{g H_i} \right)$  represent Froude Number and Reynolds Number, respectively. The influence of these two terms can be neglected in wave braking condition because the effect of viscosity is small at open channel problems and the effect of gravity is including in wave length at wave condition state, while the term  $H_i/H_t$  represent the transmission coefficient  $K_t$ , therefore, the following relationship in equations (18) dominated:

$$K_t = \frac{H_i}{H_t} = f \left( \alpha, \frac{H_i}{L}, \frac{H_i}{d}, \frac{H_i}{L_s}, \frac{H_i}{h_s}, N, \frac{H_i}{B} \right) \quad \text{Eq.18}$$

### Conclusion

Based on the results of dimensional analysis approach, Buckingham theory consider a powerful way to produce dimensionless group expression, so that, this method prove that the transmission coefficient  $K_t$  is

a function of  $\frac{H_i}{H_t}$  and state that the parameters ( $L, H_i, H_t, d, B, h_s, L_s, N, \alpha$ ) where the depth of water ( $d$ ), height of incidence wave ( $H_i$ ), height of transmission wave ( $H_t$ ), length of wave ( $L$ ), Width of the crest ( $B$ ), Step width ( $L_s$ ), Step height ( $h_s$ ), Number of steps ( $N$ ) and Inclination angle of breakwater ( $\alpha$ ) have main effect on the breaking waves over stepped mound breakwater.

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