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Earthen slope stability using dimensional analysis

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Abstract. Slope stability is an important aspect in the management of civil engineering projects. The stability of earth slopes is a significant concern for safety and the economy. There are several methods for finding the safety factor. Spencer's method is considered one of the most accurate methods for calculating the factor of safety but it depends on the others on dividing the slope geometry into the number of slices with complicated calculations. This paper introduced a new theoretical equation (using dimensional analysis) capable of defining and testing the slope stability of the natural and man-made slope of the earth without complicated calculations. The major and significant variables which affect and contribute to the stability of a slope will be listed and discussed. The Geo-SLOPE and SEEP software were used to present the stability analyses of the earth dam and compare the result. The derived equation shows the excellent relationship between the factor of safety concerning the ratio of internal friction angle over slope angle $\emptyset/\alpha$ and the hydraulic gradient. The main objective is to set a new rapid method for checking the stability of slopes instead of a complex calculation that can describe the slope stability of the natural earth slope.

Keywords: Earthen slopes, Dam, Stability, dimensional analysis, Spencer, limit equilibrium.

1. Introduction

The stability of earth slopes which are either natural or man-made such as the earth dam is significant for safety and economy [1]. The factor of safety (FS) is defined as the ratio of resisting forces generated by the shear strength of soil along the collapse surface to the sliding forces generated by the mass above the failure surface. There are many methods to find the FS. The major direction of these methods is the slice method which is the oldest numerical technique having the idea of dividing an assumed potential sliding mass into slices. The analysis of the slices method can use the Limit Equilibrium Method (LEM) [1]. The LEM is a standard analysis used to define the stability of the geotechnical structure and calculates the statistical equilibrium of the various components to find the safety factor [2], and to overcome the FS, and finally to find the equilibrium load limit [1–4].

The slice method was first employed in 1916 [4]. Thus, all researchers have tried to put some hypothesis to reduce the level of indeterminacy. At first, all interslice forces were ignored as seen in Figure 1 [5]. Then it became known as the Ordinary or Fellenius Method [4]. Bishop added interslice normal forces and satisfied overall moment equilibrium, known as Bishop Simplified Method[6]. Closed to Bishop, Junbu also continued interslice normal forces but satisfied only overall horizontal force equilibrium known as Junbu simplified method. Spencer and Morgenstern-Price Methods included interslice normal and interslice shear and satisfied all equations of statics forces and moments[4]. There are other methods such as Corps of Engineers 1&2, Low-Karafiath, and Sarma which apply for vertical slices only[6].

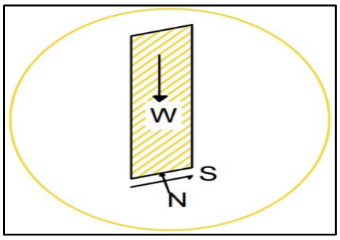


Figure 1. No interstice forces [5]

In practice, the basics are usually not well known despite the extensive usage of these different approaches. The fact that the slices approach is based on only statistics [4]. The general assumptions of these methods are the FS is a constant along the slip surface, each slice has the same FS, and then the Liquid Equilibrium Method is applied. All the previous methods for slope stability depend on the method of slices and have some assumptions to reach the static equilibrium [5]. This study will put a new approach that may be followed by the researcher in the future to set new relationships developing the equations of this phenomenon.

1.1. Dimensional analysis

Unfortunately, verey few resources are dealing with the Dimensional Analysis of earthen structures. Most of the sources are before the year 1950 for this technique for fluids searches only. The most important recent source that proposed the idea of using dimensional analysis in geotechnics was in 1999 by Roy Butterfield [7]. He discussed the idea for foundations and retaining walls.
According to Buckingham's theorem [7], every dimension homogeneity equation can be dimensionally homogeneous i.e. if it is possible to re-write all the components in groups, dimensionless groups can be produced. The dimensional analysis theorem can be defined as a method used to analyze a phenomenon by dividing or compositing a homogeneous single or dimensionless group of related parameters that affect a single premise [7]. It does not mean that this approach can obtain the complete solution, but it can minimize and control efforts and outcomes in a particular way. In general, if a phenomenon has multi-independent variables, as this paper problem, it may require a curve relation for each variable. But when the dimensional analysis is used, the associated curves may be condensed into one category of relationship curves [6]. This is one of the great advantages resulting from this approach. The main objective of this paper is to set a new rapid method for checking the stability of slopes instead of a complex calculation that can describe the slope stability of the natural earth slope.

2. Methodology

The major problem of the analysis stability of slopes for each slice is the degree of determinacy. As shown in Figure 2 [3], the driving and resisting forces are lateral earth force (E), interslice shear force (X), shear strength force (S), the weight of a slice (W), and the normal force (N). So, the total unknowns over all slices will be (4n-2) where (n) is the number of slices. Each slice has three equations which then leads to (n-2) indeterminacy [5].

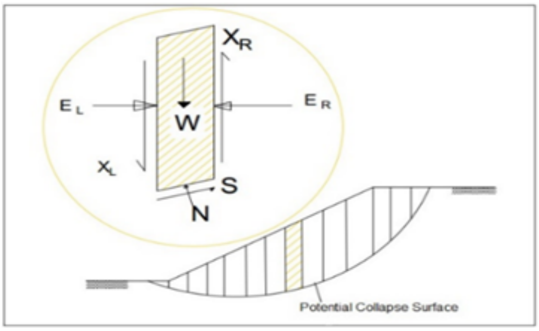


Figure 2. Forces with slice's components [5]

2.1. *Fundamental variables of slope stability*

Spencer's method is widely used and depends on exiting all these forces [8]. The general method equation for the moment and equilibrium of forces are referred to as 'equations (1-3)' as expressed by [2,6,8]:

For $\sum \text{momment} = 0$:

$$\frac{\sum[c'\beta\ R+(N-u\ \beta)R\tan\emptyset\prime]}{\sum wx-\sum N\ f\ \pm D\ d} \tag{1}$$

For $\sum forces = 0$:

$$FS = \frac{\sum[c'\beta\cos\alpha+(N-u\ \beta)R\tan\emptyset\prime\ cos\alpha]}{\sum N\sin\alpha-\ D\cos\omega} \tag{2}$$

Where: N is the normal force at the base of a slice and expressed as:

$$= \frac{W+(X_R-X_L)-\frac{c'\beta\ sin\alpha+u\ \beta\ sin\alpha\ tan\emptyset'}{FS}}{cos\alpha+\frac{sin\alpha\ \ tan\emptyset\prime}{FS}} \tag{3}$$

Where c and c' total and effective cohesion forces of the soil, $\emptyset$ and $\emptyset'$ total and effective internal friction angle, u is the pore water force, X_R and X_L are lateral right and left earth forces, R radius of collapse circle surface and all β, α, ω, D, f, d, and x are geometrical parameters (α= bottom slice inclination from horizontal (degrees), β= top slice inclination from horizontal (degrees), d =depth factor D/H and D= depth from the toe of slope to lowest point on the slip line).
By envelope of Mohr-Coulomb failure as shown in Figure 3, For any earth slope that has cohesion and internal friction angle, the total and effective shear strength, expressed by [2], are referred to 'equations (4-5)' :

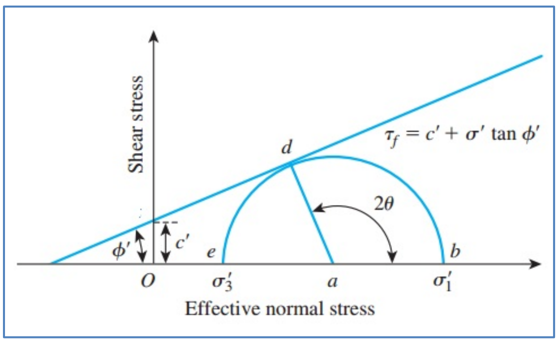


Figure 3. Failure envelop by Mohr-Coulomb [6]

$$S = C + \sigma \tan \emptyset \tag{4}$$

$$S = C' + \sigma' \tan \emptyset' \tag{5}$$

Where S is either total or effective shear strength according to other its independent variables, C and C' are total and effective cohesion stress of the soil respectively, σ and σ' total and effective normal stresses.
The soil may be saturated or partially saturated, So the effect of volumetric water content will impact clearly. Bishop set a relationship expressing the normal effective stress used as a reference to 'equation (6)' [6]:

$$\sigma' = \sigma - u_a + \aleph (u_a - u_w) \tag{6}$$

Where: $\boldsymbol{u_a}$, and $\boldsymbol{u_w}$ are pore air and water pressures respectively, and $\aleph$ is the fraction of the area of soil containing water and it is in between (0 -1) as shown in Figure 4.
The total or effective normal stresses increase with depth and, for a point, represents the weight of everything above that point and usually expressed ($\sigma = \gamma H$) where H is the depth of soil above the calculated point and ($\gamma = \rho g$) where (ρ) is the soil's mass density and (g) is gravity acceleration [2].

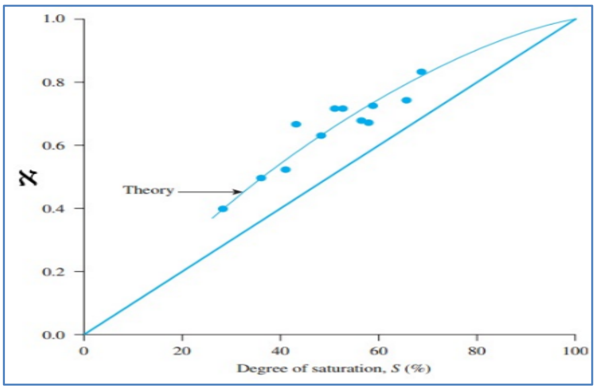


Figure 4. A fraction of the cross-sectional area with respect to degree of saturation [6]

The value of the coefficient of permeability (k) of the soil affects the slope stability for some sides such as; the generated pore pressure changing the effective stresses, and per the case of drained or undrained analysis, the seepage case, and so on [3,9–13].

The hydraulic conductivity (K) will be the dominant variable for the case of the upstream drawdown water level in deciding the slope will be drained or undrained manner [10,12].
The interslice lateral forces (X_R and X_L) are either active or passive forces. Rankine showed that the active and passive lateral earth forces depend on both cohesion and a coefficient named (ka) which depends on the internal friction angle. The relations are referred to as 'equations (7-8)' are expressed by [1]:

$$P_a = \frac{1}{2}\gamma H^2 k_a - 2CH\sqrt{k_a} \tag{7}$$

$$k_a = tan^2\left(45 - \frac{\emptyset}{2}\right) \tag{8}$$

And the same in passive lateral forces with changing of a sign but with same variables. Where (P_a) is the lateral active force, and (k_a) is the Rankine active coefficient. Coulomb expressed the relationships by different sight but his results expressing by the same variables [1].
By all the above equations (1 to 8), the fundamental and significant variables that affect the slope stability can be pointed as $\rho, g, \alpha, C, \emptyset, K, H, h_w, l$, where α is the angle of inclination of the slope,

h_w is the water depth at the upstream of the slope, and l is the average length of the seepage path from upstream to downstream.

2.2. Selection of the repeated variable

The repeated variables are chosen as an effect on the other variables [7]. So, the vertical depth (H) is very important for all relationships. The specific weight of the slope soil with its water content impact directly for normal stress calculations. The fundamental variables are $\boldsymbol{\rho}, \boldsymbol{g}, \boldsymbol{\alpha}, \boldsymbol{C}, \emptyset, \boldsymbol{k}, \boldsymbol{H}, \boldsymbol{h_w}, \boldsymbol{l}$ which can be divided into sub groups; the soil slope properties are ($\boldsymbol{\gamma}, \boldsymbol{C}, \emptyset$); the geometrical variables are $(\boldsymbol{H}, \boldsymbol{\alpha}, \boldsymbol{l})$; and the kinematic variables are (K). and repeated variables that will be taken are $\boldsymbol{\rho}$, $\boldsymbol{g}, \boldsymbol{H}$.
The number of variables will be equal to 10 as $\boldsymbol{\varphi}\,(\boldsymbol{FS}, \boldsymbol{h_w}, \boldsymbol{l}, \emptyset, \boldsymbol{\alpha}, \boldsymbol{C}, \boldsymbol{k}, \boldsymbol{\rho}, \boldsymbol{g}, \boldsymbol{H}) = \boldsymbol{0}$, and the dimensions of each one are: $[\rho] = [ML^{-3}]$; $[g] = [LT^{-2}]$; $[H] = [L]$; $[C] = [ML^{-1}T^{-2}]$; $[k] = [LT^{-1}]$; $[FS] = [M^0L^0T^0]$; $[\emptyset] = [M^0L^0T^0]$; $[\alpha] = [M^0L^0T^0]$; $[h_w] = [L]$; *and* $[l] = [L]$.
After evaluating values of a, b, and c, with re-arranging the groups, the final equation will be:

$$\mathrm{FS} = \varphi\left(\frac{\mathrm{C}}{\gamma\mathrm{H}}, \frac{\mathrm{k}}{\sqrt{\mathrm{gH}}}, \mathrm{i}, \frac{\emptyset}{\alpha}\right) \tag{9}$$

Where $\boldsymbol{i}$ is the hydraulic gradient ($\frac{\boldsymbol{h_w}}{\boldsymbol{l}}$), So the FS for the slope stability is a function of four groups as mentioned in ‘equation (9)’

3. The case studies

By this paper, which is considered the first step of a big work, will be taken specific values of $c, \boldsymbol{\gamma}, and\ K$ for the natural and man-made slope as constant for purpose of research. In another word, there are common values of them used for earthen slope. This case, for the research step, will lead to taking the values of $\frac{\boldsymbol{C}}{\boldsymbol{\gamma H}}$ and $\frac{\boldsymbol{k}}{\sqrt{\boldsymbol{gH}}}$ as constants. So, the FS is taken as a function of $\left(\boldsymbol{i}, \frac{\emptyset}{\boldsymbol{\alpha}}\right)$. It will be a steady-state case taken into account with no cases of a rapid or seismic drawdown of water level mentioned by E. Alonso and N. Pinyol [14]. The properties of soil of a real dam as shown in Table 1[12].

Table 1. The properties of soil of a real dam[12].

Sat. Hydraulic Conductivity K (m/s)	Cohesion C (kpa)	Unit weight $\boldsymbol{\gamma}$ (KN/m^3)	Friction angle $\emptyset$ (degree)
5.7x10^{-7}	12	17.95	22

The hydraulic conductivity, as it is known, is affected by the Volumatic Water Content VWC of slope soil and especially for rapid drawdown [6]. The VWC will be as shown in Figure 5 where the porosity used is 0.4 and the conductivity is shown in Figure 6.

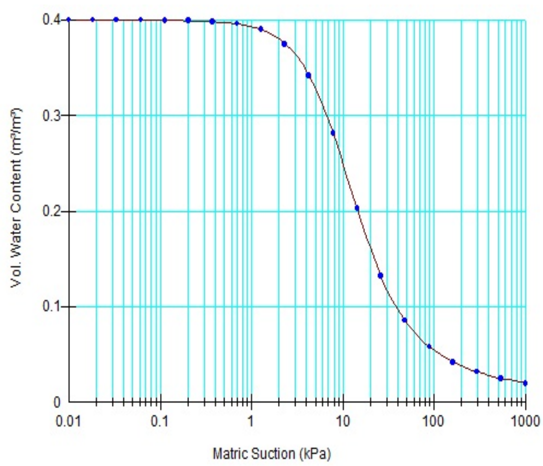


Figure 5. VWC vs Matric Suction [6]

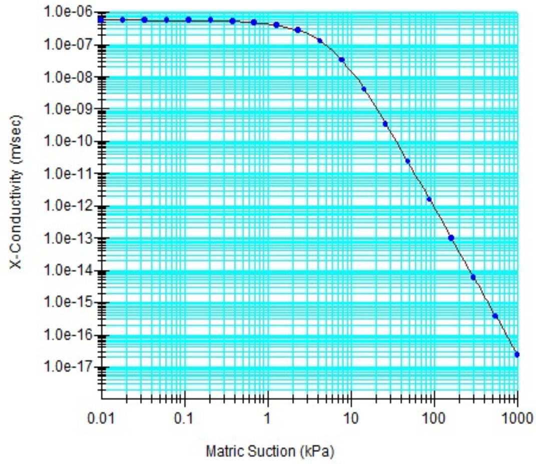


Figure 6. Conductivity vs Matric Suction [6]

This research will concentrate on the upstream with different slopes. The upstream and downstream slopes for homogeneous section are (1V:2H to 1V:3H) and (1V:2H to 1V:2.5H) [15]. So, it will be taken the range of upstream slope (1V:1.5H to 1V:3.5H) and downstream (1V:2H). The downstream water level will be constant (at zero levels). The vertical height of the slope (H) is taken as a medium height equal to 20m [14]. The crest width (a) for medium height equal to 6 m according to the 'equation. (10)' [14]:

$$a = 0.55\sqrt{H} + 0.2\,H \tag{10}$$

Finally, the initiative taken slope shape for slope stability is shown in Figure 7.

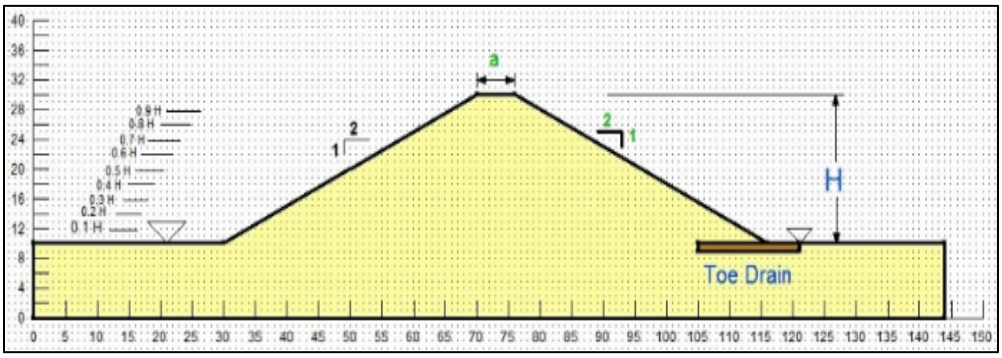
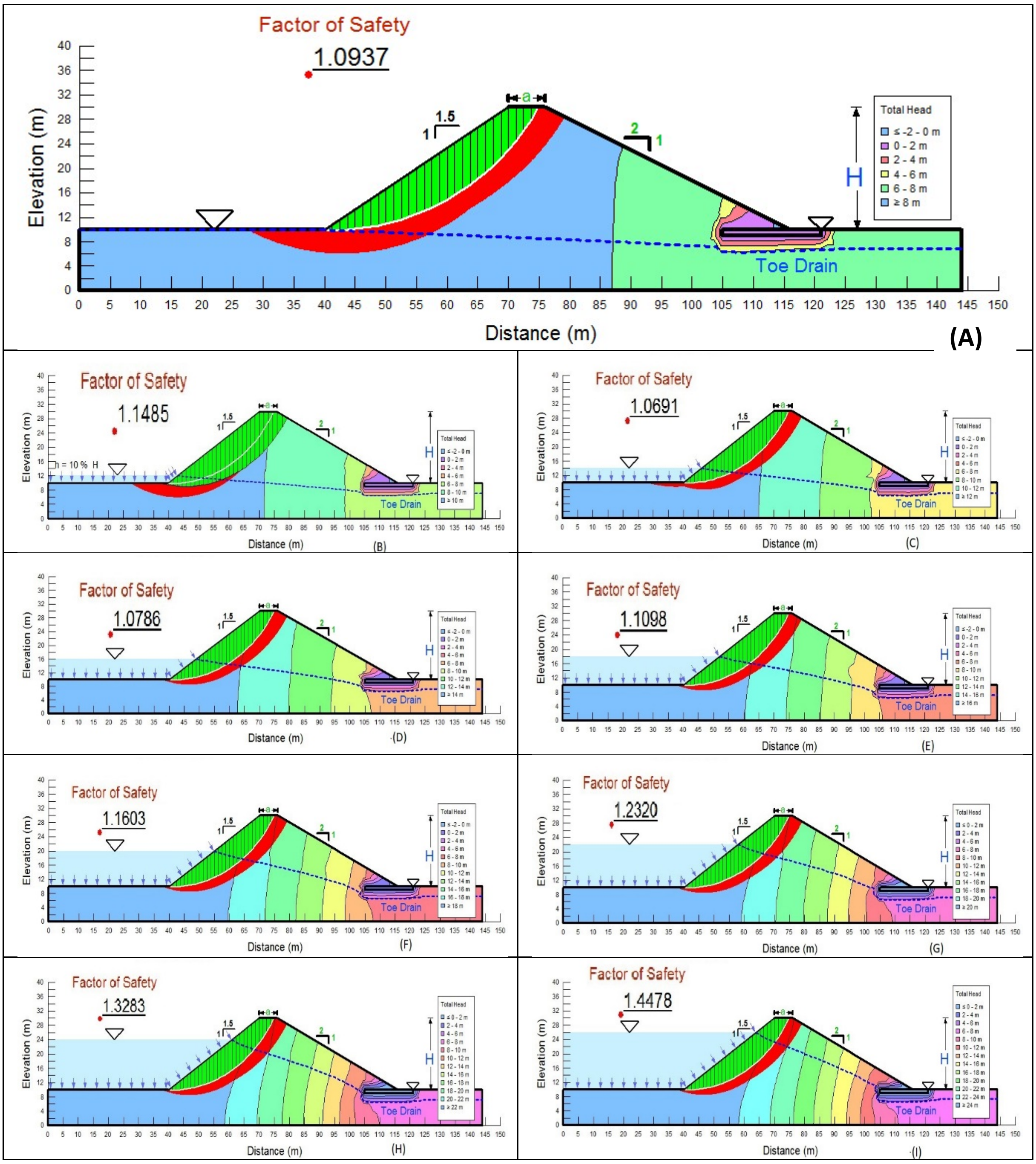


Figure 7. Initial dimensions of a studied

4. Results and discussions

This paper presents the results of the stability analyses of the homogeneous earth dam using Geo-SLOPE/W and SEEP/W software using the spencer method. Spencer's method was used to show the relationship between the factor of safety with ($\boldsymbol{i}$) and the ratio of $(\emptyset/\alpha)$**.** This method is chosen because it is composed of all forces as explained early.

The factor of safety has been calculated for the upstream slope of real earth dam using dimensional analysis. The upstream level will be varied (0, 0.1 H, 0.2 H, 0.3 H, 0.4 H, 0.5 H, 0.6 H, 0.7 H, 0.8 H, and 0.9 H) for each tested slope (Figures 8-12) , an idea from [13]. The results of different water levels with up and down steam slope 1V:1.5H are shown in Figures 8 (A-J). Then for each slope will be plotted ten graphs as shown in Figure 8. For focusing on the target, be putting one chosen graph for each other cases and the rest put as an appendix with this paper.



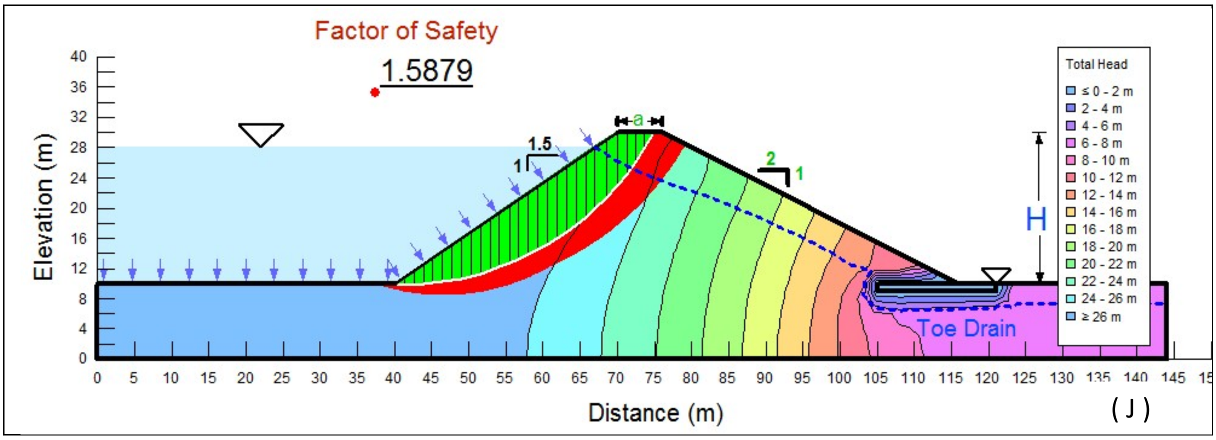


Figure 8. A different water level with upstream slope 1V:1.5H

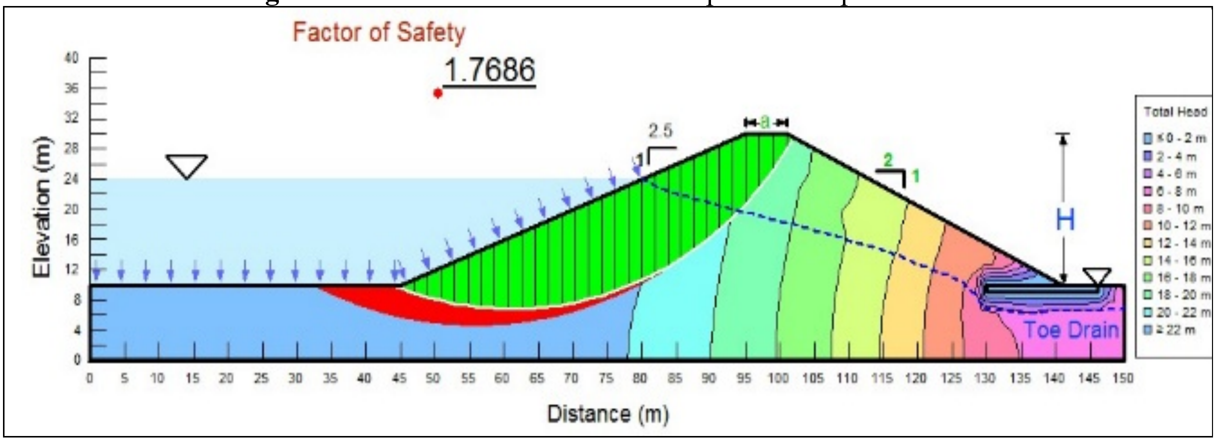


Figure 9. A chosen water level with upstream slope 1V:2H

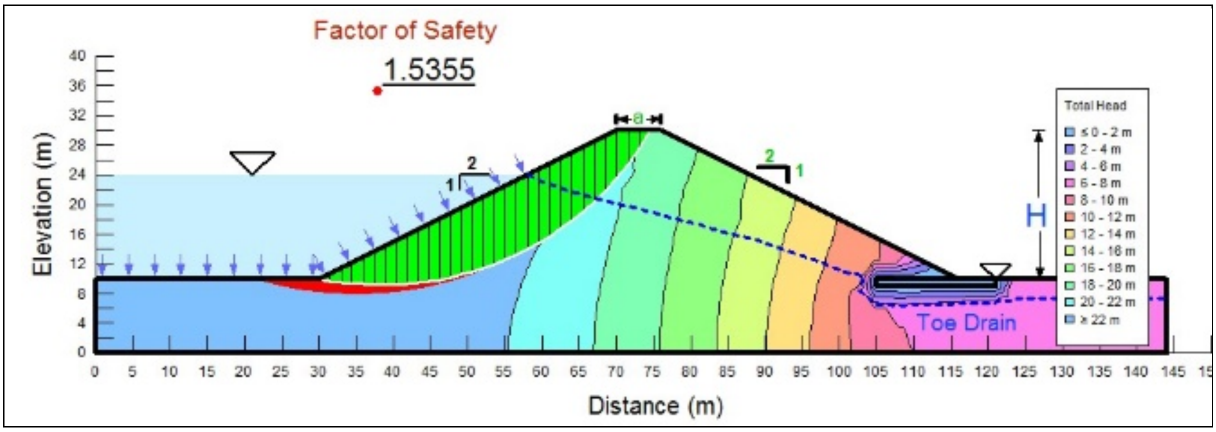


Figure 10. A chosen water level with up and downstream slope 1V:2.5H

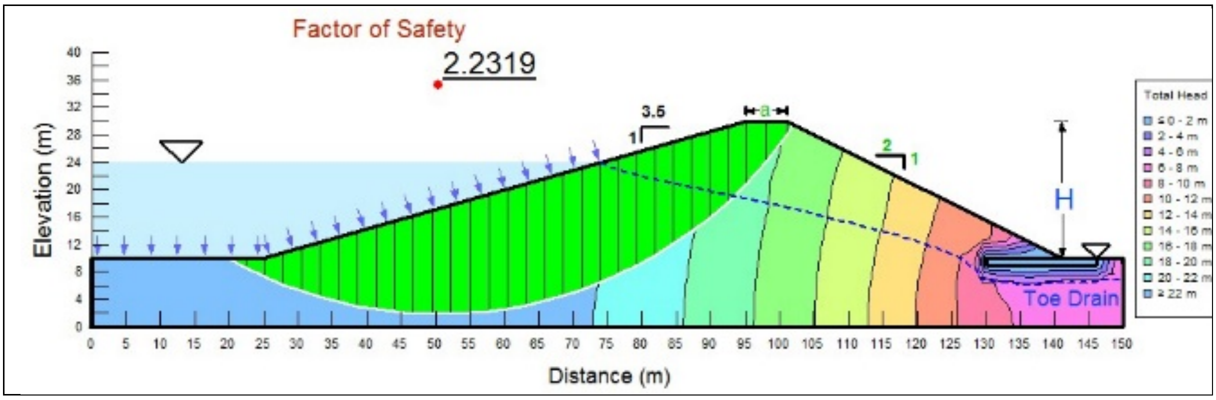


Figure 11. A chosen water level with upstream slope 1V:3H

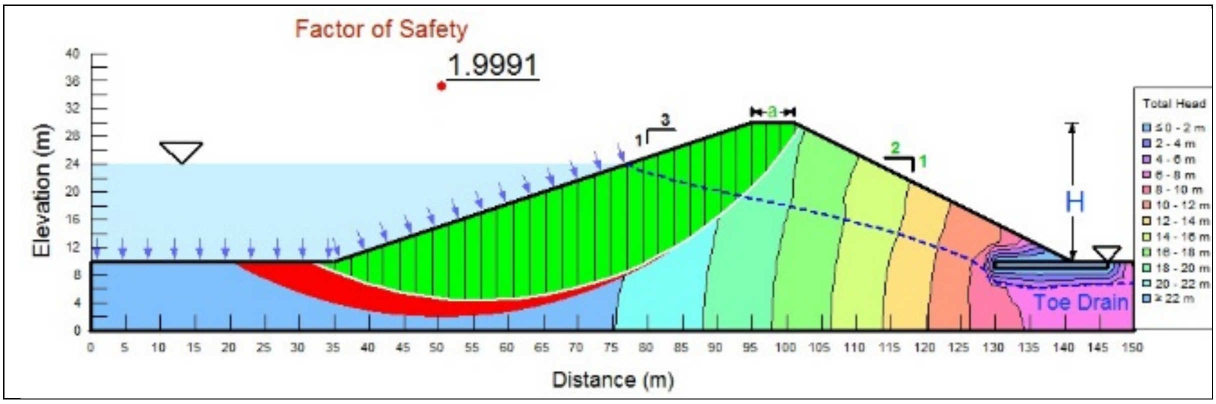


Figure 12. A chosen water level with up and downstream slope 1V:3.5H

Figures (8-12) shows that FS increases when α decreases for all zero upstream water level which confirms the strong relationship of FS with ∅. But FS appears lower values for lower water level after filling and then raise. These lower values are due to less pore water pressure which affects directly the resistance shear forces. The relationship between hydraulic gradients and the factor of safety corresponding to the ratio of ∅/α is plotted as curves as shown in figure 13.

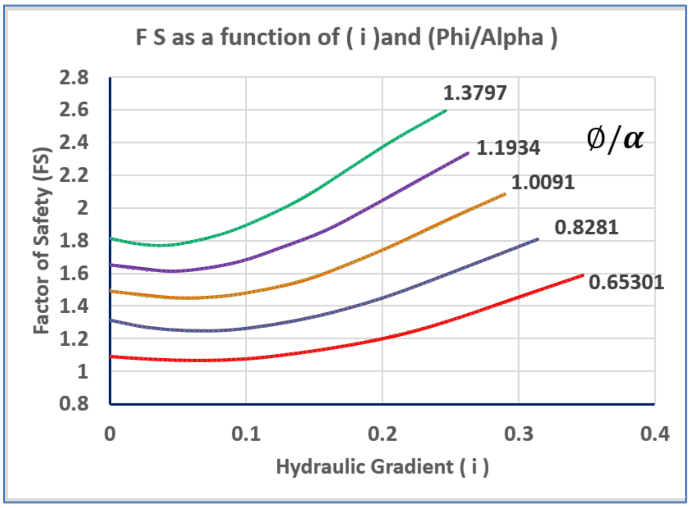


Figure 13. Factor of Safety as a function of **(i) and (∅/α)**

This figure shows that FS will be larger for a bigger value of ($\emptyset / \alpha$) which means that the dam will be safer for big $\emptyset$ with the same values of α .
The estimated values of FS that satisfy all the curves in figure (13) is derived as 'equation (11)':

$$FS = 5.728\left(\frac{h_w}{l}\right)^{2.06}\left(\frac{\emptyset}{\alpha}\right)^{2.613} + 5.366\left(\frac{h_w}{l}\right)^{2.666} + 1.576\left(\frac{\emptyset}{\alpha}\right)^{0.622} - 0.145 \quad (11)$$

The 'equation (11)' includes only two groups from the mentioned four dimensionless groups where the other two groups assumed constant with related values shown in table 1. So, this equation is considered as the first trial to know the suitability of this technique for slope stability in general. The limitations of the equation are for a homogeneous dam with Toe drained, a specific type of soil, constant downstream slope, and steady-state case.

Figure 14 shows a strong correlation between the estimated and the calculated values of FS. This result shows the changing of the coefficients is also at the same degree of inclination.
Analyzing the coefficients of the different curves, a strong correlation between the estimated and the measured values of FS is shown in Figure 14 where R^2 near to one.

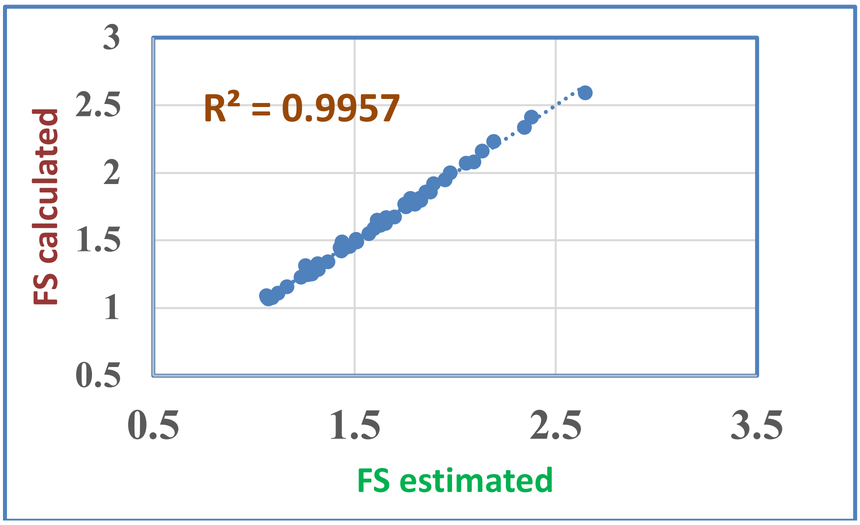


Figure 14. FS estimated vs FS calculated

The values of α between (1V:2H to 1V:2.5H) are safer for the FS when it's more than 1.5H. This means the slope stability of the earth dams increases when the dam holds more water upstream. During the filling process, the FS values are reduced due to the increase of the pore water pressure within the dam structure. FS values rise as the water level increases, thus increasing the protection of the upstream slope face by the water forces. The FS is greater than the internal friction angle ratio for a higher inclination. This means that the greater $\emptyset$, the greater the protection factor at zero water level. Also, there are negligible variations between the FS values measured as compared to the estimated.

5. Conclusion

To conclude, the slope stability of the natural and man-made slope of the earth dam can be defined and measured using Dimensional Analysis Techniques. A general empirical equation can represent all the derived empirical equations since there are negligible variations between the FS values measured as compared to the estimated. This technique will reduce the effort, time, and cost expected compared to other different methods. Finally, it's recommended to experiment to verify the relationship obtained and to take into account the general derived equation with all four dimensionless groups to be a general one.

6. Appendix

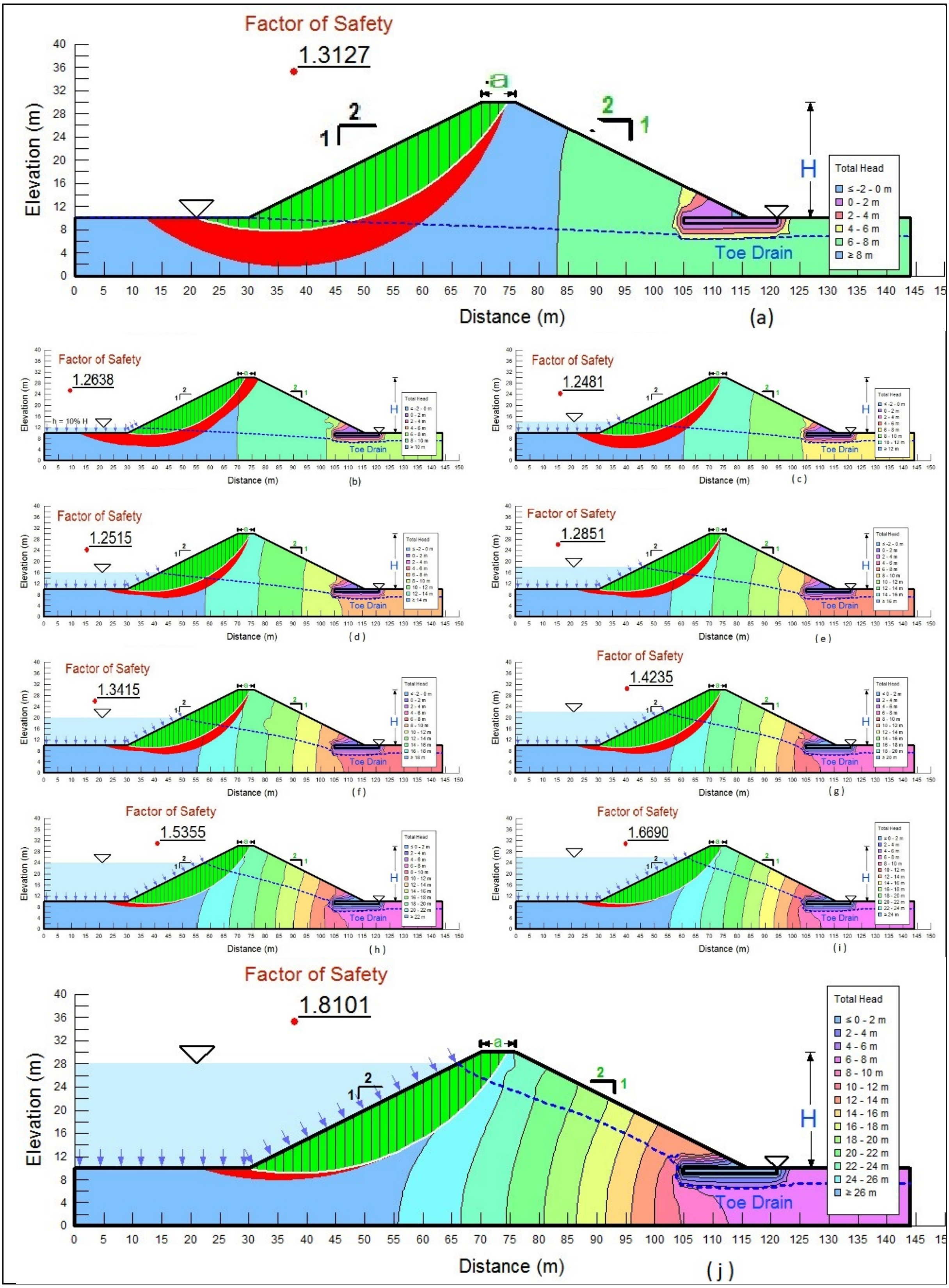

Factor of Safety
1.3127
Elevation (m)
Distance (m)
Total Head
≤ -2 - 0 m
0 - 2 m
2 - 4 m
4 - 6 m
6 - 8 m
≥ 8 m
Toe Drain
H
(a)
Factor of Safety
1.2638
h = 10% H
(b)
Factor of Safety
1.2481
(c)
Factor of Safety
1.2515
(d)
Factor of Safety
1.2851
(e)
Factor of Safety
1.3415
(f)
Factor of Safety
1.4235
(g)
Factor of Safety
1.5355
(h)
Factor of Safety
1.6690
(i)
Factor of Safety
1.8101
Total Head
≤ 0 - 2 m
2 - 4 m
4 - 6 m
6 - 8 m
8 - 10 m
10 - 12 m
12 - 14 m
14 - 16 m
16 - 18 m
18 - 20 m
20 - 22 m
22 - 24 m
24 - 26 m
≥ 26 m
Toe Drain
(j)

Figure A1. A different water level with up an downstream slope 1V:2H

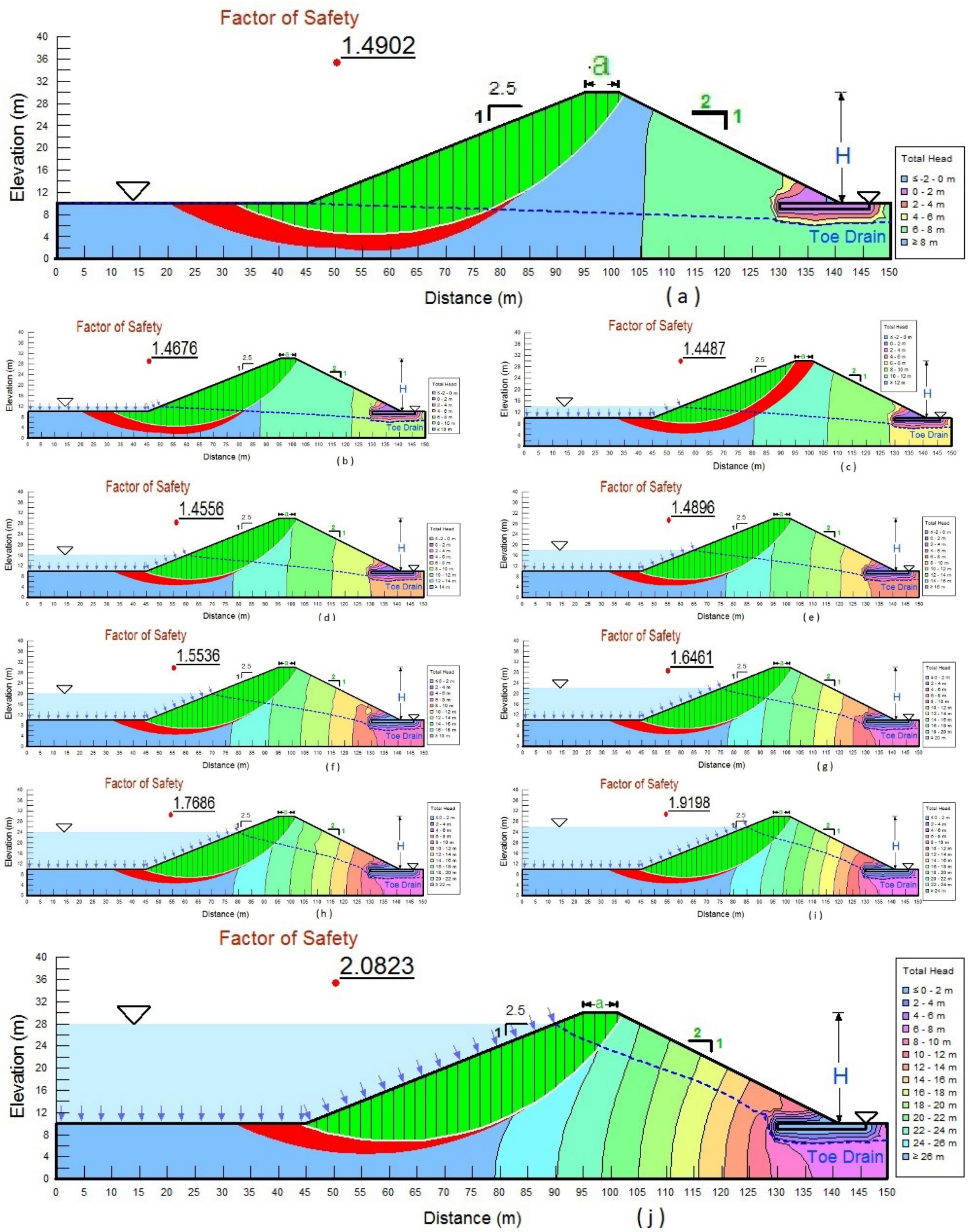


Figure A2. A different water level with upstream slope 1V:2.5H

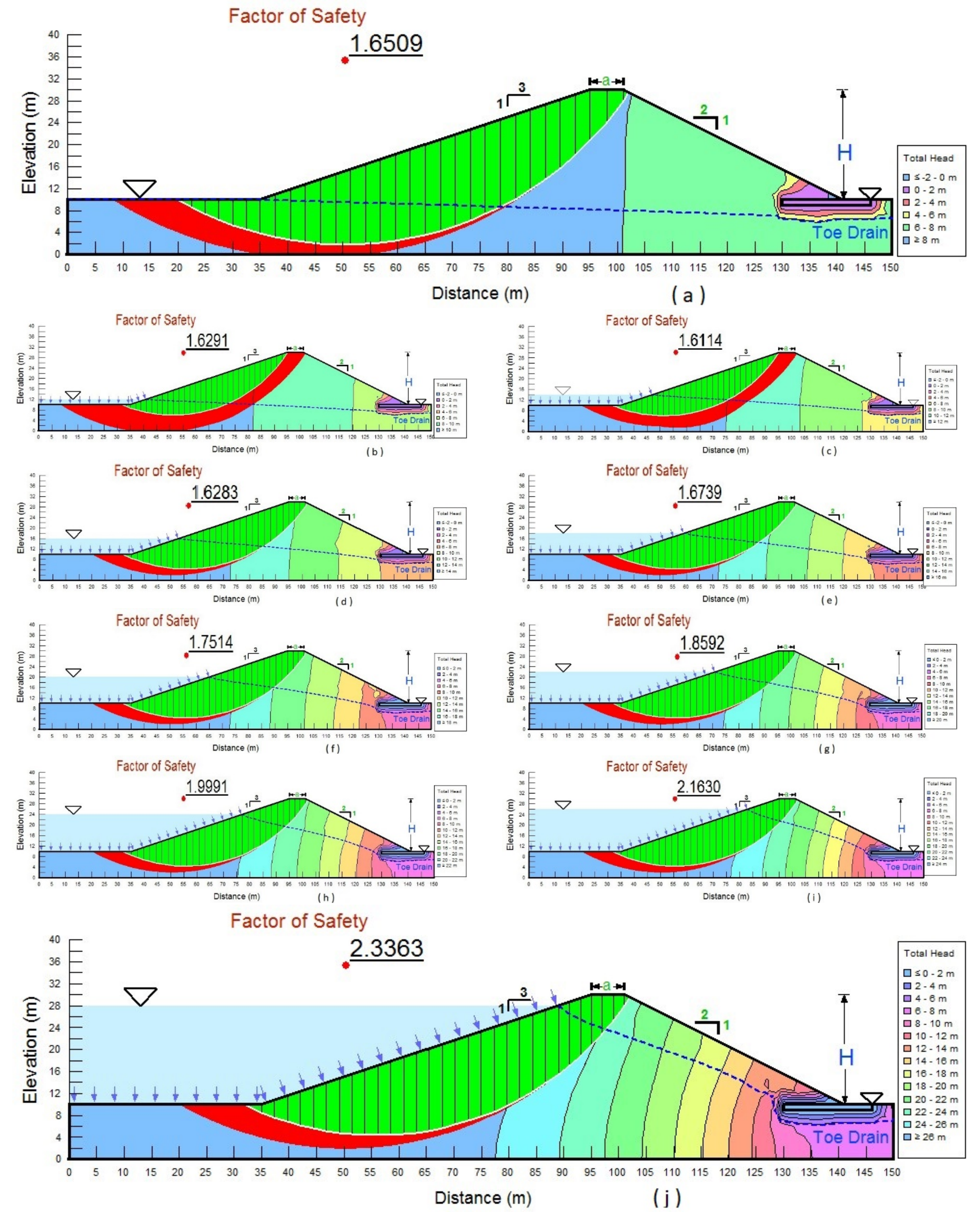


Figure A3. A different water level with upstream 1V:3H

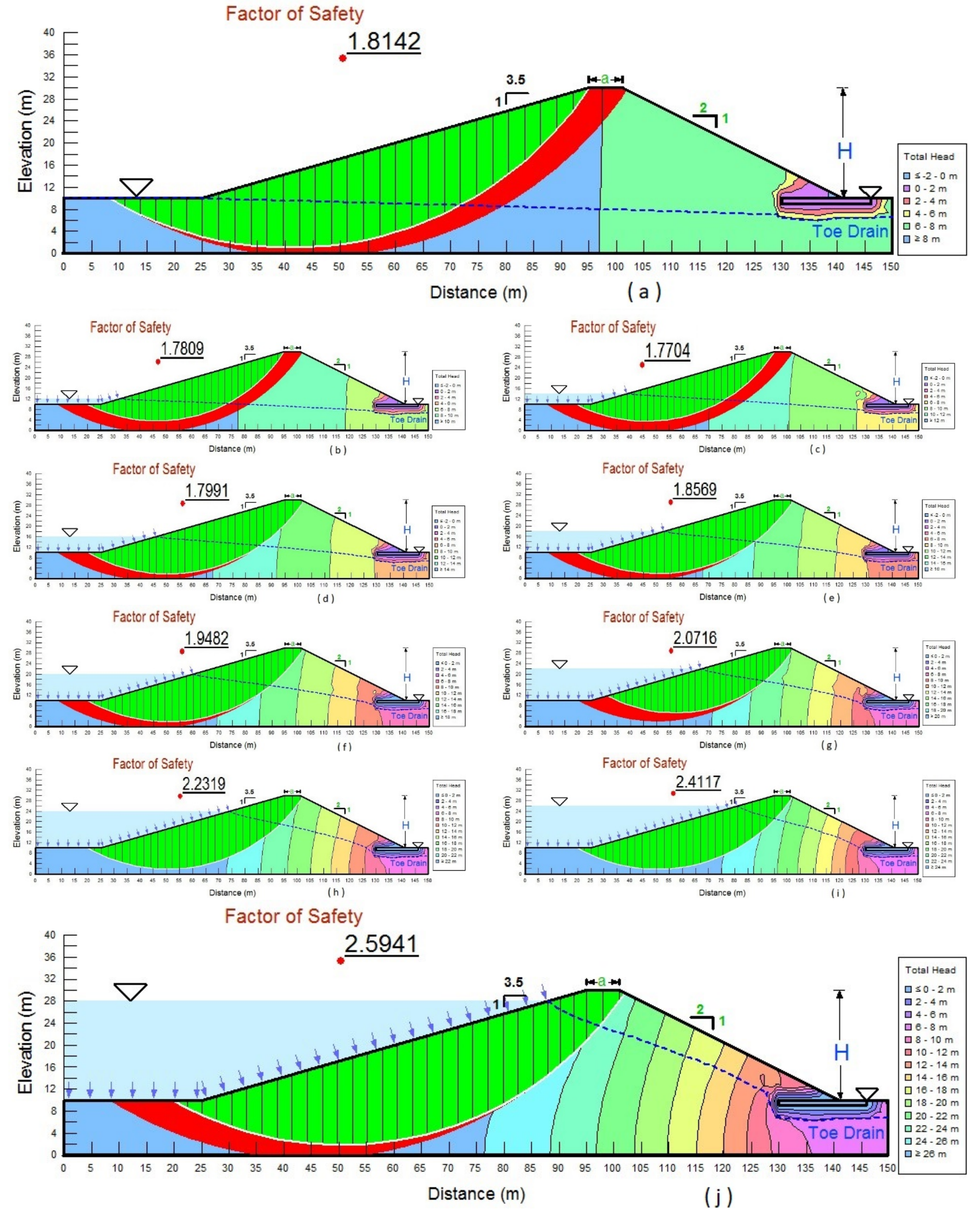


Figure A4. A different water level with upstream 1V:3.5H

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