Fixed point results for multivalued mappings in partial Hausdorff metric spaces

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Abstract

The main purpose of this paper is to introduce and study fixed point (F.P) under a contractive condition satisfying Geraghty-type by using the concept of partial Hausdorff metric spaces. Our results improve and unify a multitude of (F.P) theorems and generalized some recent results in partial metric spaces (P.M.S).

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1. Introduction

In (1969), Nadler [1] Proved the multivalued version of Banach contraction principle (B.C.P). where he extended (B.C.P) from case single-valued map to case multivalued map by using the concept Hausdorff metric. In (1992), Matthews [2] introduced the notion of the (P.M.S) as a generalization of metric space in which each object does not necessarily have a zero distance from itself, where it was a very useful to study of denotational semantics of dataflow networks. This notion was so useful to solve some hardness of the domain theory. in (1994), Matthews [3] extended (B.C.P) to (P.M.S). Thereafter, several authors proved some (F.P) theorems using these concepts see for instance ([4]-[11]).

In (2012), Aydi et al. [12] introduced the notion of a partial Hausdorff metric. Where they proved the existence of (B.C.P) for multivalued maps in complete (P.M.S). Thereafter, several authors proved some (F.P) theorems using this concept (see [13]-[17]). In this paper, we prove (F.P) theorem in the setting of (P.M.S) by using a partial Hausdorff metric. Our results generalized and extend some of the known results.

2. Preliminaries:

We recall some basic definitions and results in P.M.S which are needed in this paper.

Definition (2.1) [2][3]

Let M be a nonempty set , then a partial metric on M is a function $p: M^2 \to \Re^+$ (where $\Re^+$ is the set of all nonnegative real number), such that the following axioms hold for all $m,n,r \in M$.

(pm_1) $m = n \Leftrightarrow p(m,m) = p(n,n) = p(m,n)$, (separation axiom)

(pm_2) $0 \le p(m,m) \le p(m,n)$, (non-negatively and small self – distance)

(pm_3) $p(m,n) = p(n,m)$, (symmetry)

(pm_4) $p(m,n) \le p(m,r) + p(r,n) - p(r,r)$, (triangular inequality)

Then (M, p) is said to be a P.M.S.

It is clear if $p(m,n) = 0$ then from (pm_1) and (pm_2) it follows that $m = n$ But the converse not hold in general sees [2].

It is remarkable that for each partial metric p on the set M, the functions $d_p, p^w : M^2 \to \Re^+$ are defined by

$$d_p(m,n) = 2p(m,n) - p(m,m) - p(n,n).$$

$$p^{w}(m,n) = \max\{p(m,n) - p(m,m), p(m,n) - p(n,n)\}$$
$$= p(m,n) - \min\{p(m,m), p(n,n)\}$$

are ordinary metrics on M .

Each partial metric p on M generates a T_0 −Topology $\tau(p)$ on M whose base is the family of the open p − ball $\{B_p(m;\varepsilon), m \in M, \varepsilon > 0\}$, where
$B_p(m,\varepsilon) = \{n \in M : p(m,n) < p(m,m) + \varepsilon\}$, for all $m \in M$ and $\varepsilon > 0$.

Example (2.2) [3][5]

(1) The pair $(\Re^+, p_i)$, $i = 1,2$ where

$$p_1(m,n) = Max\{m,n\} \ \forall m,n \in \Re^+$$

$$p_2(m,n) = d(m,n) + \alpha \ \forall m,n \in \Re^+$$ and $\alpha \geq 0$, is a P.M.S.

(2) Let $p : M \times M \to \Re^+$, $M \subset \Re^+$

$$p(m,n) = \min\{m,n\} \ \forall m,n \in M \subset \Re^+$$

Since (pm_2) is fail if $m > n$. Thus, (M, p) is not P.M.S.

Definition (2.3) [3][9]

1- A sequence $\{q_n\}$ in a P.M.S (M, p) is said to be converge to the point $q \in M \Leftrightarrow \lim_{n \to \infty} p(q, q_n) = p(q,q)$.

2- A sequence $\{q_n\}$ in a P.M.S (M, p) is said to be Cauchy $\Leftrightarrow \lim_{m,n \to \infty} p(q_m, q_n)$ be exists (and is finite).

3- A P.M.S (M, p) is said to be complete if every Cauchy sequence $\{q_n\}$ in M converges, with respect to $\tau(p)$, to a point $q \in M$ such that $p(q,q) = \lim_{n,m \to \infty} p(q_n, q_m)$.

Lemma (2.4) [3]

Let (M, p) be a P.M.S. Then

1- A sequence $\{q_n\}$ is Cauchy in a P.M.S if and only if $\{q_n\}$ is a Cauchy in a metric space (M, d_p),

2- A P.M.S (M, p) is complete if, and only if , a metric space (M, d_p) is complete .In addition, $\lim_{n \to \infty} d_p(q_n, q) = 0 \Leftrightarrow p(q,q) = \lim_{n \to \infty} p(q_n, q)$

Lemma (2.5) [11]

Let (M,p) be a P.M.S. , If $\{q_n\}\subset M, q_n \to q$ as $n\to\infty$ and $p(q,q)=0$ then $\lim_{n\to\infty} p(q_n,r)=p(q,r)$.

Lemma (2.6) [8]

Let (M,p) be a complete P.M.S. Then

(i) If $p(m,n)=0 \Rightarrow m=n$

(ii)If $m\neq n \Rightarrow p(m,n)>0$

Definition (2.7) [12]

Suppose that (M,p) be a P.M.S. suppose $CB^p(M)$ be the family of all nonempty closed and bounded subsets of P.M.S (M,p). For all $U,V\in CB^p(M)$ and $m\in M$, define $\delta_p(V,U)=\sup\{p(v,U): v\in V\}$ and $\delta_p(U,V)=\sup\{p(u,V): u\in U\}$ where $p(v,U)=\inf\{p(v,u): u\in U\}$.

The mapping $H_p: CB^p(M)\times CB^p(M)\to[0,+\infty)$ defined by

$$H_p(U,V)=Max\{\delta_p(V,U),\delta_p(U,V)\}$$

is called the partial Hausdorff metric induced by p.

Proposition (2.8) [12]

Let (M,p) be a P.M.S. Then the following is holds; for all $U,V,W\in CB^p(M)$

(1) $\delta_p(U,U)=\sup\{p(u,u): u\in U\}$;

(2) $\delta_p(U,U)\le\delta_p(U,V)$;

(3) $\delta_p(U,V)=0\Rightarrow U\subseteq V$;

(4) $\delta_p(U,V)\le\delta_p(U,W)+\delta_p(W,V)-\inf_{w\in W} p(w,w)$.

Proposition (2.9) [12][13]

Let (M,p) be a P.M.S. Then the following is holds; for all $U,V,W\in CB^p(M)$

(1) $H_p(U,U)\le H_p(U,V)$;

(2) $H_p(U,V)=H_p(V,U)$;

(3) $H_p(U,V)\le H_p(U,W)+H_p(W,V)-\inf_{w\in W} p(w,w)$;

(4) $H_p(U,V)=0\Rightarrow U=V$.

The converse of (4) may not true in general as the following example

Example (2.10) [12]

Let $M=[0,1]$ be endowed with a partial metric $p:M\times M\rightarrow\Re^{+}$ defined by

$p(m,n)=Max\{m,n\}$

From (1) of proposition (2.8), we have

$H_p(M,M)=\delta_p(M,M)=\sup\{m:0\leq m\leq 1\}=1\neq 0$.

Lemma (2.11) [12]

Suppose (M,p) be a P.M.S, $U,V\in CB^{p}(M)$, $h>1$. Then for all $u\in U$, there exist $v=v(u)\in V$ such that $p(u,v)\leq hH_p(U,V)$.

We remark that if U,V are compact then $p(u,v)\leq H_p(U,V)$.

Lemma (2.12) [10]

Suppose (M,p) be a P.M.S and $\{q_n\}$ be a sequence of M such that $\lim_{n\to\infty}p(q_{n+1},q_n)=0$. If $\{q_n\}$ is not Cauchy sequence in (M,p), then there exists $\varepsilon>0$ and two sequence $\{m(k)\},\{n(k)\}$ of positive integers, with $m(k)<n(k)$, such that the following sequences $\{p(q_{n(k)},q_{m(k)})\}$, $\{p(q_{m(k)},q_{n(k)+1})\}$, $\{p(q_{m(k)-1},q_{n(k)+1})\}$ and $\{p(q_{m(k)-1},q_{n(k)})\}$ tend to ε as $k\rightarrow+\infty$.

Remark (2.13)[18]

Let S denote the class of the functions $\beta:[0,\infty)\rightarrow[0,1)$ which satisfy the condition $\beta(t_n)\rightarrow 1\Rightarrow t_n\rightarrow 0$

Main Results

Theorem (3.1)

Suppose (M,p) be a complete P.M.S and $F:M\rightarrow C(M)$ {$C(M)$ is the family of all compact subsets of M } be a multivalued map . Suppose that there exists $\beta\in S$ and $L\geq 0$ such that for all $m,n\in M$,

$$H_p(Fm,Fn)\leq\beta(M_P(m,n))M_P(m,n)+LN_P(m,n) \quad (1)$$

where

$$M_P(m,n)=\max\{p(m,n),p(m,Fm),p(n,Fn),\tfrac{1}{2}[p(m,Fn)+p(n,Fm)]\}$$
$$N_P(m,n)=\min\{p^w(m,Fm),p^w(n,Fn),p^w(m,Fn)+p^w(n,Fm)\}$$

Then F has a (F.P) q, i.e. $q\in Fq$. Moreover $p(q,q)=0$

Proof:

Let $q_0\in M$ be an arbitrary point, construct the sequence $\{q_n\}$ in M such that $q_{n+1}\in Fq_n$ for each $n\in N$

If $p(q_n,q_{n+1})=0$ for some $n\in N$ then $q_n=q_{n+1}\in Fq_n$, q_n is a (F.P) of F.

Assume $p(q_n,q_{n+1})>0$ for all $n\in N$

We claim $\{p(q_n,q_{n+1})\}$ is decreasing and tends to 0 as $n\to\infty$

By condition (1), we have

$$0<p(q_{n+2},q_{n+1})\le H_p(Fq_{n+1},Fq_n)$$
$$\le \beta(M_p(q_{n+1},q_n))M_p(q_{n+1},q_n)+LN_p(q_{n+1},q_n) \quad (2)$$

Since

$$M_p(q_{n+1},q_n)=\max\{p(q_{n+1},q_n),p(q_{n+1},Fq_{n+1}),p(q_n,Fq_n),\tfrac{1}{2}[p(q_n,Fq_{n+1})+p(q_{n+1},Fq_n)]\}$$

$$M_p(q_{n+1},q_n)=\max\{p(q_{n+1},q_n),p(q_{n+1},q_{n+2}),\tfrac{1}{2}[p(q_n,q_{n+2})+p(q_{n+1},q_{n+1})]\}$$

Since $\tfrac{1}{2}[p(q_n,q_{n+2})+p(q_{n+1},q_{n+1})]\le\tfrac{1}{2}[p(q_n,q_{n+1})+p(q_{n+1},q_{n+2})]$

$$M_p(q_{n+1},q_n)=\max\{p(q_{n+1},q_n),p(q_{n+1},q_{n+2}),\tfrac{1}{2}[p(q_n,q_{n+1})+p(q_{n+1},q_{n+2})]\}$$

Since $\tfrac{1}{2}[p(q_n,q_{n+1})+p(q_{n+1},q_{n+2})]\le\max\{p(q_{n+1},q_n),p(q_{n+1},q_{n+2})\}$

$$M_p(q_{n+1},q_n)=\max\{p(q_{n+1},q_n),p(q_{n+1},q_{n+2})\}$$
$$N_p(q_{n+1},q_n)=\min\{p^w(q_{n+1},Fq_{n+1}),p^w(q_n,Fq_n),p^w(q_{n+1},Fq_n),p^w(q_n,Fq_{n+1})\}$$
$$=\min\{p^w(q_{n+1},q_{n+2}),p^w(q_n,q_{n+1}),p^w(q_{n+1},q_{n+1}),p^w(q_n,q_{n+2})\}$$

Since $p^w(q_{n+1},q_{n+1})=0$ it follows that $N_p(q_{n+1},q_n)=0$

If $M_p(q_{n+1},q_n)=p(q_{n+1},q_{n+2})$ then

$$p(q_{n+1},q_{n+2})\le H_p(Fq_n,Fq_{n+1})\le\beta(p(q_{n+1},q_{n+2}))p(q_{n+1},q_{n+2})<p(q_{n+1},q_{n+2})$$

Which is a contradiction

So $M_p(q_{n+1},q_{n+2})=p(q_{n+1},q_n)$ it following

$$0<p(q_{n+2},q_{n+1})\le\beta(p(q_{n+1},q_n))p(q_{n+1},q_n)<p(q_{n+1},q_n) \tag{3}$$

Hence the sequence $\{p(q_{n+1},q_n)\}$ is decreasing and bounded below, thus it converges to some $a\ge 0$. Indeed $a=0$. If we suppose $a>0$. From (3) we have

$$\frac{p(q_{n+2},q_{n+1})}{p(q_{n+1},q_n)}\le\beta(p(q_{n+1},q_n))<1,\quad \forall n\in N$$

Which yields that $\lim_{n\to\infty}\beta(p(q_{n+1},q_n))=1$ and since $\beta\in S$, we have

$\lim_{n\to\infty}p(q_{n+1},q_n)=0$, that is $a=0$ a contradiction to the assumption $a>0$

Hence $a=0$ and $\lim_{n\to\infty}p(q_{n+1},q_n)=0$.

Now to show and $\{P(q_{m(k)-1},q_{n(k)+1})\}$ $\{q_n\}$ is a Cauchy sequence in (M,p), suppose $\{q_n\}$ is not a Cauchy and by using lemma (2.11) there exists $\varepsilon>0$ and two sequence $\{m(k)\},\{n(k)\}$ of positive integers, with $m(k)<n(k)$, such that $\{p(q_{m(k)},q_{n(k)})\}$, $\{p(q_{m(k)},q_{n(k)+1})\}$, $\{p(q_{m(k)-1},q_{n(k)})\}$ and $\{p(q_{m(k)-1},q_{n(k)+1})\}$ tends to ε as $k\to\infty$.

Putting in condition (1) $m=q_{m(k)-1}$ and $n=q_{n(k)}$, it follows that.

$$p(q_{m(k)},q_{n(k)+1})\le H_p(Fq_{m(k)-1},Fq_{n(k)})$$

$$\le\beta(M_p(q_{m(k)-1},q_{n(k)}))M_p(q_{m(k)-1},q_{n(k)})+LN_p(q_{m(k)-1},q_{n(k)}) \tag{4}$$

Where

$$M_p(q_{m(k)-1},q_{n(k)})=\max\{p(q_{m(k)-1},q_{n(k)}),p(q_{m(k)-1},Fq_{m(k)-1}),p(q_{n(k)},Fq_{n(k)}),$$
$$\tfrac{1}{2}[p(q_{m(k)-1},Fq_{n(k)})+p(q_{n(k)},Fq_{m(k)-1})]\}$$

$$=\max\{p(q_{m(k)-1},q_{n(k)}),p(q_{m(k)-1},q_{m(k)}),p(q_{n(k)},q_{n(k)+1}),$$
$$\tfrac{1}{2}[p(q_{n(k)-1},q_{n(k)+1})+p(q_{m(k)},q_{n(k)})]\}$$

$$N_p(q_{m(k)-1}, q_{n(k)}) = \min\{p^w(q_{m(k)-1}, Fq_{m(k)-1}), p^w(q_{n(k)}, Fq_{n(k)}), p^w(q_{n(k)}, Fq_{m(k)-1}), p^w(q_{m(k)-1}, Fq_{n(k)})\}$$

$$= \min\{p^w(q_{m(k)-1}, q_{n(k)}), p^w(q_{n(k)}, q_{n(k)+1}), p^w(q_{n(k)}, q_{n(k)}), p^w(q_{n(k)-1}, q_{n(k)+1})\}$$

Since $p^w(q_{n(k)}, q_{n(k)}) = 0$ it follows that $N_p(q_{m(k)-1}, q_{n(k)}) = 0$

Letting $k \to \infty$ we get

$$\lim_{m,n\to\infty} M_p(q_{m(k)-1}, q_{n(k)}) = \varepsilon \qquad (5)$$

and since $N_p(q_{m(k)-1}, q_{n(k)}) = 0$,then from (4) we have

$$\frac{p(q_{m(k)}, q_{n(k)+1})}{M_p(q_{m(k)-1}, q_{n(k)})} \le \beta(M_p(q_{m(k)-1}, q_{n(k)})) < 1 \text{ for all } n \in N$$

Letting $k \to \infty$ we get

$\lim_{k\to\infty} \beta(M_p(q_{m(k)-1}, q_{n(k)})) = 1$, since $\beta \in S$ we have

$\lim_{k\to\infty} M_p(q_{m(k)-1}, q_{n(k)}) = 0$ Which is contradiction to (5).

Therefore $\{q_n\}$ is a Cauchy in (M, p). since (M, p) is complete it follow that $\{q_n\}$ converges to $q \in M$ and

$$p(q,q) = \lim_{n\to\infty} p(q_n, q) = \lim_{n,m\to\infty} p(q_n, q_m) = 0 \qquad (6)$$

Now we show that $q \in M$ is a (F.P) of F i.e $q \in Fq$

If $p(q, Fq) > 0$ by using (p_4) and condition (1) we get

$$p(q,Fq) \le p(q,q_{n+1}) + p(q_{n+1},Fq) - p(q_{n+1},q_{n+1})$$
$$p(q,Fq) \le p(q,q_{n+1}) + p(q_{n+1},Fq) \le p(q,q_{n+1}) + H_p(Fq_n, Fq)$$
$$p(q,Fq) \le p(q,q_{n+1}) + \beta(M(q_n,q))M_p(q_n,q) + LN_p(q_n,q) \qquad (7)$$

Where

$$M_p(q_n,q)=\max\{p(q_n,q),p(q_n,Fq_n)p(q,Fq),½\,p(q_n,Fq)+p(q,Fq_n)]\}$$
$$N_p(q_n,q)=\min\{p^w(q_n,Fq_n),p^w(q,Fq),p^w(q_n,Fq),p^w(q,Fq_n)\}$$

as $n\to\infty$

$$\lim_{n\to\infty}M_p(q_n,q)=p(q,Fq) \tag{8}$$
$$\lim_{n\to\infty}N_p(q_n,q)=0$$

Letting $n\to\infty$ in (7) we have

$p(q,Fq)\le\beta(p(q,Fq))p(q,Fq)<p(q,Fq)$ Which is contradiction $p(q,Fq)>0$

Thus $p(q,Fq)=0$ and $q\in Fq$ hence q is a (F.P) of F.

By taking $L=0$ in Theorem (3.1), we obtain the following results.

Corollary (3.2)

let (M,p) be a complete P.M.S and $F:M\to C(M)$ be a multivalued map . Suppose that there exists $\beta\in S$ such that for all $m,n\in M$,

$$H_p(Fm,Fn)\le\beta(M_P(m,n))M_P(m,n) \tag{9}$$

where $M_P(m,n)=\max\{p(m,n),p(m,Fm),p(n,Fn),½[p(m,Fn)+p(n,Fm)]\}$.

Then F has (F.P) q, Moreover $p(q,q)=0$.

If in Theorem (3.1) we put $\beta(t)=\lambda$, $\lambda\in[0,1)$. Then we have the following corollary.

Corollary (3.3)

Suppose (M,p) be a complete P.M.S and $F:M\to C(M)$ be a multivalued map. suppose that there exists $\lambda\in[0,1)$ and $L\ge0$ such that

$$H_p(Fm,Fn)\le\lambda M_P(m,n)+LN_P(m,n) \tag{10}$$

for all $m,n\in M$, where

$$M_P(m,n)=\max\{p(m,n),p(m,Fm),p(n,Fn),½[p(m,Fn)+p(n,Fm)]\}$$
$$N_P(m,n)=\min\{p^w(m,Fm),p^w(n,Fn),p^w(m,Fn)+p^w(n,Fm)\}.$$

Then F has (F.P) q, i.e. $q\in Fq$. Moreover $p(q,q)=0$

We remark that in the case of single -valued mappings

Theorem (3.1) is a generalization of theorem (3) of M. Dinarvand [11]

Corollary (3.2) is a generalization of theorem (3.1) of Dukic et al [19]

Also, by taking $L=0$ in corollary (3.3) we obtain the Cric (F.P) theorem [20] in the setting of a metric space.

Now we give an example to support our main result. In this example there is a partial metric and a contractive condition (1) satisfying the hypotheses of Theorem (3.1) but do not satisfy in the setting of usual metric d.

Example (3.4)

Let $M=\{0,\frac{1}{2},1\}$ be endowed with partial metric space $p:M^2\to\Re^+$ defined by

$$p(0,0)=p(\tfrac{1}{2},\tfrac{1}{2})=0$$

$$p(1,1)=\tfrac{1}{4}\quad p(0,\tfrac{1}{2})=\tfrac{1}{3}\quad p(0,1)=\tfrac{11}{24}\quad p(\tfrac{1}{2},1)=\tfrac{1}{2}$$

and $p(m,n)=p(n,m)$ for every $m,n\in M$, then (M,P) is a complete partial metric space.

Define $F:M\to C(M)$ such that $F(0)=F(\frac{1}{2})=\{0\}$, $F(1)=\{\frac{1}{2}\}$

and let the map β be defined by $\beta(t)=\frac{3}{4}$ for all $t\geq 0$. We shall show that for all $m,n\in M$ the condition (1) is satisfied. For this, we have the following cases

(i) $H_p(F\tfrac{1}{2},F1)=H_p(F0,F1)=H_p(\{0\},F\{\tfrac{1}{2}\})=p(0,\tfrac{1}{2})=\tfrac{1}{3}$

On other hands

$$M_p(0,1)=\max\{p(0,1),p(0,F0),p(1,F1),\tfrac{1}{2}[p(0,F1)+p(1,F0)]\}$$

$$\max\{p(0,1),p(0,0),p(1,\tfrac{1}{2}),\tfrac{1}{2}[p(0,\tfrac{1}{2})+p(1,0)]\}=\{\tfrac{11}{24},0,\tfrac{1}{2},\tfrac{1}{2}[\tfrac{1}{3}+\tfrac{11}{24}]\}=\tfrac{1}{2}$$

Then $M_p(0,1)=\frac{1}{2}$ and $\beta(M_p(0,1))M_p(0,1)=\frac{3}{8}$

Since $LN_p(0,1)=0$,we have $\beta(M_p(0,1))M_p(0,1)+LN_p(0,1)=\frac{3}{8}$ and thus condition (1) is satisfied.

$(ii)\, H_p(F0,F\tfrac{1}{2})=H_p(\{0\},\{0\})=0\leq\beta(M_p(0,1))M_p(0,0)+LN_p(0,0)$

(iii) For all $m = n$, $m,n \in \{0, \tfrac{1}{2}, 1\}$ we have, $H_p(Fm, Fn) = 0$ Thus all conditions of Theorem (3.1) are satisfied. Hence 0 is a fixed point of F. On the other hand, the metric d_p induced by the partial metric p is given by

$$d_p(m,n) = 2p(m,n) - p(m,m) - p(n,n)$$
$$d_p(0,0) = d_p(\tfrac{1}{2}, \tfrac{1}{2}) = p(1,1) = 0$$
$$d_p(\tfrac{1}{2}, 1) = d_p(1, \tfrac{1}{2}) = \tfrac{3}{4}$$
$$d_p(0,1) = d_p(1,0) = \tfrac{2}{3}$$
$$d_p(0, \tfrac{1}{2}) = d_p(\tfrac{1}{2}, 0) = \tfrac{2}{3}$$

Now, we show that Theorem (3.1) is not applicate in the setting of usual metric spaces. We have,

$$H(F0, F1) = H(\{0\}, \{\tfrac{1}{2}\}) = \max\{\sup\{d_p(\{0\}, \{\tfrac{1}{2}\})\}, \{\sup\{d_p(\{\tfrac{1}{2}\}, \{0\})\}\}$$
$$= d_p(0, \tfrac{1}{2}) = \tfrac{2}{3}$$
$$M_d(0,1) = \max\{d_p(0,1), d_p(0, F0), d_p(1, F1), \tfrac{1}{2}[d_p(0, F1) + d_p(1, F0)]\}$$
$$= \max\{d_p(0,1), d_p(0, \{0\}), d_p(1, \{\tfrac{1}{2}\}), \tfrac{1}{2}[d_p(0, \{\tfrac{1}{2}\}) + d_p(1, \{0\})]\}$$
$$= \max\{\tfrac{2}{3}, 0, \tfrac{3}{4}, \tfrac{2}{3}\} = \tfrac{3}{4}$$
$$\beta(M_d(0,1))M_d(0,1) = (\tfrac{3}{4})(\tfrac{3}{4}) = \tfrac{9}{16}$$

Since $LN_d(0,1) = 0$ then $H(F0, F1) \geq \beta(M_d(0,1))M_d(0,1) + LN_d(0,1)$

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حول نتائج النقطة الصامدة للدوال متعددة القيم في الفضاء المتري الجزي الهوزدورفي

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المستخلص

يتناول هذا البحث استعراض ودراسة النقاط الصامدة تحت شرط جيرتي باستخدام الفضاءالمتري الجزئي. النتائج التي حصلنا عليها هي تحسين وتوحيد العديد من النتائج في مبرهنات النقطة الصامدة وتعميم بعض النتائج الحديثة في الفضاء المتري الجزئي المرتب