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# Slow Light Using Tunneling-Induced Transparency in Triple Semiconductor Conical Quantum Dot Molecule

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**Abstract.** The electronic states of InAs/GaAs quantum dot has investigated for triple conical quantum dots molecule (CQDM). We calculated Eigen energies as a function of external voltage. Tunneling-induced transparency (TIT) in triple InAs/GaAs quantum dots using tunneling instead of pump laser, analogous to electromagnetically induced transparency (EIT) in atomic systems, has studied. The interdot quantum coupling strength is tuned by static electric fields. For parameters appropriate to a 100 Gbits/s optical network, slow down factor (SDF) as  $10^9$  can be achieved. The scheme is expected to be useful to construct a variable semiconductor optical buffer based on TIT in triple InAs/GaAs quantum dots controlled by electric fields.

**Keywords:** quantum dot, Tunneling-induced transparency, slow down factor.

## INTRODUCTION

During the last few years, triple quantum dot (TQD) is one of the multi-QDs that has been subject of intensively interest due to interesting features such as internal degrees of freedom that emerge over and done with orbital gesture along the triangular shape. Also, the system has several dissimilar possible attentions of inhabiting electrons [1], [15]. Moreover, TQD are receiving much attention, due to extra regulatory factors cannot be create in two QDs because the former has multilevel structure. Transmission-dispersion spectrum, multiple transparency windows and cavity linewidth narrowing Kerr nonlinearity and resonance fluorescence spectrum of TDQ are investigated theoretically [2]. Due to its triangular arrangement, TQDs have attracted interest in the fundamental coherence phenomena, such as spin entanglers, CPT, Kondo effect, dark state and spin-polarized currents [3].

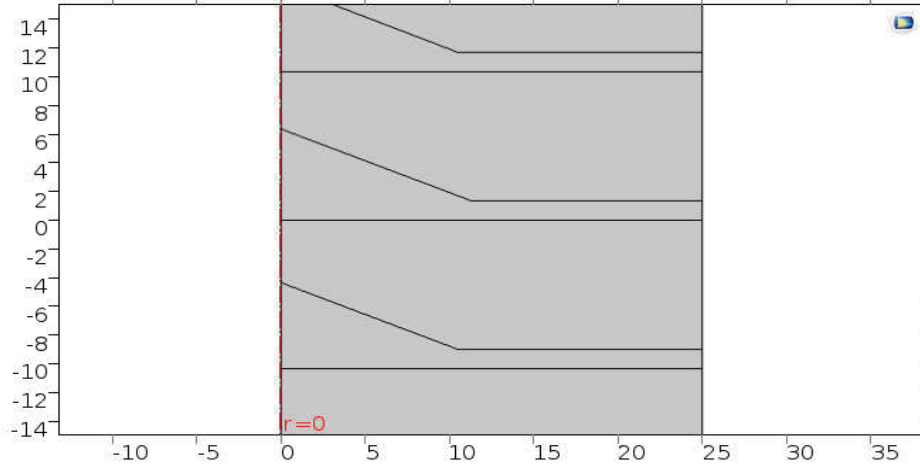
Extensive theoretical studies of the electronic and optical properties of QDs have been performed by several groups [4], [5], [6], [7], [8]. However, no theoretical studies of the electron tunneling rate for conical quantum dots of real geometry have been reported to date. In light of the promising IR detector application of QD systems, it is desirable to investigate the electron tunneling rate, so that the dark current of the QD device can be assessed.

Slow light phenomena have potential applications such as optical communication (all optical buffers, ultrasensitive switches) [9], nonlinear optics with low optical intensity, and quantum information storage [10]. The design of slow light devices using semiconductor nanostructures is of great interest because the use of semiconductor components have widespread in optoelectronics and these devices can be potentially integrated with other components in an optical communication systems. The “quantum dot molecule” QDM consisting of a vertically stacked pair of InAs/GaAs islands formed via strain driven self-assembly is a promising system for communication. Recently, interdot coupling controlled by applying an external electric field in an individual QDM was observed [11].

Due to the geometrical complexity, a numerical method of finite element has been adopted to solve Schrodinger's equation with strained potential. The calculations are done by using "Comsol" framework and homemade MATLAB codes. Energy eigenvalues has been calculated as a function of each of the following parameters: external voltage, wetting layer and QDM spacer where other parameters of a single conical-shaped are

held constant. The maximum obtainable slow down factor (SDF) (which it is a measure of the group-velocity reduction), was examined. The SDF is a figure of merit relevant for optical storage. In this paper, we investigate the interaction of a triple QDM hold through the tunneling coupling in its place of coupling laser. We examine the stationary optical response of a TQD molecules controlled through the tunneling coupling and thereafter the produced slow light propagation. We can use the density matrix and get a general analytical expression for the linear susceptibility of the signal-laser field. CQD molecule structure has been selected and optimized for lasting quantum coherences and calculate the group-velocity SDF for this structure. We use proper parameters of strained QDs from our electronic band structure model using COMSOL Multiphysics software and homemade Matlab codes for estimating SDF.

As mention in chapter two we do the same steps taking into account triple instead of one QD, as shown in Fig. 1

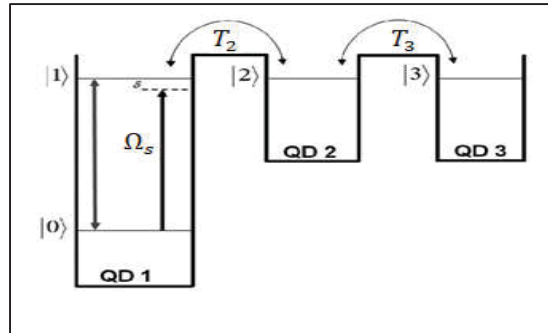


**FIGURE 1.** 2D geometry of a triple quantum dot with a wetting layer.

The confined electronic states of the self-assembled InAs/GaAs vertically-aligned CQDM structures with WL are obtained through solving the stationary state Schrödinger equation. With this technique, QDs are grown up in layers separated by a spacer. For this kind of QD molecules we tend to study the effect of the external voltage and analyze the resulting energy spectra and dipole moments

## THE METHOD

The schematic illustration of energy levels is shown in Fig. 2. At interdot nanoscale parting, the electron state is rather delocalized and the hole states are localized in the QD.



**FIGURE 2.** Diagram of TQDs (QD<sup>(1)</sup>, QD<sup>(2)</sup>, and QD<sup>(3)</sup>).

Initially, there are no excitons confidential totally the QDs, the grand state (G.S)  $|0\rangle$ . Exciton is created directly when the QD is illuminated by a laser beam, which relates to state  $|1\rangle$ . The external voltage changes the band structure, allowing the electron to shafts of QD(1) to the QD(2) and then to QD(3) creating excitons in direct, which referred as states

$|2\rangle - |3\rangle$ . Via registration a door electrode among them near QD the channel barrier in a TQDs may loosened. The Hamiltonian of this system is given by

$$H = \sum_{j=0}^3 E_j |j\rangle\langle j| + [(\Omega_s e^{-i\omega_s t} |0\rangle\langle 1| + T_2 |2\rangle\langle 1| + T_3 |3\rangle\langle 2| + H.C)], \quad (1)$$

Where  $T_2$  and  $T_3$  are the tunneling coupling. The system has been investigated by using for density matrix technique. From the Liouville Eq. we get the next Eq.s for the density-matrix elements

$$\dot{\rho}_{01} = -i[\Omega_s(\rho_{11} - \rho_{00}) + T_2\rho_{02} + T_3\rho_{03}] + (i\Delta_s + \gamma_{01})\rho_{01}, \quad (2a)$$

$$\dot{\rho}_{02} = -i(T_2\rho_{01} - \Omega_s\rho_{12}) + \left[\frac{i}{2} + (\Delta_1 + \Delta_2) - \gamma_{02}\right]\rho_{02}, \quad (2b)$$

$$\dot{\rho}_{03} = -i(T_3\rho_{01} - \Omega_s\rho_{13}) + \left[\frac{i}{2}(\Delta_1 + \Delta_3) - \gamma_{03}\right]\rho_{03}, \quad (2c)$$

$$\dot{\rho}_{11} = -i[\Omega_s(\rho_{10} - \rho_{01}) + T_2(\rho_{12} - \rho_{21}) + T_3(\rho_{13} - \rho_{31})] - \Gamma_{10}\rho_{11}, \quad (2d)$$

$$\dot{\rho}_{12} = -i[T_2(\rho_{11} - \rho_{22}) + \Omega_s\rho_{02} + T_3\rho_{32}] + \left[\frac{i}{2}(\Delta_2 + \Delta_1) - \gamma_{12}\right]\rho_{12}, \quad (2e)$$

$$\dot{\rho}_{13} = -i[T_3(\rho_{11} - \rho_{33}) + \Omega_s\rho_{03} + T_2\rho_{23}] + \left[\frac{i}{2}(\Delta_3 + \Delta_1) - \gamma_{13}\right]\rho_{13}, \quad (2f)$$

$$\dot{\rho}_{22} = -iT_2(\rho_{21} - \rho_{12}) - \Gamma_{20}\rho_{22}, \quad (2g)$$

$$\dot{\rho}_{23} = -i(T_3\rho_{21} - T_2\rho_{13}) + \left[\frac{i}{2}(\Delta_3 + \Delta_2) - \gamma_{23}\right]\rho_{23}, \quad (2h)$$

$$\dot{\rho}_{33} = -iT_3(\rho_{31} - \rho_{13}) - \Gamma_{30}\rho_{33}, \quad (2i)$$

$$\dot{\rho}_{ij} = -\dot{\rho}_{ji}^* \quad (2j)$$

$$\rho_{00} + \rho_{11} + \rho_{22} + \rho_{33} = 1. \quad (2k)$$

The detunings are defined as  $\Delta_1 = \omega_{01} - \omega$ ,  $\Delta_2 = \Delta_1 + 2\omega_{21}$ , and  $\Delta_3 = \Delta_1 + 2\omega_{31}$ , where  $\Gamma_{10}$ ,  $\Gamma_{20}$  and  $\Gamma_{30}$  are represent of radiative decay rate of populaces starting  $|1\rangle \rightarrow |0\rangle$ ,  $|2\rangle \rightarrow |0\rangle$  also  $|3\rangle \rightarrow |0\rangle$ , and  $\gamma_1, \gamma_2$ , and  $\gamma_3$  are the pure dephasing rate. Therefore the  $\gamma_{mn}$  between level  $|m\rangle$  and  $|n\rangle$  container be got

$$\gamma_{0n} = \gamma_{n0} = \frac{1}{2}(\Gamma_{n0} + \gamma_n), (n = 1, 2, 3) \quad (3a)$$

$$\gamma_{mn} = \gamma_{nm} = \frac{1}{2}(\Gamma_{m0} + \Gamma_{n0} + \gamma_n + \gamma_m), (m \neq n; n = 1, 2, 3) \quad (3b)$$

Assuming that the system in G.S  $|0\rangle$  at  $t = 0$ , i.e.,  $\rho_{00}(0) = 1$ . Acceptable to inspect the  $\chi'$  and  $\chi''$  properties of a feeble signal-laser field coupling states  $|0\rangle$  and  $|1\rangle$  we determine the steady-state 1st order of susceptibility, with (absorption -refractive index) determined via the( imaginary -real) part of the susceptibility. linear susceptibility can be expressed as

$$\chi(\Delta_1) = -\frac{\Gamma_{opt} |\mu_{01}|^2}{V \varepsilon_0 \Omega_s} \rho_{01}(t \rightarrow \infty) \quad (4)$$

Assuming that  $\rho_{00}(t) \approx 1$  used for all times. Solving Eq. (2) for the steady-state limit, and using the first order approximation solve for  $\rho_{01}(t)$ . Linear susceptibility reads in Eq.

$$\chi(\Delta_1) = -\frac{\Gamma_{opt} |\mu_{01}|^2}{V \varepsilon_0} \frac{1}{\Delta_1 + i\gamma_{01} - \frac{T_2^2}{\Delta_1 - \omega_{21} + i\gamma_{02}} - \frac{T_3^2}{\Delta_1 - \omega_{31} + i\gamma_{03}}} \quad (5)$$

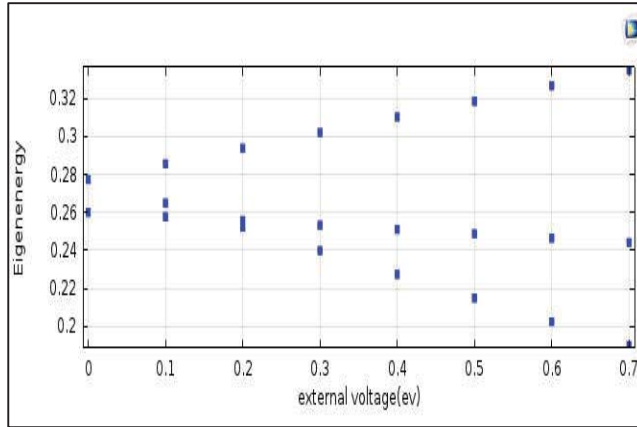
The group velocity is define in the Eq. [3]

$$v_g = \frac{c}{1 + \frac{1}{2}Re(\chi) + \frac{\omega_s dRe(\chi)}{2 d\Delta_1}} \quad (6)$$

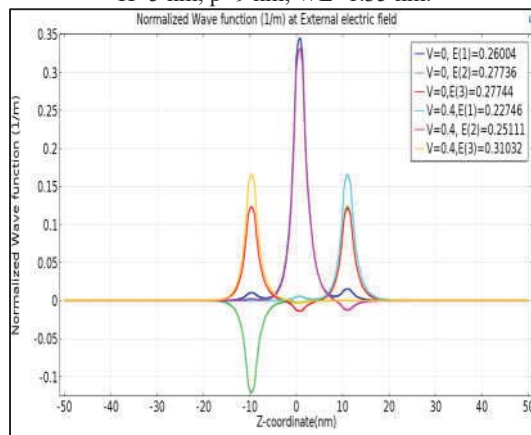
## RESULTS AND DISCUSSION

### Energy Levels as a Function of External Voltage

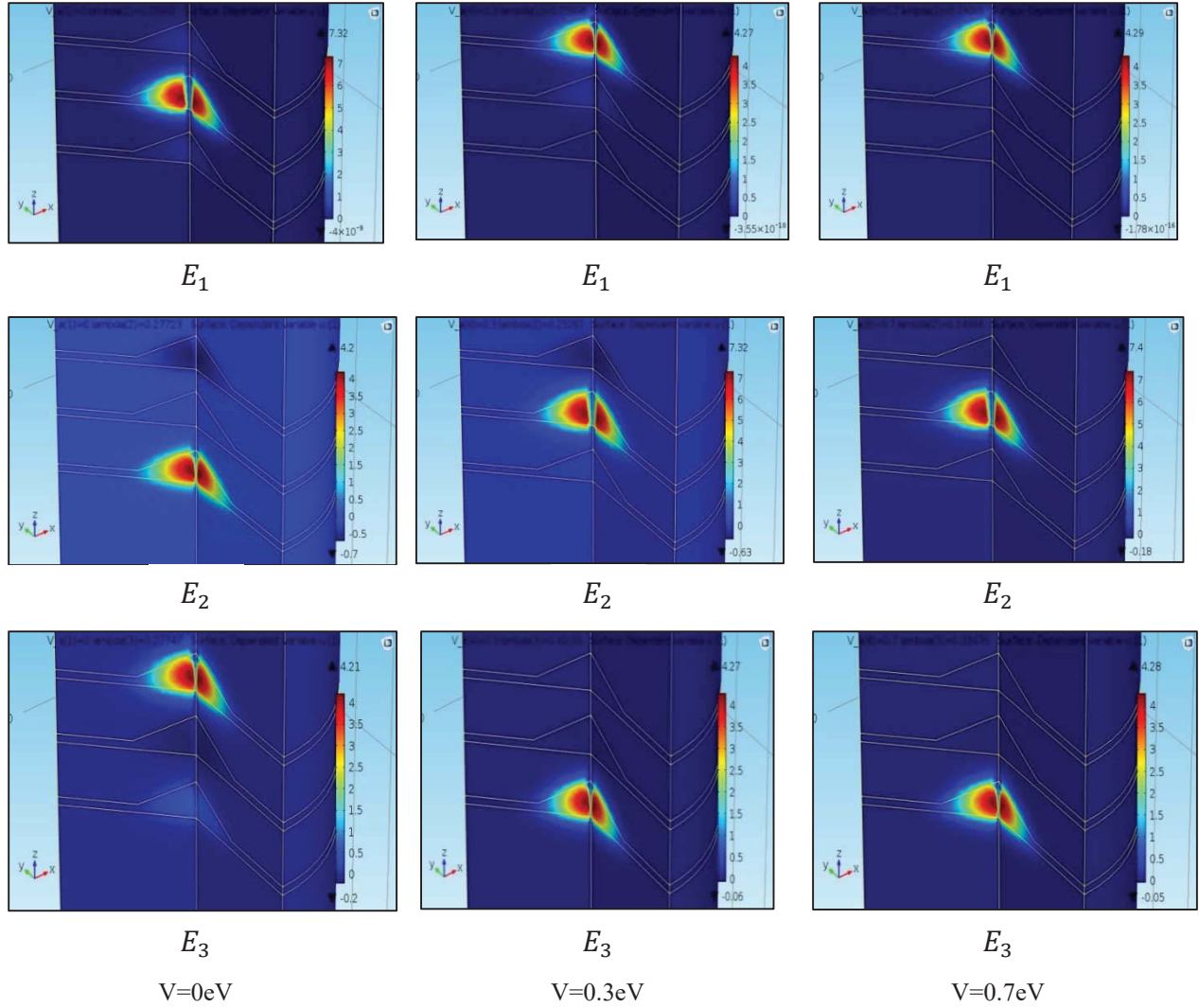
We have made changes to the external voltage values of the TCQD applied within the range (0-0.7) eV. The energy eigenvalues corresponding to excited states, normalized wave function and in 3D are shown in Fig. 3, Fig. 4, Fig. 5 respectively.



**FIGURE 3.** Energy eigenvalues corresponding to 1st excited states QDM as a function of the external voltage for  $R=11.5$  nm,  $H=5$  nm,  $p=9$  nm,  $WL=1.35$  nm.



**FIGURE 4.** Normalized wave functions at different external electric fields.



**FIGURE 5.** Energy eigenvalues of electrons in the C.B at difference external voltage in 3D.

Symmetrical QD is reflected to asymmetrical in case a present of external electrical field where, if the electric field is strong and increasing then, the energy of a bound state is lowered as long as the energy level remains in quadrangular region of the well. The energy level is transferred into triangular region of the well (that describes the increasing of bound state energy of the well middle) through an electrical field increasing. The dipole interband elements for the electronic states are calculated. For TQD just diagonal transitions between electronic states for the intersubband matrix element would be dipole allowed is shown in Table 1.

**TABLE 1.** Conical triple QD interband  $\mu_{01}$  for TQDM of radius 11.3 nm and WL thickness 1.3nm at different external fields.

External Voltage	$\mu_{01}/e$
$V_a(\text{eV})$	( $\text{\AA}$ )
0	0.693
0.1	0.714
0.2	0.734
0.3	0.755
0.4	0.775
0.5	0.796
0.7	0.816
0.8	0.837

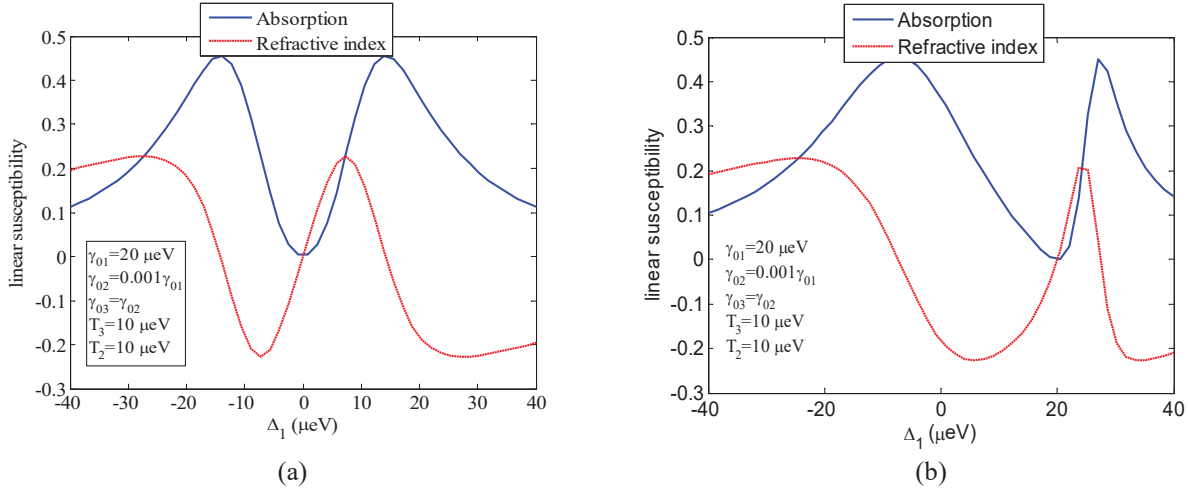
## The Susceptibility

For our investigation, we used the parameter from Ref [12] such as dephasing rate and the radiative decay rate of populations. The other parameters for example the optical confinement factor can be found in Ref. [13], momentum matrix element  $\frac{\mu_{01}}{e} = 0.693\text{\AA}$  and CQD with radius=12.5 nm and height=5.56 nm are used. By the barrier characteristics may be exact of the tunneling couplings and also by the external voltage are selected from Ref. [1]. For such consideration, by resolving numerically the attached Eq.s(5) we investigate refractive index and the absorption disposal of the intermediate with attendance of the two  $T_e$  couplings. The tunneling couplings may be controlled through the external electric field. First, we assume the state of  $\omega_{21} = \omega_{31}$ . The real and imaginary (red dotted line and blue solid line) parts of the susceptibility got mathematically with of  $\Delta_1$  is shown in Fig. 6. When  $\omega_{31} = 0$  we can see one symmetrical transparency window happens at the position of  $\Delta_1 = 0$  with a Fig. [blue solid line in Fig. 6a]. However the refractive index curve [red dotted line in Fig. 6a] shows a sharp difference around  $\Delta_1 = 0$ , which appears in the refractive index a big positive derivative. In Fig. 6b, we offer the state of  $\omega_{31} \neq 0$ . The position of the transparency window occurs for  $\Delta_1 = \omega_{31}$  with asymmetrical shape (blue solid line), and the refractive index curve also displays a sharp difference around  $\Delta_1 = \omega_{31}$  (red dotted line). Lastly from Eq. (6) we can see the slow light depends on  $[d\text{Re}(\chi)/d\omega] > 0$ . The slow light signals get when frequencies fall into the transparency windows joined of the refractive index. Next, we consider the case of  $\omega_{21} \neq \omega_{31}$ . The real and imaginary (red dotted line and blue solid line) parts of the optical susceptibility got numerically with  $\Delta_1$  is shown in Fig. 7. When  $\omega_{21} = -\omega_{31}$ , we have two transparency windows at the position of  $\Delta_1 = -\omega_{21}$  and  $\Delta_1 = -\omega_{31}$  with a symmetrical shape [blue solid line in Fig. 7a]. That is the transparency window in a double QD system will be separated into two transparency windows for TQD. Concurrently, the refractive index curves [red dotted line in Fig. 7a] at the center have same slope of the two transparency windows can propagations subliminal. This symmetrical spectrum has advantage that two signals may propagate with the similar group velocity with changed frequencies and slowed down with equal SDF. By varying  $\omega_{21}$ , the  $\chi'$  can be changed at the duple transparency windows, as showed in Fig. 7b, The two transparency windows located at the position of  $\Delta_1 = -\omega_{21}$  or  $\Delta_1 = -\omega_{31}$  can be made very narrow by selecting the suitable parameters. As can be seen, the curve of refractive index presents slope at their center of the transparency windows that agree to with high value of SDF. We could controlled the two weak pulses by change group velocities various with central frequencies spreading. And by selecting a suitable parameters, an ultra-slow light can be realized. The greater value of slope for refractive index is, the slower the propagating of light is. To get much greater slope of the refractive index curves, ultra-narrow transparency windows must be created. Via setting tunneling coupling and also the external voltage, the transparency windows could been made very narrow. These results display that the transparency window, in addition to the positive derivative in the  $\chi''$  could be controlled through varying the electric field ( $\omega_{21}$  or  $\omega_{31}$ ) and tuning the laser detuning, for example the condition  $\omega_{21} + \Delta_1 = 0$  or  $\omega_{31} + \Delta_1 = 0$  is fulfilled. In addition, as can be seen from Fig. 8, the absorption peak between  $\omega_{21} = \Delta_1$  and  $\omega_{31} = \Delta_1$  can become very narrow if the tunneling coupling  $T_2$  and  $T_3$  are increased [see Fig. 8a] or the value of  $\omega_{21} - \omega_{31}$  are decreased [see Fig. 8b], while the outboard one near  $\omega_{21} = \Delta_1$  or  $\omega_{31} = \Delta_1$  when we select a large value became quite narrow of  $\omega_{31}$  or  $\omega_{31}$  [see Fig. 8c].

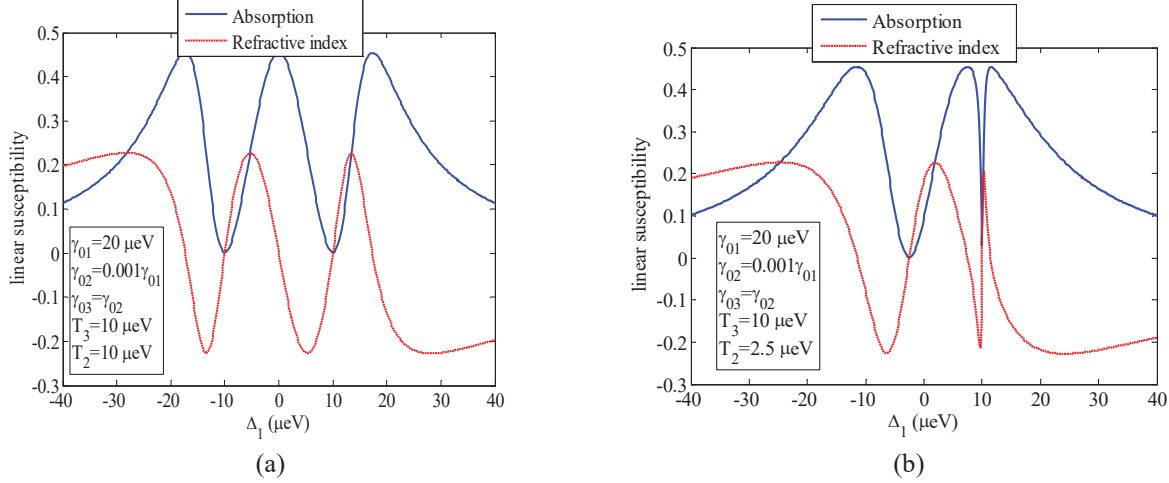
The curves of dispersion separated into 1<sup>st</sup> normal dispersion and 2<sup>nd</sup> anomalous dispersion systems, that's well known. In the 1<sup>st</sup> dispersion (normal) system the signal field is a slow light because the group velocity in medium smaller than velocity of light in space i.e. ( $V_g < c$ ); but in the anomalous dispersion system we find the group

velocity in medium greater than velocity of light in space i.e. ( $V_g > c$ ), that's lead to the signal field is superluminal.

From Fig. 7b, it is clearly that the normal dispersion system agree with to the TIT transparency window i.e. the signal field has a negligible absorption (slow light), while the anomalous dispersion system agree with absorption peaks i.e.; the signal field has a superluminal (strong absorption). This result proves that it is certainly to obtain double switching through find the sign of  $\frac{\partial[ReK(\omega)]}{\partial\omega}$  has changed with the enhancement of the interdot tunneling coupling strength from negative to positive value in which switching from the anomalous dispersion system to the normal dispersion system.

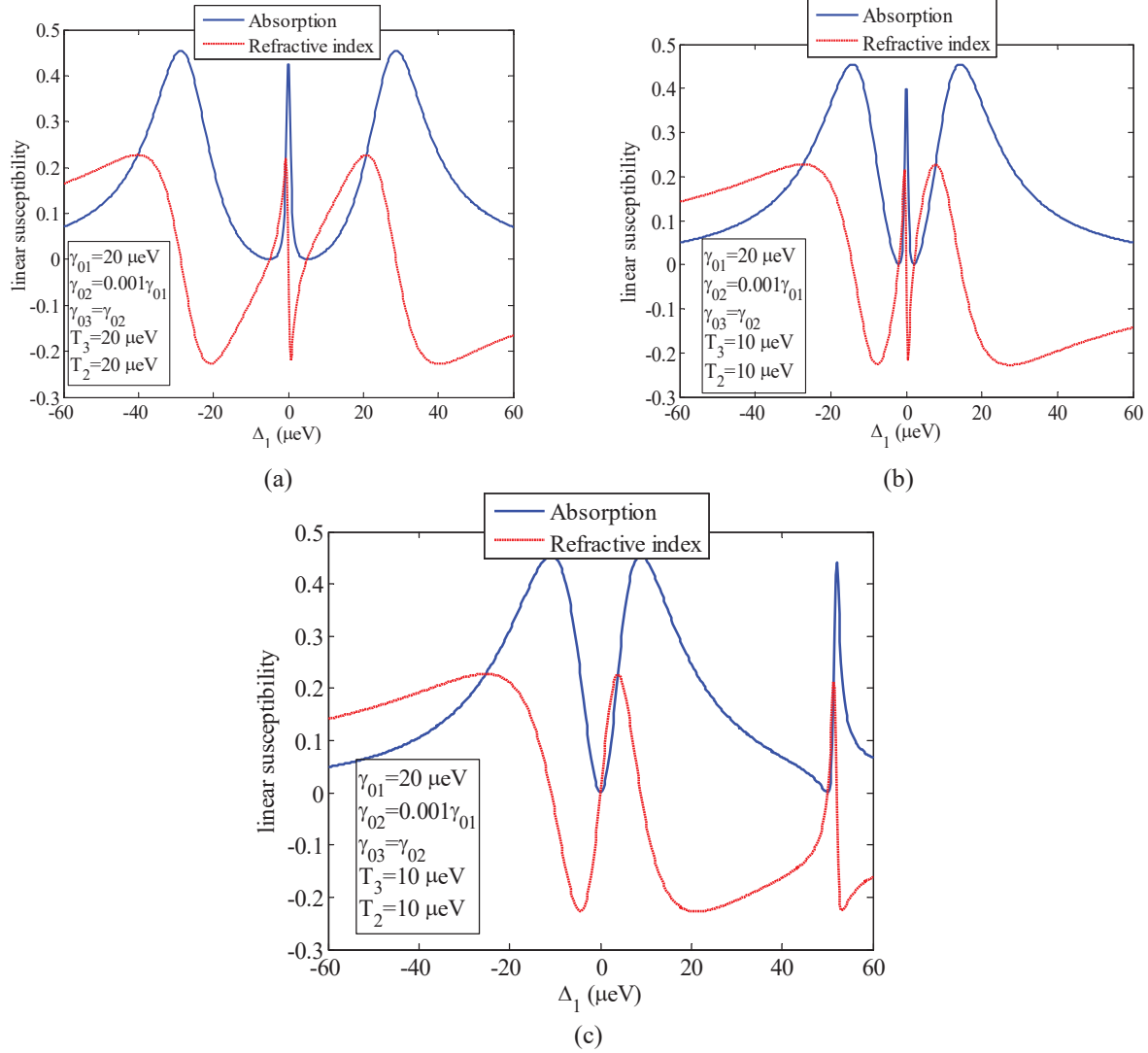


**FIGURE 6** The refractive index spectra (curves is red dotted) and absorption (curves is blue solid) as a function of detuning  $\Delta_1$  for a TQDs.(a)  $\omega_{21} = \omega_{31} = 0$ , (b)  $\omega_{21} = \omega_{31} = 20\mu\text{eV}$ .



**FIGURE 7.** The refractive index spectra (curves is red dotted) and absorption (curves is blue solid) as a function of detuning  $\Delta_1$  for a TQDs.(a)  $\omega_{21} = 10\mu\text{eV}$ ,  $\omega_{31} = -10\mu\text{eV}$ , (b)  $\omega_{21} = 10\mu\text{eV}$ ,  $\omega_{31} = -2.5\mu\text{eV}$ .



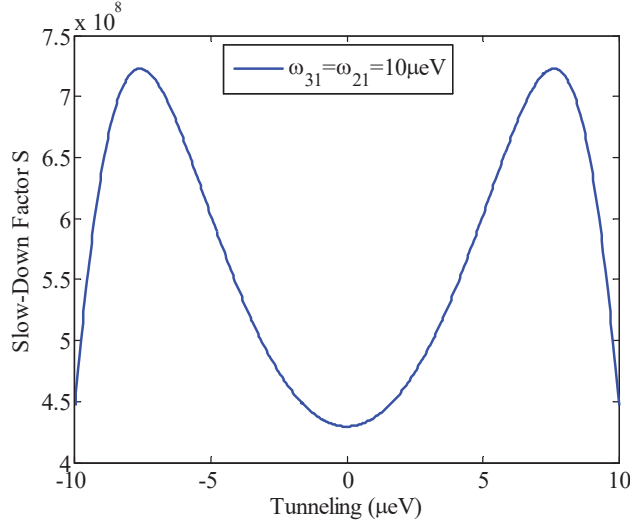


**FIGURE 8.** The refractive index spectra (curves is red dotted) and absorption (curves is blue solid) as a function of detuning  $\Delta_1$  for a TQDs.(a)  $\omega_{21} = 5\mu eV, \omega_{31} = -5\mu eV$ ,(b)  $\omega_{21} = 2\mu eV, \omega_{31} = -2\mu eV$ ,(c)  $\omega_{21} = 50\mu eV, \omega_{31} = 0$ .

Study the steady-state behavior in a TQDs medium of a low signal field with external voltage. Its's note properties of absorption and the dispersion in the medium can be controlled through tunneling between neighboring QDs and has also the medium can be used as an optical switch by adjusting a proper external voltage can be through it control of the propagation by the laser pulse. TQD system can be used for many application in processing and transmission information of optical for example high-speed optical switch for quantum networks.

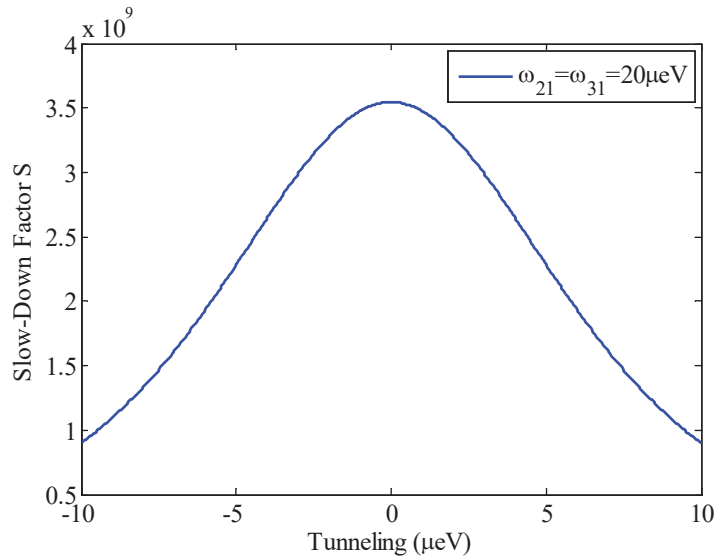
### Calculation Slow-Down Factor

By using Eq.(6) we can calculated SDF at  $T_3=10\mu eV$  is shown in Fig. 9 and Fig. 10, using parameters defined as follows:  $\Gamma = 6 \times 10^{-3}$ , momentum matrix element  $\frac{\mu_{01}}{e} = 0.693 \text{ \AA}$  and CQD of radius=12.5 nm and height=5.56 nm,  $\omega_{21} = \omega_{31} = 10 \mu eV$ ,  $\omega_{21} = \omega_{31} = 20 \mu eV$ . Control of the tunneling couplings can be take through the external electric field and the barrier feature.



**FIGURE 9.** SDF with the tunneling when  $\omega_{21} = \omega_{31} = 10$

Figure 9 displays that the calculated SDF for  $\Delta_1=20 \mu\text{eV}$  is equal to  $10^8$  that meaning the value of group velocity equal to 3 m/s. This result gives a bandwidth of 100 Gbits/s using in semiconductors for communication applications [14].



**FIGURE 10.** Group velocity SDF with the tunneling  $T_2 \mu\text{eV}$  when  $\omega_{21} = \omega_{31} = 20 \mu\text{eV}$

Figure 10 displays that the calculated SDF for  $\Delta_1=20 \mu\text{eV}$  is around  $10^9$  that meaning, the group velocity is equal to 0.3 m/s

## CONCLUSIONS

In conclusion, we have studied slow light using TIT in the triple conical QD molecules and the effect of applied electric field on SDF. The absorption peak and transparency window can be tuned by the applied electric field. The bandwidth of the transparency windows can also be controlled by the applied electric field. In our model, with  $T_e=10\mu\text{eV}$ , we can get SDF about  $10^8$  that meaning the value of group velocity equal to 3 m/s and SDF for  $\Delta_1=20 \mu\text{eV}$  is around  $10^9$  that meaning, the group velocity is equal to 0.3 m/s. Such a system may be applied in all optical buffers, optical switching and filters. Depended on the results obtain, when the voltage of QD is increased, the slope

of real part of refractive index ( $\chi'$ ) curve increases and then, a big value of SDF is achieved. When the values of the external voltages decrease the SDF is increase

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