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17-Brauer trees of spin characters for S_n, n=17, 18, 19, 20

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Abstract : In this paper, we calculate the Brauer trees and the decomposition matrices of the symmetric groups S_n when n= 17, 18, 19, 20 for p=17 by using (r,$\bar{r}$)-inducing.

Keywords: Brauer tree, spin characters, decomposition matrices.

1- Introduction

The Brauer tree is a graph connection between the irreducible ordinary characters in the block of defect one [1]. Brauer expanded the theory of representation from the field with the characteristic zero to the field with the characteristic p>0 and proving that for each ordinary representation, there is a modular representation on p-regular classes. We will found the Brauer trees by calculating the decomposition matrix of spin characters for the symmetric group S_n when n=17,18,19,20 and p=17 and the theoretical is depended on [2].

Notation

A_i	The principle indecomposable spin characters of S_{17}
B_j	The principle indecomposable spin characters of S_{18}
C_k	The principle indecomposable spin characters of S_{19}
D_l	The principle indecomposable spin characters of S_{20}
$\langle \alpha \rangle^*$	The double spin characters
$\langle \alpha \rangle, \langle \alpha \rangle'$	The associate spin characters

2- Brauer trees of spin characters for S_{17}:

In the symmetric group S_{17} there are 57 irreducible spin characters and the covering group has 56 of 17-regular classes then the decomposition matrix of S_{17} has 57 rows and 56 columns.

The symmetric group S_{17} has one block B_1 of defect one and the other spin blocks of defect zero.

2.1 The block B_1:

This block has the irreducible spin characters $\langle 17\rangle^*$, $\langle 16,1\rangle$, $\langle 16,1\rangle'$, $\langle 15,2\rangle$, $\langle 15,2\rangle'$, $\langle 14,3\rangle$, $\langle 14,3\rangle'$, $\langle 13,4\rangle$, $\langle 13,4\rangle'$, $\langle 12,5\rangle$, $\langle 12,5\rangle'$, $\langle 11,6\rangle$, $\langle 11,6\rangle'$, $\langle 10,7\rangle$, $\langle 10,7\rangle'$, $\langle 9,8\rangle$, $\langle 9,8\rangle'$ which is an associate[3].

Lemma (2.1.1):

The Brauer tree of B_1 is:

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          ‹16,1› - ‹15,2› - ‹14,3› - ‹13,4› - ‹12,5› - ‹11,6› - ‹10,7› - ‹9,8›
        /
‹17›*
        \
          ‹16,1›′- ‹15,2›′- ‹14,3›′ - ‹13,4›′ - ‹12,5›′ - ‹11,6›′- ‹10,7›′ - ‹9,8›′
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Proof :

deg{‹17›*, ‹15,2› , ‹15,2›′, ‹13,4› , ‹13,4›′ , ‹11,6› , ‹11,6›′, ‹9,8›, ‹9,8›′ } ≡ 1 mod 17
deg{‹16,1›, ‹16,1›′, ‹14,3›, ‹14,3›′, ‹12,5› ,‹12,5›′ ,‹10,7› ,‹10,7›′ } ≡ −1 mod 17 [4]
by the (r,$\bar{r}$)-inducing of ‹16› , ‹16›′ , ‹15,1›* , ‹14,1›* , ‹13,3›* , ‹12,4›* , ‹11,5›* , ‹10,6›* and ‹19,7›* in S_{16} to S_{17} we have the columns A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 , A_8 , A_9 , A_{10} , A_{11} , A_{12} , A_{13} , A_{14} , A_{15} and A_{16} respectively[5].
we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_1															
1	‹17›*	1	1														
-1	‹16,1›	1		1													
-1	‹16,1›′		1		1												
1	‹15,2›			1		1											
1	‹15,2›′				1		1										
-1	‹14,3›					1		1									
-1	‹14,3›′						1		1								
1	‹13,4›							1		1							
1	‹13,4›′								1		1						
-1	‹12,5›									1		1					
-1	‹12,5›′										1		1				
1	‹11,6›											1		1			
1	‹11,6›′												1		1		
-1	‹10,7›													1		1	
-1	‹10,7›′														1		1
1	‹9,8›															1	
1	‹9,8›′																1
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}

(table 1)

3- Brauer trees of spin characters for S_{18}:

In the symmetric group S_{18} there are 69 irreducible spin characters and the covering group has 68 of 17-regular classes then the decomposition matrix of S_{18} has 69 rows and 68 columns.

The symmetric group S_{18} has one block B_1 of defect one and the other spin blocks of defect zero.

3.1- The block B_1:

This block has the irreducible spin characters ‹18› , ‹18›′, ‹17,1›*, ‹15,2,1›, ‹15,2,1›′, ‹14,3,1›, ‹14,3,1›′,‹13,4,1›, ‹13,4,1›′,‹12,5,1›, ‹12,5,1›′, ‹11,6,1›, ‹11,6,1›′, ‹10,7,1›, ‹10,7,1›′, ‹9,8,1›, ‹9,8,1›′ which is an associate [3].

Lemma (3.1.1):

The Brauer tree of B_1 is:

‹18› \ ‹15,2,1› - ‹14,3,1› - ‹13,4,1› - ‹12,5,1› - ‹11,6,1› - ‹10,7,1› - ‹9,8,1›
\ /
‹17,1›*
/ \
‹18›′ / \ ‹15,2,1›′ - ‹14,3,1›′ - ‹13,4,1›′ - ‹12,5,1›′ - ‹11,6,1›′ - ‹10,7,1›′ - ‹9,8,1›′

Proof:

deg{ ‹18›, ‹18›′, ‹15,2,1› , ‹15,2,1›′, ‹13,4,1›, ‹13,4,1›′ , ‹11,6,1› , ‹11,6,1›′, ‹9,8,1›, ‹9,8,1›′ } ≡ 1 mod 17

deg{ ‹17,1›*, ‹14,3,1› , ‹14,3,1›′ ,‹12,5,1› , ‹12,5,1›′, ‹10,7,1› ,‹10,7,1›′ } ≡ −1 mod 17[4]

by the $(r,\bar{r})$-inducing of A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 , A_8 , A_9 , A_{10} , A_{11} , A_{12} , A_{13} , A_{14} , A_{15} and A_{16} in S_{17} to S_{18} we have the columns B_1 , B_2 , B_3 , B_4 , B_5 , B_6 , B_7 , B_8 , B_9 , B_{10} , B_{11} , B_{12} , B_{13} , B_{14} , B_{15} and B_{16} respectively [5]. we get the following decomposition matrix

The degree mod 17	Spin characters	The decomposition matrix for B_1															
1	‹18›	1															
1	‹18›′		1														
-1	‹17,1›*	1	1	1	1												
1	‹15,2,1›			1		1											
1	‹15,2,1›′				1		1										
-1	‹14,3,1›					1		1									
-1	‹14,3,1›′						1		1								
1	‹13,4,1›							1		1							
1	‹13,4,1›′								1		1						
-1	‹12,5,1›									1		1					
-1	‹12,5,1›′										1		1				
1	‹11,6,1›											1		1			
1	‹11,6,1›′												1		1		
-1	‹10,7,1›													1		1	
-1	‹10,7,1›′														1		1
1	‹9,8,1›															1	
1	‹9,8,1›′																1
		B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}

(table 2)

4- Brauer trees of spin characters for S_{19}:

In the symmetric group S_{19} there are 81 irreducible spin characters and the covering group has 79 of 17-regular classes then the decomposition matrix of S_{19} has 81 rows and 79 columns.
The symmetric group S_{19} has one block B_1 of defect one and the other spin blocks of defect zero.

4.1- The block B_1:

This block has the irreducible spin characters ‹19›*, ‹17,2› , ‹17,2›$'$, ‹16,2,1›*, ‹14,3,2›*, ‹13,4,2›*, ‹12,5,2›*, ‹11,6,2›*, ‹10,7,2›*, ‹9,8,2›* which is a double [3].

Lemma (4.1.1):

The Brauer tree of B_1 is:

‹19›*- ‹17,1› = ‹17,1›$'$ - ‹16,2,1›*- ‹14,3,2›*- ‹13,4,2›*- ‹12,5,2›*- ‹11,6,2›*-‹10,7,2›*- ‹9,8,2›*

Proof :

deg{ ‹19›*, ‹16,2,1›*, ‹13,4,2›*, ‹11,6,2›*, ‹9,8,2›*} ≡ 2 mod 17

deg{(‹17,2› + ‹17,2›$'$) , ‹14,3,2›* , ‹12,5,2›*, ‹10,7,2›*} ≡ −2 mod 17[4]

by the $(r,\bar{r})$-inducing of B_1 , B_3 , B_5 , B_7 , B_9 , B_{11} , B_{13} and B_{15} in S_{18} to S_{19} we have the columns C_1 , C_2 , C_3 , C_4 , C_5 , C_6 , C_7 and C_8 respectively[5].
we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_1							
2	‹19›*	1							
-1	‹17,2›	1	1						
-1	‹17,2›$'$	1	1						
2	‹16,2,1›*		1	1					
-2	‹14,3,2›*			1	1				
2	‹13,4,2›*				1	1			
-2	‹12,5,2›*					1	1		
2	‹11,6,2›*						1	1	
-2	‹10,7,2›*							1	1
2	‹9,8,2›*								1
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8

(table 3)

5- Brauer trees of spin characters for S_{20}:

In the symmetric group S_{20} there are 96 irreducible spin characters and the covering group has 93 of 17-regular classes then the decomposition matrix of S_{20} has 96 rows and 93 columns.
The symmetric group S_{20} has two blocks B_1 , B_2 of defect one and the other spin blocks of defect zero.

5.1- The block B_1:

This block has the irreducible spin characters ‹19,1›*, ‹18,2›* , ‹17,2,1›, ‹17,2,1›$'$, ‹14,3,2,1›*, ‹13,4,2,1›*, ‹12,5,2,1›*, ‹11,6,2,1›*, ‹10,7,2,1›*, ‹9,8,2,1›* which is a double [3].

Lemma (5.1.1):

The Brauer tree of B_1 is :

‹19,1›* - ‹18,2›* - ‹17,2,1›= ‹17,2,1›′ - ‹14,3,2,1›* -‹13,4,2,1›* - ‹12,5,2,1›* -‹11,6,2,1›* - ‹10,7,2,1›* - ‹9,8,2,1›*

Proof :

deg{ ‹19,1›*, (‹17,2,1› + ‹17,2,1›′) , ‹13,4,2,1›*, ‹11,6,2,1›*,‹9,8,2,1›*} ≡ 2 mod 17

deg{ ‹18,2›*, ‹14,3,2,1›* , ‹12,5,2,1›*, ‹10,7,2,1›*} ≡ −2 mod 17[4]

by the $(r,\bar{r})$-inducing of ‹18,1› , C_2 , C_3 , C_4 , C_5 , C_6 , C_7 and C_8 in S_{19} to S_{20} we have the columns D_1 , D_2 , D_3 , D_4 , D_5 , D_6 , D_7 and D_8 respectively [5].

we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_1							
2	‹19,1›*	1							
-2	‹18,2›*	1	1						
1	‹17,2,1›		1	1					
1	‹17,2,1›′		1	1					
-2	‹14,3,2,1›*			1	1				
2	‹13,4,2,1›*				1	1			
-2	‹12,5,2,1›*					1	1		
2	‹11,6,2,1›*						1	1	
-2	‹10,7,2,1›*							1	1
2	‹9,8,2,1›*								1
		D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8

(table 4)

5.2- The block B_2:

This block has the irreducible spin characters ‹20› , ‹20›′, ‹17,3›*, ‹16,3,1›, ‹16,3,1›′, ‹15,3,2›, ‹15,3,2›′, ‹13,4,3› , ‹13,4,3›′, ‹12,5,3›, ‹12,5,3›′, ‹11,6,3›, ‹11,6,3›′, ‹10,7,3›, ‹10,7,3›′, ‹9,8,3›, ‹9,8,3›′ which is an associate [3].

Lemma (5.2.1):

The Brauer tree of B_2 is :

‹20›   ‹16,3,1› - ‹15,3,2› - ‹13,4,3› - ‹12,5,3› - ‹11,6,3› - ‹10,7,3› - ‹9,8,3›

    \\   /

     ‹17,3›*

    /   \\

‹20›′   ‹16,3,1›′- ‹15,3,2›′ - ‹13,4,3›′ - ‹12,5,3›′ - ‹11,6,3›′ - ‹10,7,3›′ - ‹9,8,3›′

Proof :

deg{ ‹20› , ‹20›′, ‹16,3,1›, ‹16,3,1›′ , ‹13,4,3› , ‹13,4,3›′ , ‹11,6,3› , ‹11,6,3›′ , ‹9,8,3› , ‹9,8,3›′} ≡ 2 mod 17

deg{ ‹17,3›*, ‹15,3,2› , ‹15,3,2›′, ‹12,5,3› , ‹12,5,3›′, ‹10,7,3› ,‹10,7,3›′} ≡ −2 mod 17[4]

by the $(r,\bar{r})$-inducing of C_1 , ‹16,3› , ‹16,3›′, C_3 , C_4 , C_5 , C_6 , C_7 and C_8 in S_{19} to S_{20} we have the columns D_9 , D_{10} , D_{11} , D_{12} , D_{13} , D_{14} , D_{15} , D_{16} , D_{17} , D_{18} , D_{19} , D_{20} , D_{21} , D_{22} , D_{23} and D_{24} respectively [5].

we get the following decomposition matrix:

The degree mod 17	Spin characters	The decomposition matrix for B_2															
2	‹20›	1															
2	‹20›′		1														
-2	‹17,3›*	1	1	1	1												
2	‹16,3,1›			1		1											
2	‹16,3,1›′				1		1										
-2	‹15,3,2›					1		1									
-2	‹15,3,2›′						1		1								
2	‹13,4,3›							1		1							
2	‹13,4,3›′								1		1						
-2	‹12,5,3›									1		1					
-2	‹12,5,3›′										1		1				
2	‹11,6,3›											1		1			
2	‹11,6,3›′												1		1		
-2	‹10,7,3›													1		1	
-2	‹10,7,3›′														1		1
2	‹9,8,3›															1	
2	‹9,8,3›′																1
		D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}

(table 5)

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17-أشجار براور للمشخصات الاسقاطية لـ S_n عندما n=17, 18, 19, 20

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المستخلص:

في هذا البحث تم حساب اشجار براور ومصفوفات التجزئة للزمر التناظرية عندما $n = 17,18,19,20$ بحيث p=17 باستخدام $(r,\bar{r})$ - المولدة

الكلمات المفتاحية :

شجرة براور ، المشخصات الاسقاطية ، مصفوفات التجزئة