Numerical Study of Mixed Convection Flows in Lid- Driven Cavity Utilizing Nanofluid

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Abstract

A numerical investigation of study laminar mixed convection flows through a Copper-water nanofluid in a lid –driven cavity has been executed. In the present study, the vertical left and the inclined left walls are heated at constant temperature while the vertical right and the inclined right walls are maintained at constant cold temperature. The bottom wall is insulated and moving with uniform velocity. The study has been carried out for Rayleigh number Ra= 10^4 – 10^6, Reynolds number Re=20-100 and solid volume fraction of Cu nanoparticles Ø= 0-0.05. The effective viscosity and thermal conductivity of nanofluid have been calculated by Brinkman and Maxwell-Garnett models, respectively. The results indicated that, the Nusselt number increases with increasing Ra, Re and Ø.

Keywords: Mixed convection, Nanofluid, Lid-driven.

دراسة نظرية للحمل المختلط لجريان مائع نانوني في فجوة دوارة ذات جدار متحرك

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الخلاصة

تم في هذا البحث دراسة نظرية لجريان المائع (نحاس_ماء) النانوني المختلط في فجوة متحركة ذات جدار متحرك. في هذه الدراسة الجدار الأيسر والجدار المائل الأيسر مسخنة عند درجة حرارة ثابتة أما العمودي الأيمن والجدار الأيمن المائل مبردان بدرجة حرارة ثابتة. الجدار السفلي يكون معزول ويتحرك بسرعة منتظمة. نُفذت هذه الدراسة ضمن رقم رايلي (10^4-10^6) , رقم رينولدز (20-100) و النسبة الحجمية للجسيمات النانونية (0-0.05) الموصلية الحرارية واللزوجة الفعالة قد حُسبت من نماذج ماكسويل- كارنت وبرانكمان على التوالي. أشارت النتائج إلى أن رقم نسلت يزداد مع زيادة Ra, Re ,Ø.

Nomenclature	
g	Gravitational field (m s^{-2})
H	Height of enclosure (m)
k	Thermal conductivity (W $m^{-1}K^{-1}$)
L	Length of base enclosure (m)
n	Normal vector
Nu_{av}	average Nusselt number
Ra	Rayleigh number $Ra = \beta g H^3 \Delta T/(\upsilon\alpha)$
Re	Reynolds number $Re = \rho U_o H/\mu$
T	Temperature (K)
u	Velocity component along x-direction (m s^{-1})
v	Velocity component along y-direction (m s^{-1})
U	Dimensionless velocity component along x-direction
V	Dimensionless velocity component along y-direction
x,y	Cartesian coordinates (m)
X,Y	Dimensionless Cartesian coordinates
Greek symbols	
α	Thermal diffusivity (m^2 s^{-1})
β	Thermal expansion coefficient (K^{-1})
ϕ	Nanoparticles volume fraction
μ	Dynamic viscosity (Pa.s)
υ	Kinematic viscosity (m^2/s)
θ	Dimensionless temperature
ρ	Density (kg m^{-3})
Ψ	Dimensionless stream function
Subscripts	
c	Cold
f	Fluid
h	Hot
nf	Nanofluid
p	Solid nanoparticles

Introduction

Mixed convection has received a noticeable attraction in research due to its wide application in industry. Like , solar collectors, storage of grains, disposing of waste materials, colloing of electronics, etc. the performance of these device can be improved by using nanofluids rather than regular fluids. A nanofluid is a base liquid with suspended metallic or non-metallic nanoparticles. Because traditional fluids used for heat transfer applications such as water, mineral oils and ethylene glycol have a rather low thermal conductivity ,nanofluids with relatively higher thermal conductivities have attracted enormous interest from researchers due to their potential in enhancement of heat transfer with little or no penalty in pressure drop .

Chamkha & Abu-Nada [1] studied laminar mixed convection flow in single and double – Lid driven square cavities filled with a water – Al_2O_3 nanofluid .The left and right walls of the cavity were kept insulated while the bottom and top walls were maintained at constant temperatures with the top surface being the hot wall. It's found that, for small Richardson number causes reductions in the average Nusselt number. Abbasian et al. [2] investigated numerically mixed convection laminar flow around an adiabatic body in a lid-driven enclosure filled with nanofluid (Al_2O_3 –water) using variable thermal conductivity and viscosity. The vertical enclosure's wall are maintained at constant cold temperature and the horizontal bottom enclosure's wall is kept at constant hot temperature. The top wall of the enclosure is insulated and moving with uniform velocity. The ratio of body's length to the enclosure's length is kept constant at 1/3. The study has been carried out for Ri (0.01-100), Ø (0-0.06) and Gr (10^4). The results showed that, the average Nu increased by increasing Ø and reduction with Ri. Ridha et al. [3] studied numerically modeling of mixed convection in lid- driven partially heated cavities. The results show that, for Ri >1, the average Nu relatively low and for Ri<1 the forced convection becomes dominate, the natural convection weak, as a result of which Nu is relatively higher. The heat transfer rate increases with increasing the solid volume fraction of the nanoparticles.

Siranandam et al. [4] indicated convection flow and heat transfer behavior of nanofluids with different nano-particles in a square cavity. The hot left wall temperature is varied linearly with height whereas the cold right wall temperature is kept constant. It's found that, the heat transfer rate increases on increasing the volume fraction of the nanofluid for all types of nanoparticles considered. The increment of Nu is strongly dependent on the nanoparticles chosen. Ahmed and Mansour [5] indicated numerically mixed convection flows in a square lid-driven cavity partially heated from below using Cu-water, Ag-water ,AL_2O_3-water and TiO_2- water nanofluid. The results showed that, the increasing in Ø leads to decreasing both of activity of the fluid motion and the fluid temperature, however, it increases the corresponding Nu by adding TiO_2 nanoparticles to the base fluid this gives large value of Nu on the contrary, by adding silver (Ag) nanoparticles to the base fluid this gives small values of Nu.

Aminossadati S.M. and Ghasemi B. [6] studied numerically natural convection cooling of a heat source embedded on the bottom wall of an enclosure filled with nanofluids. The top and vertical walls of the enclosure are maintained at a relatively low temperature. The results showed that, the increase at Rayleigh numbers strengthens the natural convection flows which results in the reduction of heat source temperature. Sheikhzadeh G.A. et al. [7] presented the fluid flow and heat transfer in lid-driven enclosures filled with Cu –nanofluid numerically. The moving vertical walls of the enclosure are maintained at constant temperature. The results show that by increasing the volume fraction of nanoparticles, the variation of average Nu number on the hot wall is linear in two cases.

Farhad Talibi et al. [8] studied numerically laminar mixed convection flows through a cu-water nanofluid in a square lid-driven cavity. The top and bottom walls are insulated while the vertical walls are different constant temperature. Its found that at the fixed Re number the solid concentration affects on the flow pattern and thermal behavior particularly for a higher Ra number. Abu-nada and chamkha [9] studied natural convection heat transfer characteristics in a differentially heated enclosure filled with a CuO-EG-Water nanofluids for different published variable thermal conductivity and variable viscosity models. The results showed that ,the effect of thermal conductivity models was less significant than the viscosity models at high Rayleigh number. Mostafa Mahmoodi and Seyed Mohammad Hashemi [10] indicated natural convection fluid flow and heat transfer inside C-shaped enclosures filled with Cu-water nanofluid. The results found that the mean Nusselt number increased with increase in Rayleigh number and volume fraction of Cu nanoparticles regardless aspect ratio of the enclosure.

It can be concluded from the survey above, that all the studied geometries are rectangular enclosures . thus, the present study is concerned with mixed convection in such a hooded rectangular enclosure with base is being moving in either direction. This geometry can be considered as a simulation for a modified solar collector.

Theoretical Analysis

A schematic diagram of the considered model is shown in **Fig. 1**. It is a two – dimensional enclosure of height H and based length L,L=H filled with Cu-water nanofluid. The insulated lid-driven bottom wall is sliding at a constant speed [+ u_o for case I and with – u_o for case II]. The left vertical and inclined walls are heated at constant temperature (T_h) while the right vertical and inclined walls are kept at constant temperature (T_c). The Cu-water nanofluids is assumed to be Newtonian, study, laminar, incompressible, in thermal equilibrium, and the nanoparticles are kept uniform in shape and size. The thermo-physical properties of the base fluid and the nanoparticles, presented in table-1, are considered to be constant with the exception of it's density which varies according to the Boussinesq approximation. The viscosity of the nanofluid is assumed to be a function of volume fractions of nanoparticles by using Brinkman model [11] .

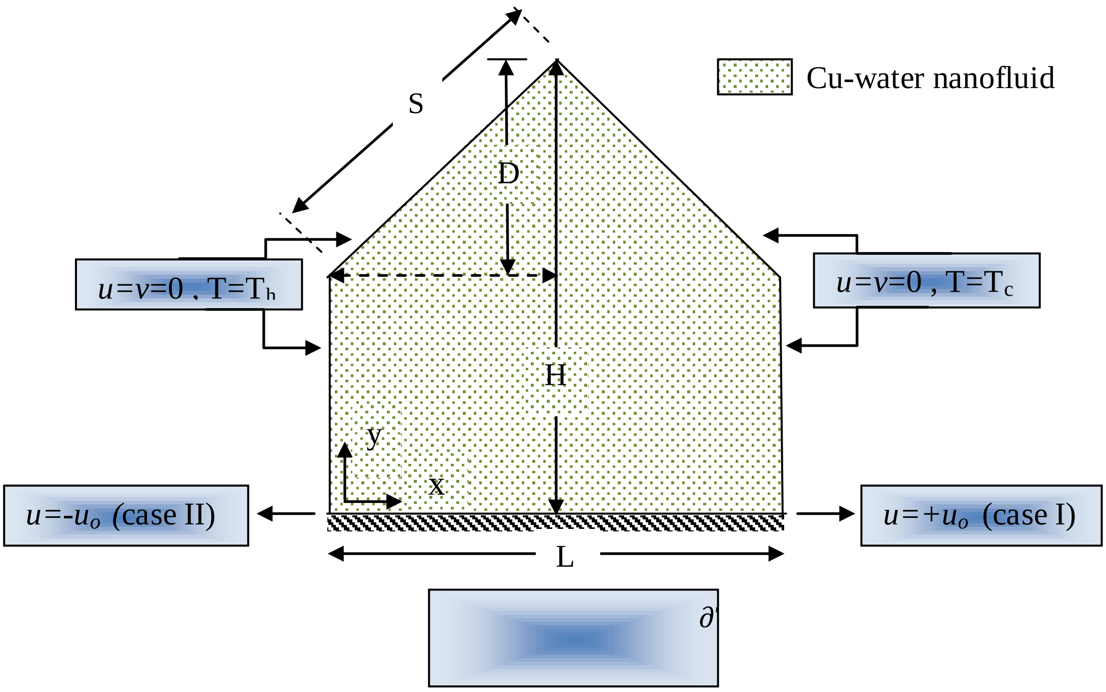


Fig. 1 schematic diagram of the enclosure

Table 1
Thermo physical properties of water and copper [8].

Property	Water	Copper
c_p	4179	383
ρ	997.1	8954
k	0.6	400
β	$2.1*10^{-4}$	$1.67*10^{-5}$

The governing equations for laminar, steady-state lid driven convection in an enclosure filled with Cu –water nanofluids are given as:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation in x-direction:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\upsilon_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

Momentum equation in y-direction:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y}+\upsilon_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$+\frac{g}{\rho_{nf}}(T-T_C)\left[\emptyset\,\rho_s\beta_s+(1-\emptyset)\rho_f\beta_f\right] \qquad (3)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

Where : $\alpha_{nf} = k_{nf}/(\rho c_p)_{nf}$

The effective density of nanofluid at the reference temperature can be define as:

$$\rho_{nf} = (1-\emptyset)\rho_{nf}+\emptyset\,\rho_s \qquad (5)$$

The heat capacitance of nanofluid can be given as:

$$(\rho C_P)_{nf} = (1-\emptyset)(\rho C_P)_f+\emptyset(\rho C_P)_s \qquad (6)$$

The effective thermal conductivity of the nanofluid is approximated by the Maxwell – Garnett model [12]

$$K_{nf} = k_f\left[\frac{\left(k_{np}+2k_f\right)-2\emptyset\left(k_f-k_{np}\right)}{\left(k_{np}+2k_f\right)+\emptyset\left(k_f-k_{np}\right)}\right] \qquad (7)$$

Equations (1)-(4) can be converted to the dimensionless forms by definition of the following parameters as:

$$X=\frac{x}{H}\,, \qquad Y=\frac{y}{H}\,, U=\frac{u}{U_0}\,, V=\frac{v}{U_0}\,, \theta=\frac{T-T_c}{T_h-T_c}\,, P=\frac{p}{\rho_{nf}{U_0}^2} \qquad (8)$$

Therefore using the above parameters leads to dimensionless form of the governing equations as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (9)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{1}{(1-\emptyset)^{2.5}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (10)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{1}{(1-\emptyset)^{2.5}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$
$$+\frac{Ra}{Re^2.Pr}\frac{\rho_f}{\rho_{nf}}\left(1-\emptyset+\emptyset\frac{\rho_s\beta_s}{\rho_f\beta_f}\right)\theta \qquad (11)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{k_{nf}}{k_f}\frac{(\rho C_P)_f}{(\rho C_P)_{nf}}\frac{1}{Re.Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (12)$$

Where: $Pr = \upsilon_f/\alpha_f$, $Re= \rho U_o H/\mu$, $Ra=\beta g H^3 \Delta T/(\upsilon\alpha)$

The boundary conditions are:

θ =1 , *U=V=0* for left vertical and inclined walls

θ =0 , *U=V=0* for right vertical and inclined walls (13)

$\partial\theta/\partial Y$=0 ,$V=0$, $\begin{cases} U=+1 & case\ I \\ U=-1 & case\ II \end{cases}$ for bottom wall

Numerical approach

The dimensionless governing equations (9) - (12) with the corresponding boundary conditions given in equation (13) are solved based on the finite element method by using Flex PDE software package. Other useful equations such as Nusselt number for heated walls can be calculated after solving the governing equations for U,V and θ. The local Nusselt number for the left vertical and inclined walls are defined as:

$$Nu_l=\frac{K_{nf}}{K_f}\frac{\partial\theta}{\partial X}\Big|_{\text{left vertical wall}} \qquad (14)$$

$$Nu_{in}=\frac{K_{nf}}{K_f}\frac{\partial\theta}{\partial N}\Big|_{\text{left inclind wall}} \qquad (15)$$

Thus, the average Nusselt number for each side of the heated walls is determined by integrating the local Nusselt number along the heated walls :

$$Nu_{avl}=\frac{1}{l}\int_0^l Nu_l\,.dy \qquad (16)$$

$$Nu_{avin}=\frac{1}{0.5}\int_{0.5}^1 Nu_{in}\,.ds \qquad (17)$$

The total average Nusselt number for the heated walls can given by :

$$Nu_{avt}=\int_0^{0.5} Nu_l\,.dy+\int_{0.5}^1 Nu_{in}\,.dy \qquad (18)$$

Validation and comparison of the study :

The studied geometry in this paper is an obstructed solar collector cavity ; therefore several grid size sensitivity tests together with the continuity equation ($\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$) are achieved. The obtained results showed an exactly validation of the velocity distribution for grid size obtained by imposing an accuracy of 10^{-3}.

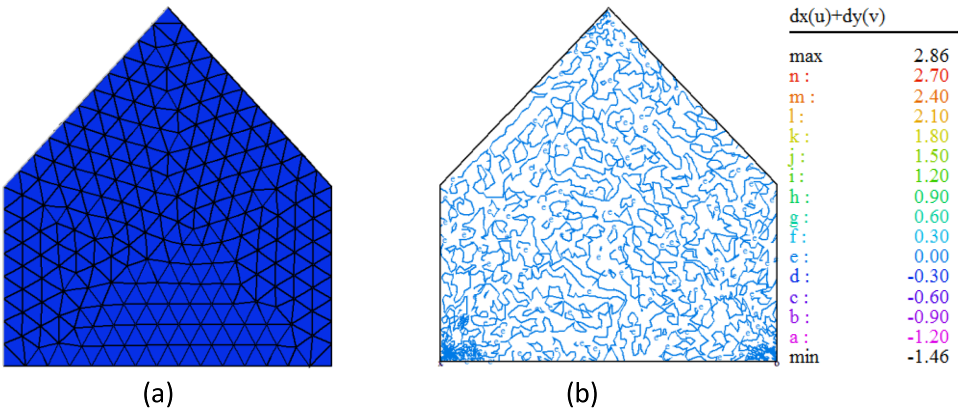


Fig.2 (a) grid distribution over the domain (b) validation of continuity equation.

This accuracy is a compromised vale between the result accuracy and the time consumed in each run. The grid domain for Ra= 10^4, Re =20 and Ø=0 is shown in fig. (2-a) and the distribution of $\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)$ over the domain is presented in fig.(2-b). Table -2 referred to the comparison of the present values of Nusselt average (Nu_{av}) for (Ra=10^4,$10^5$and 10^6) with the values of ref. [8] (third column) and with ref. [13] (fourth column). It is obvious that good agreement is obtained. As a result, the confidence in the present numerical solution is enhanced.

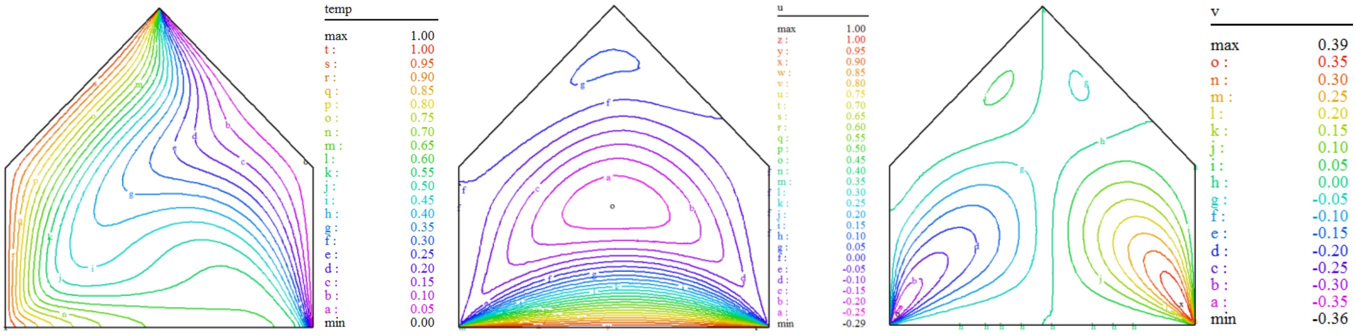
Table 2 Testing the used code with others

Ra	Nu_{av}		
	Present work	Farhad Talebi et al.[8]	De Vahl Davis [13]
10^4	2.262	2.248	2.242
10^5	4.549	4.503	4.523
10^6	9.183	9.147	9.035

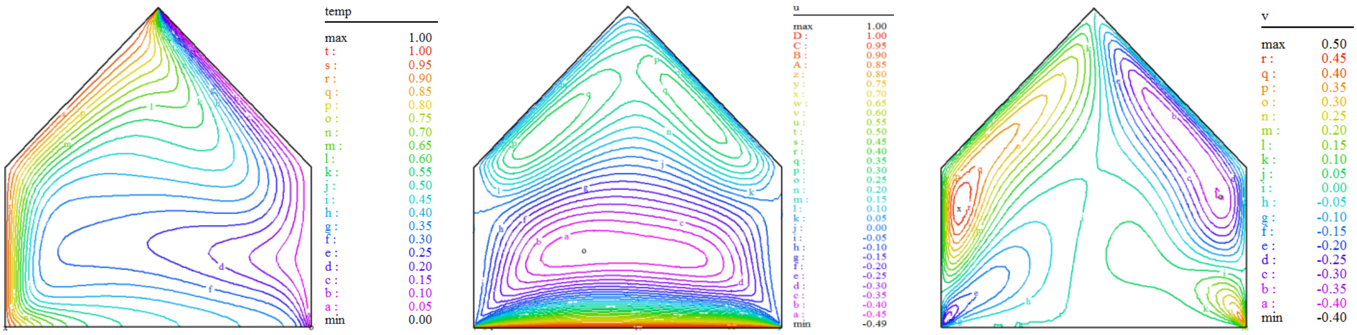
Results and Discussion :

The present study is carried out for copper-water nanofluid at Rayleigh number Ra=10^4-10^6 , Reynolds number Re=1-100 and solid volume fraction Ø=0-0.05. The discussion is established on two cases: case I and case II . In case I, the bottom wall is moving to the right while the case II, the bottom wall is moving to the left.

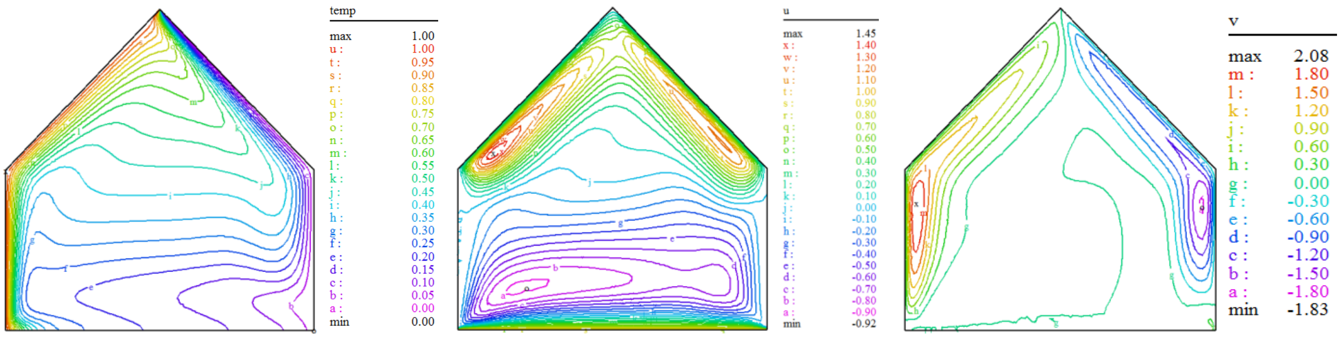
Fig.3 shows isotherms contour (on the left), U-velocity contour (in the middle) and V-velocity contour (on the right) for various Ra numbers , Re=20 and Ø=0.05 . It is noticed that, at lower Ra number the solid concentration has more effect to increase the heat penetration; because the conduction heat transfer effect decreases



(a) Ra=10^4



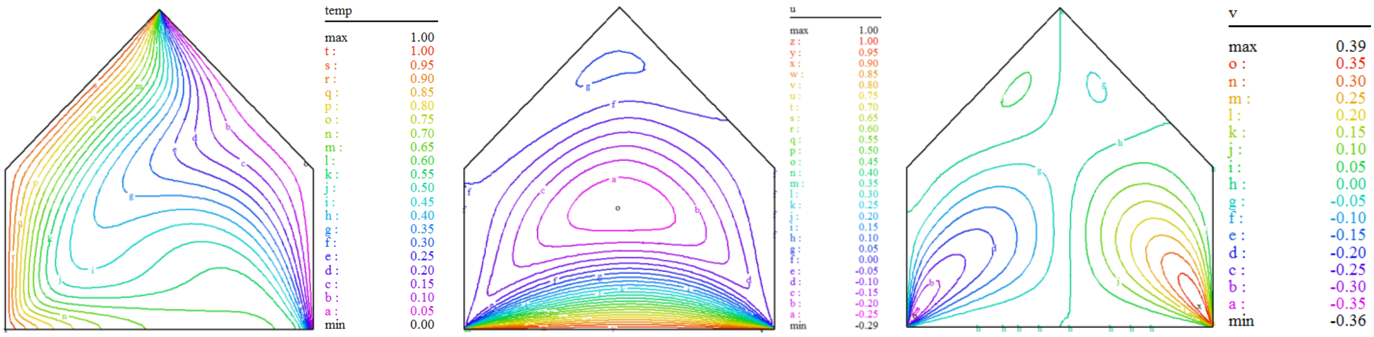
(b) Ra=10^5



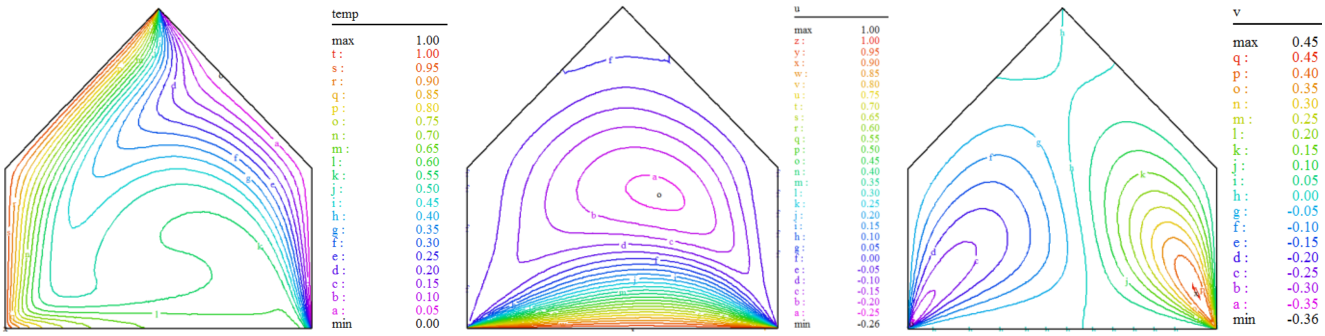
(c) Ra=10^6

Figure .3. Isotherms (left) , U-velocity (middle) and V-velocity (right) for Re= 20, Ø=0.05 and U=+1 at a) Ra=10^4, b) Ra=10^5 c) Ra=10^6

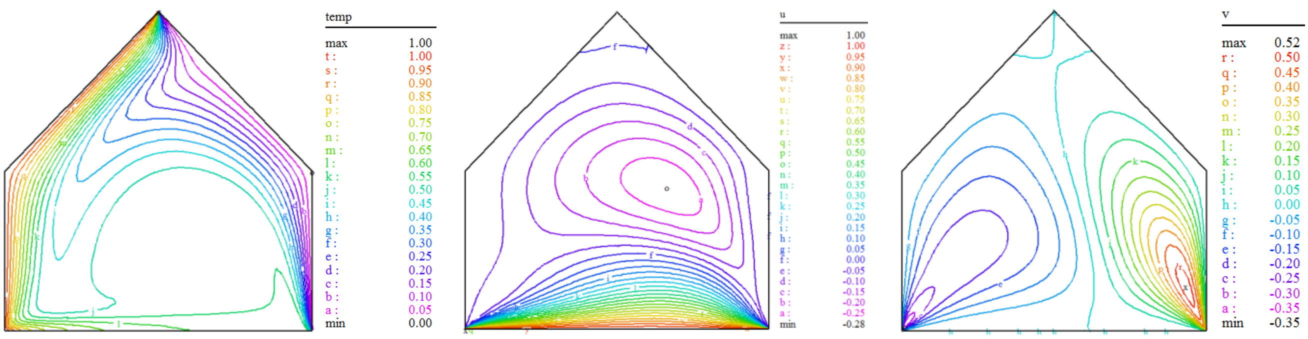
with increasing Ra number, so the solid concentration has smaller effect on thermal distribution. When Ra number decreases the intensity of U –velocity contour increases near the bottom wall due to its moving and buoyancy effect. Also, V-component increases with increasing Ra number. Fig. 4 indicates the effect of Re number on this contours at Ra=10^4 and Ø=0.05, for isotherms as can be seen that when Re number smaller, the effect of lid- driven is in significant and the intensity of the isotherms are concentrated close to the heated wall.



(a) Re=20



(b) Re=50



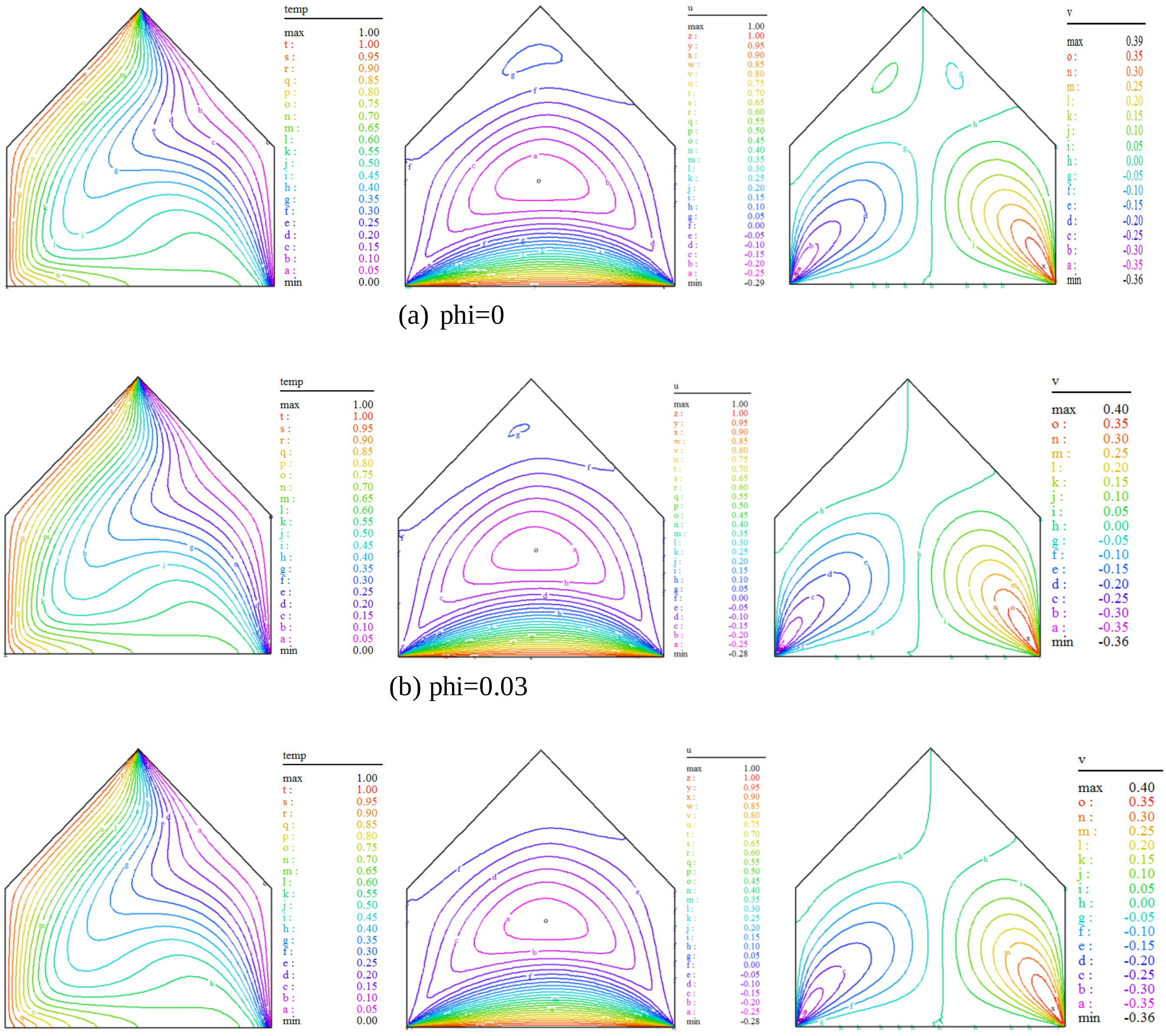
(c) Re=100

Figure .4. Isotherms (left) , U-velocity (middle) and V-velocity (right) for Ra= 10^4, phi=0.05 and U=+1 at a) Re=20, b) Re=50 c) Re=100.

However , as Re number increases the effect of lid-driven increases and hence forced convection flow. It is noticed also that with increasing Re, an isothermal zone is localized above the moving bottom wall. At Re=20, the vortex core of U-velocity contour is in the center of the cavity. When Re number increases the vortex core moves to the right side of the cavity due to increasing forced convection. When Re number increases V-velocity increases also. Fig.5 demonstrates that the effect of solid volume fraction on this contours at Ra=$10^4$and Re= 20, the increase in solid concentration does not have considerable effect on isotherm contour but the intensity of U and V-velocities increases with increasing solid volume fraction due to lid driven flow.

Figures 6&7 shows for case II, where Fig.6 demonstrated the effect of variation of Ra number on the isotherms (left) U-velocity(middle) and V-velocity (right)for U=-U, Re=20 and $\varnothing$=0.05. As can be seen for the isotherms, intensity of curves increases near the hot walls with increasing Ra number. This is an indication of increasing of heat transfer. The isotherms become mostly horizontal with higher Ra which means the dominance of natural convection. While U-velocity contours at small Ra number increases close to the lid driven wall and the focus slopes to the left heated wall and with increasing Ra number U- velocity increases close to the two upper sides of the cavity and became semi uniform near the bottom wall. V-component increases with increasing Ra number especially near the hot left sides. All these observation are due to the strengthen of the natural convection with increasing Ra. Figure 7. indicates the effect of the height of the hood of the cavity (D). The isotherms contour do not varied but U-velocity decreased with decreasing value of D while V-velocity increases with decreasing values of D. The attribution of these variations can be referred to the following, when D is decreased, large free path for the vertical component will be a available, therefore, less drag in the vertical path which leads to enhance the V-component. On the other hand, the behavior of U- component is due to the dominance of V over U components.

Figure 8. presents that the overall heat transfer (Nu_{av}) is an increasing functions of Ra, Re and $\varnothing$. More over, this figure tell us that when the bottom wall is moving to the left (U= -1), the average Nusselt number manifests greater values. This is quit reasonable, because the direction of the lid coincides with the natural movement of nanofluid which is generated from the left walls towards the upper zone of the enclosure, i.e. the aiding mechanism will be obtained U= -1, and the opposing mechanism is obtained when U= +1. Figure 8. a-left shows the variation of Nu_{av} on Ra number for the second case. As a result Nu_{av} increases with Ra number for two cases but the values of Nu_{av} for cases II larger than case I due to the secondary flow. Fig 8.a-right indicates the effect of θ_{av} on Ra for two cases . θ_{av} decreases with increasing Ra number for two cases. Fig.8. b- left shows the effect of Nu_{av} on Re number . as can be see that Nu_{av} increases linearly approximately with Re number for two cases. Fig.8.b-right represents the variation of θ_{av} on Re for two cases. The increases in Re number leads to decreasing θ_{av} for two cases. Fig. 8.c-left refers to the effect of of Nu_{av} on $\varnothing$. As can be see that Nu_{av} increases $\varnothing$ for two cases. Also, that talk for θ_{av} at fig.8.c-right.



(a) phi=0

(b) phi=0.03

(c) phi=0.05

Figure .5. Isotherms (left) , U-velocity (middle) and V-velocity (right) for Ra= 10^4, Re=20 and U=+1 at a) Ø= 0, b) Ø=0.03, c) Ø=0.05.

Conclusions:

The mixed convection heat transfer of lid driven enclosure filled with Cu-water is studied numerically using finite element method implemented in Flex PDE software package. Two cases for lid driven wall [+ 1 and –1] are studied with different ranges or Ra, Re and ϕ and D. The obtained results led us to the following conclusions:

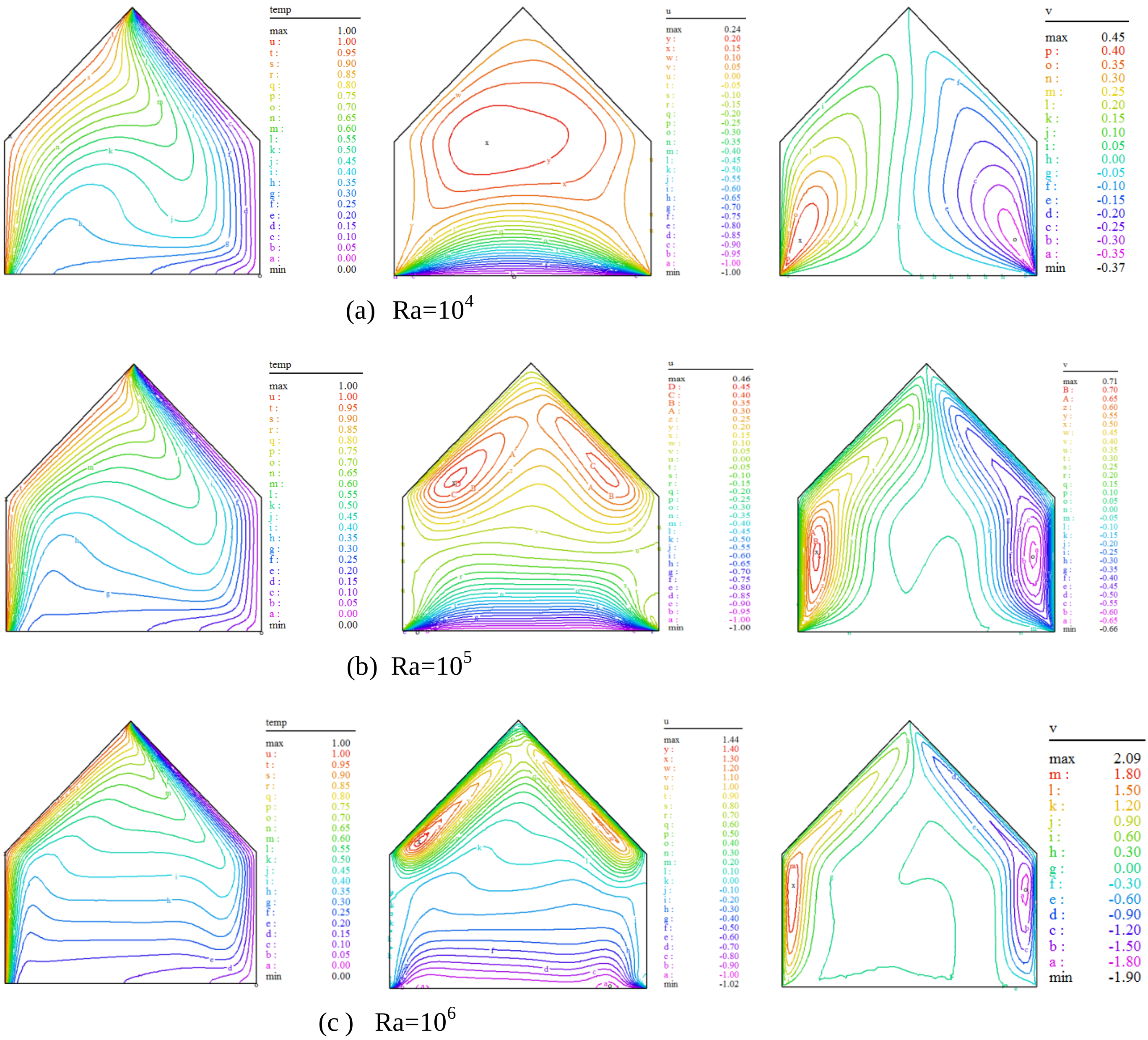
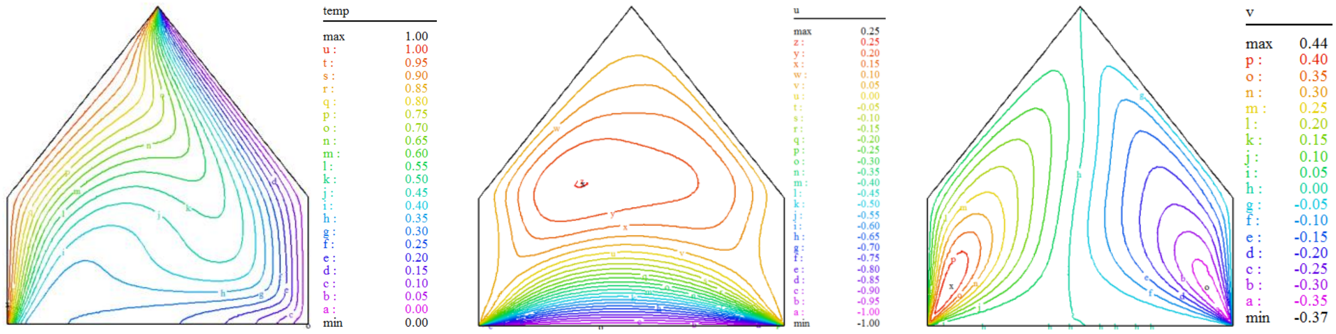
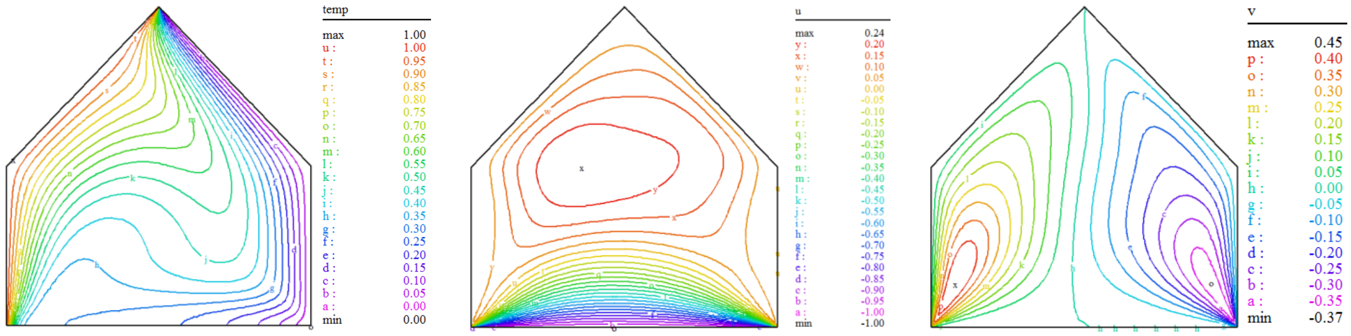


Figure .6. Isotherms (left) , U-velocity (middle) and V-velocity (right) for Re= 20, phi=0.05 and U= -1 at a) $Ra=10^4$, b) $Ra=10^5$ c) $Ra=10^6$.

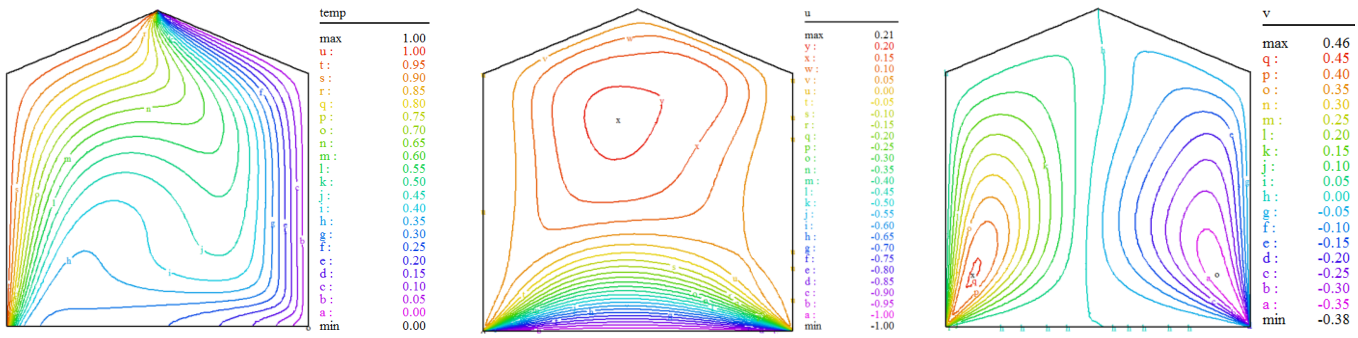
1. Nu_{av} increases with increasing Ra number , Re number and Ø.
2. The heat transfer inside the enclosure can be enhanced when the bottom wall is being lid to the left, and can be retarded when the bottom wall is being lid to the right (U=+1) .
3. θ_{av} increases with increasing Re number and Ø for two cases, but decreases with increasing Ra number.



(a) D=0.6



(b) D=0.5



(c) D=0.2

Figure .7. Isotherms (left) , U-velocity (middle) and V-velocity (right) for $Ra=10^4$, Re= 20, Ø=0.05 and U= -1 at a) D=0.6, b) D= 0.5 c) D=0.2.

References

[1] Ali J. Chamkha & Eiyad Abu-Nada , "Mixed convection flow in single and double lid-driven square cavities filled with water- Al_2O_3 nanofluid : Effect of viscosity models", European Journal of Mechanics B/ fluids, 36 ,2012, pp.82-96.

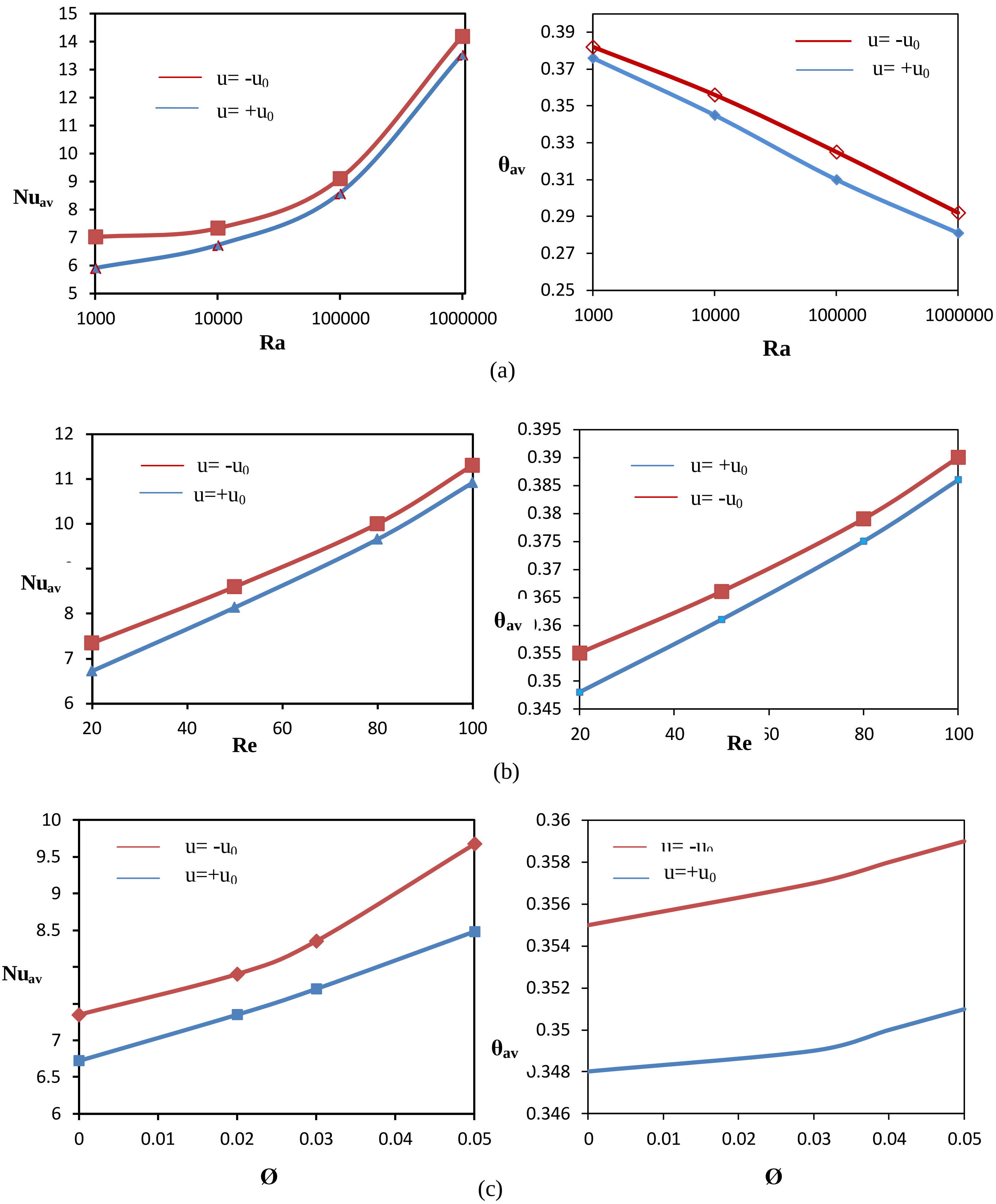


Fig. 8. Variation of Nu_{av} (left) and θ_{av} (right) with a) Ra , b) Re, c) Ø .

[2] Abbasian A.A. , Sheikhzadeh G.A. , Heidary R. , Hajialigol N. & M. Ebrahim Qomi," Numerical study of mixed convection in a lid-driven enclosure with a centered using nanofluid variable properties" , Journal of Nanostructures , JNS 2, 2012,pp. 51-60.

[3] Jami Ridha, Brahim Ben-Beya & Taieb Lili, "Numerical study of mixed convection of nanofluid in lid-driven enclosure partially heated" , International Conference on Control, Engineering & Information Technology (CEIT ' 13), Vol.4, 2013,pp.223-226.

[4] Sivanandam Sivasankarn , Thangaraj Aasaithambi & Subbarayagaunder Rajan , "Natural convection of nanofluids in a cavity with linearly varying wall temperature", Maejo Int. J.Sci. Technol. , 4(03), 2010,pp. 468-482.

[5] Sameh E. Ahmed and Mansour M.A., " Numerical study of mixed convection in partially heated lid- driven cavities filled with nanofluids ", Int. J. of Appl. Math. and Mech., 8(8),2012, pp. 34-54.

[6] Aminossadati S.M. and Ghasemi B., "Natural convection cooling of a localized heat source at the bottom of a nanofluid- filled enclosure" , European Journal of Mechanics B/ fluids, 28 ,2009, PP.630-640.

[7] Sheikhzadeh G.A., Hajialigol N. ,Ebrahim Qomi M. and Fattahi A.," Laminar mixed convection of Cu-water nanofluid in two-sided lid-driven enclosures", Journal of Nanostructures", 2012,pp. 44-53.

[8] Farhad Talebi, Amir Houshang Mahmoudi and Mina Shahi, " Numerical study of mixed convection flows in a square lid-driven cavity utilizing nanofluid", International communications in heat and mass transfer", 37,2010,pp. 79-90.

[9] Eiyad Abu-Nada and Ali J. Chamkha," Effect of nanofluid variable properties on natural convection in enclosures filled with a CuO- EG-Water nanofluid", International Journal of thermal sciences, 49, 2010,pp. 2339-2352.

[10] Mostafa Mahmoodi and Seyed Mohammad Hashemi," Numerical study of natural convection of a nanofluid in C-shaped enclosures", International Journal of thermal sciences, 55, 2012,pp. 76-89.

[11] Brinkman H.C.," The viscosity of concentrated suspensions and solutions", J. Chem. Phys. 20, 1952,pp571-581.

[12] Maxwell-Garnett, "Colours in metal glasses and in metallic films", Philos. Trans. Roy. Soc. A.203, 1904, pp. 385-342.

[13] De Vahl Davis G.," Natural convection of air in a square cavity a bench mark numerical solution ,"International Journal of Numerical Methods of Fluids 3, 1983, pp. 249- 264.