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Linear arrays with variable interelement spacings

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Abstract

The synthesis of uniformly excited equally and unequally spaced array employing physical optics technique to achieve sidelobe reduction and narrow beamwidth by varying the elements number (N) and positions, with equal current magnitude of antenna. The elements of this array are considered to be short backfire antennas (SBFAs) fed by coaxial waveguide and excited by TE_{11}-mode. It is observed that by employing the unequally spaced array synthesis technique, the first sidelobe level (FSLL) is reduced by 8.85 dB over that of an equally spaced array patterns for N=15.

In this paper an empirical relation for the unequally spaced antenna arrays was found. This relation expresses the distribution and the height of sidelobes depending on the space broadening factor.

Key Words: Linear array, Antennas arrays, Short backfire antenna

1-Introduction

It is well-known that the use of antenna arrays rather than single antenna element provides more opportunities of high gain, narrow beamwidth, low side lobe ratio, etc. as required by the designer. It has been accepted since the early 1960s that the positions of radiating elements of an array can be used as an additional set of parameters for advanced array design [1].

Most array antennas employ equal spacings between adjacent elements. The theory is well understood, and convenient analytical procedures are available for antenna design and radiation pattern synthesis. It is possible, however, to operate array antennas with nonuniform, or unequal, spacings between adjacent elements. The element spacings provide another parameter, in addition to the amplitude and phase of the element current, with which to control the radiation pattern. Unequally spaced linear arrays may be used to obtain radiation patterns with low peak side lobes without the need for an amplitude taper. This might be of importance in applications where it is not convenient to individually adjust the amplitude of the current at the elements. The beam width of the unequally spaced array may be as narrow as that of the equally spaced array. The unequally spaced array permits the antenna to operate over a wide frequency range without the appearance of grating lobes. It can also be scanned over a wide angle without the formation of the grating lobes that could appear with the equally spaced array [2].

In this paper, we propose the synthesis problem formulation for the SBFA array with a uniform supply distribution in amplitude and phase. The only parameter which can modify the radiation pattern, in this case, is the distribution of the space sources. The radiated field from the array at a given point in space is the vector sum of the radiated fields from the individual elements. In other words, the far field of the array is determined by superposition [3].

Short backfire antennas are highly efficient radiators of simple and compact construction, and highly directional radiator. The SBFA consists of a leaky cavity resonator formed from two plane reflectors of different diameter, spaced a half-wavelength apart, with a feed placed between them. Here, we shall illustrate the calculation of the array factor and the element factor to determine the radiation field generated by linear array of antennas with uniform current distribution and unequally distance between elements.

2- Array Technique

The basic idea of array theory is the principle of superposition of the fields. Not only the field but also other array quantities such as directivity and gain may be written as the product of an element-dependent part and a dimensionless array-dependent part.

The array factor displays the features in the radiation pattern which are determined by the number and the arrangement of the element in the array. We utilize the fact that in most applications the distance between the transmitter and the receiver is very large compared to both the wavelength and the dimensions of the antenna, and thus, for simplicity consider a single antenna and derive expression for its far-field.

For unequally spaced linear array of N identical elements the far-field is given by [3]:-

$$E(\theta,\varphi)=g(\theta,\varphi)\cdot f(\theta,\varphi) \qquad \ldots\ldots(1)$$

where $g(\theta,\varphi)$ and $f(\theta,\varphi)$ are the element factor and array factor, respectively.

2-1 Array factor

The array factor of a linear array with N identical radiating elements with arbitrary positions can be written as [2] :-

$$f(\theta,\varphi)=\sum_{m=0}^{N-1} a_m e^{jm\psi} \qquad \ldots\ldots(2)$$

with

$$\psi=\beta x_m \sin\theta\cos\varphi \qquad \ldots\ldots(3)$$

$$f(\theta,\varphi)=\sum_{m=0}^{N-1} a_m e^{ju\sin\theta}=\sum_{m=0}^{N-1} a_m \sum_{n=-\infty}^{\infty} e^{jn\theta} J_n(u_m) \ldots (4)$$

Since $f(\theta,\varphi)$ is periodic in θ,

$$f(\theta,\varphi)=\sum_{n=-\infty}^{\infty} C_n e^{jn\theta} \qquad \ldots (5)$$

where

$$C_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta,\varphi)e^{-jm\theta}\,d\theta \qquad \ldots(6)$$

Comparing equations (4) and (5) gives:-

where a_m is the excitation coefficient current of the mth element located at x_m, β is the propagation phase constant. This array factor is a function of ψ and may be recognized as a Fourier series and it is a dimensionless quantity. Let $\left(u_m=\beta x_m\cos\varphi\right)$, then the associated polynomial is:-

$$C_n=\sum_{m=0}^{N-1} a_m J_n(u_m) \qquad \ldots\ldots(7)$$

Equation (7) gives the coefficients C_n of the Fourier series expansion of the array factor of equation (5) in terms of the element current a_m and the distribution of the elements along the x-axis. By using the matrix notation, equation (7) can be written as follows :-

$$\begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix} = \begin{bmatrix} J_0(\beta x_0\cos\varphi) & J_0(\beta x_1\cos\varphi) & \ldots\ldots & J_0(\beta x_{N-1}\cos\varphi) \\ J_1(\beta x_0\cos\varphi) & J_1(\beta x_1\cos\varphi) & \ldots\ldots & J_1(\beta x_{N-1}\cos\varphi) \\ J_2(\beta x_0\cos\varphi) & J_2(\beta x_1\cos\varphi) & \ldots\ldots & J_2(\beta x_{N-1}\cos\varphi) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ J_n(\beta x_0\cos\varphi) & J_n(\beta x_1\cos\varphi) & \ldots\ldots & J_n(\beta x_{N-1}\cos\varphi) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{N-1} \end{bmatrix} \qquad \ldots\ldots\ldots(8)$$

Progressive phase delay α_x can be used to move the beam of an unequally spaced linear array just as it is used for the equally spaced linear array. For the unequally spaced linear array the phase required at the mth element to tilt the beam to the angle (θ_o,φ_o) is :-

$$\alpha_x=\beta x_m\sin\theta_o\cos\varphi_o \qquad \ldots(9)$$

It is sometimes convenient to take the geometrical center of the array as the phase reference as shown in Fig.(1).

A novel approach for an empirical relation is found in this work for the unequally spaced antenna arrays, which is given by:-

$$f(\theta,\varphi) = I_o\left[1 + \sum_{m=-(N-1)/2}^{-1} e^{j[\beta(mx_m - 2^{-m}\Delta)\sin\theta\cos\varphi - m\alpha_x]} + \sum_{m=1}^{(N-1)/2} e^{j[\beta(mx_m + 2^{m}\Delta)\sin\theta\cos\varphi - m\alpha_x]}\right]$$

, for odd N ....(10-a)

$$f(\theta,\varphi) = I_o\left[2\cos\left(\frac{\beta x_o \sin\theta\cos\varphi - \alpha_x}{2}\right) + \sum_{m=-(N/2)}^{-2} e^{j[\beta((m+\frac{1}{2})x_m - 2^{-(m+1)}\Delta)\sin\theta\cos\varphi - (m+\frac{1}{2})\alpha_x]}\right.$$
$$\left. + \sum_{m=2}^{(N/2)} e^{j[\beta((m-\frac{1}{2})x_m + 2^{(m-1)}\Delta)\sin\theta\cos\varphi - (m-\frac{1}{2})\alpha_x]}\right]$$

, for even N ....(10-b)

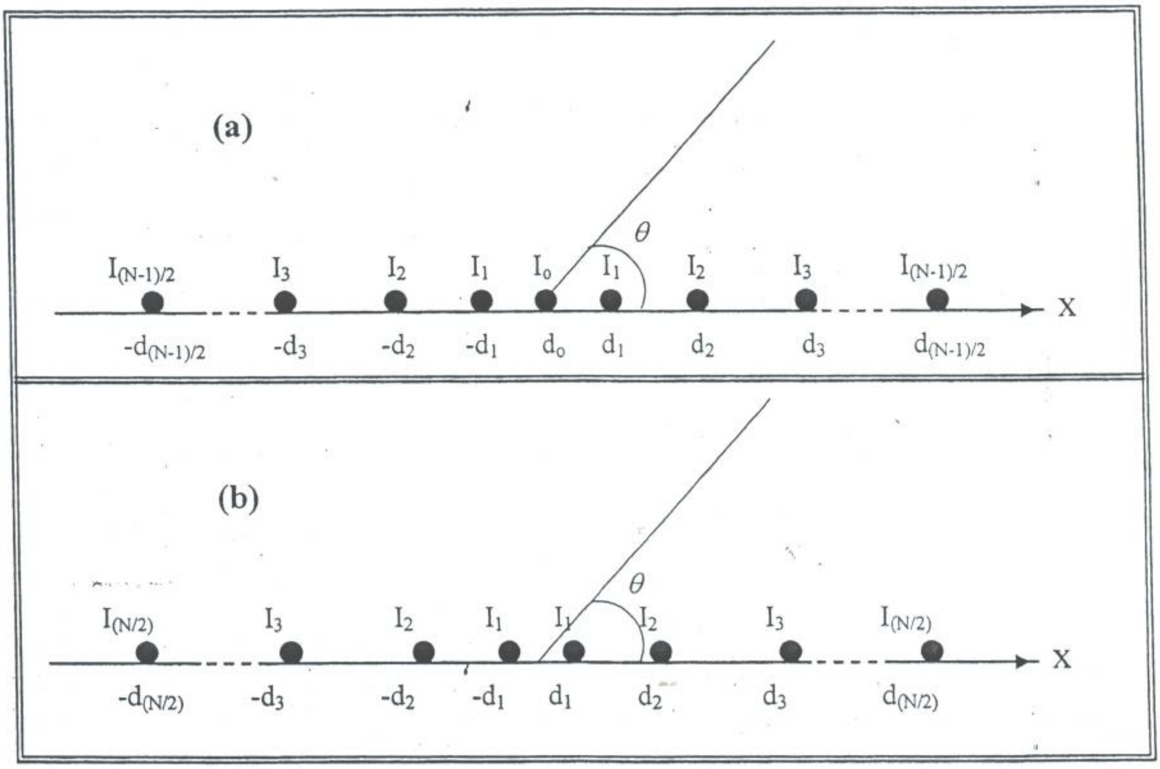


Fig.(1): Geometry of unequally spaced N-element array (a) odd number

The spacing between adjacent elements of the array is constrained by the space broadening factor (Δ). One of the more common array type is one in which the element currents have equal amplitudes, so :-

$$I_{-(N-1)/2} = I_{-(N-2)/2} = \ldots\ldots = I_{-1} = I_o = I_1 = \ldots\ldots = I_{(N-2)/2} = I_{(N-1)/2}$$, for odd N ....(11)

where I_o is the current amplitude for the center element.

2-2 Element Array

Short backfire antenna consists of two parallel circular metal plates of unequal diameters. These were placed perpendicular and co-axial to the central axis of the antenna and they are spaced by a half wavelength. A coaxial waveguide excited by TE_{11}-mode, used as a feeder to excite the antenna with a microwave energy, was positioned midway between and parallel to the plates [4]. The front small plate which is called sub-reflector acts as a mirror to reflect the wave incident from the excitor back toward the large plate called main-reflector which in turn reflects the wave back again to an observation point located in the front of the antenna as shown in Fig.(2) [5].

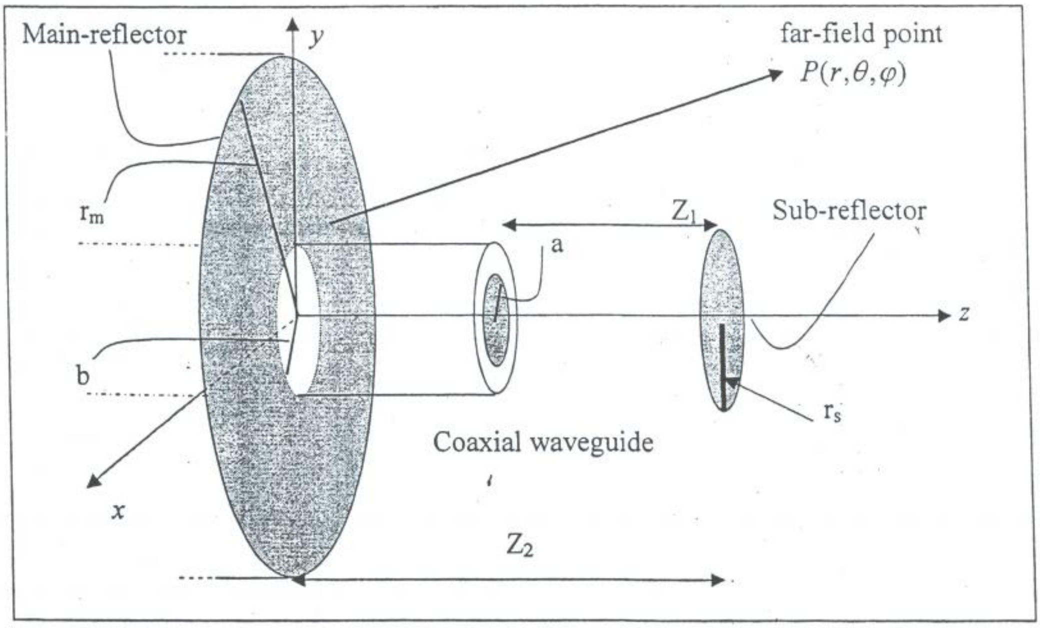


Fig. (2): Sketch of SBFA.

3- Results and discussion

Equations (10-a) and (10-b) have been applied to achieve the appropriate adjacent element spacings in H-plane. Fig.(3) describes the variation of first sidelobe level (FSLL) and peak sidelobe level (PSLL) with the space broadening factor for N=15. Fig.(4) depicts the effect of the space broadening factor on the beamwidth of the nonuniform array for the same number of elements.

Figures.(5) and (6) shows the comparison between the uniform and nonuniform array patterns for different values of $(\Delta = 0.0, 0.01\lambda, 0.02\lambda)$.

The synthesized array spacings are presented in Table(1) for nonuniform and uniform arrays. A perusal of the results yields the following conclusions:-

1) The value of the FSLL and PSLL decreases continually as the value of Δ ranges from 0.01λ to 0.02λ ; however, the corresponding value of the FSLL and PSLL increases as Δ increasing.

2) It is observed that by employing the nonuniform array synthesis technique, the FSLL is reduced by 8.85 dB over that of a uniform array patterns for N=15. Fig.(3), shows that the minimum FSLL and PSLL of the space broadening factor $(\Delta = 0.01\lambda, 0.02\lambda)$ is 23.25 dB.

3) The value of the 3 dB beamwidth decreases continually as the value of the space broadening factor ranges from 0.01λ to 0.2λ. The 3 dB beamwidth remains unchanged after these values of the space broadening factor (Δ).

4) The value of the 10 dB beamwidth decreases continually as the value of Δ ranges from 0.01λ to 0.07λ.

5) Similar figures for the FSLL, PSLL, 3dB and 10dB beamwidths were found regardless the number of the elements of the array.

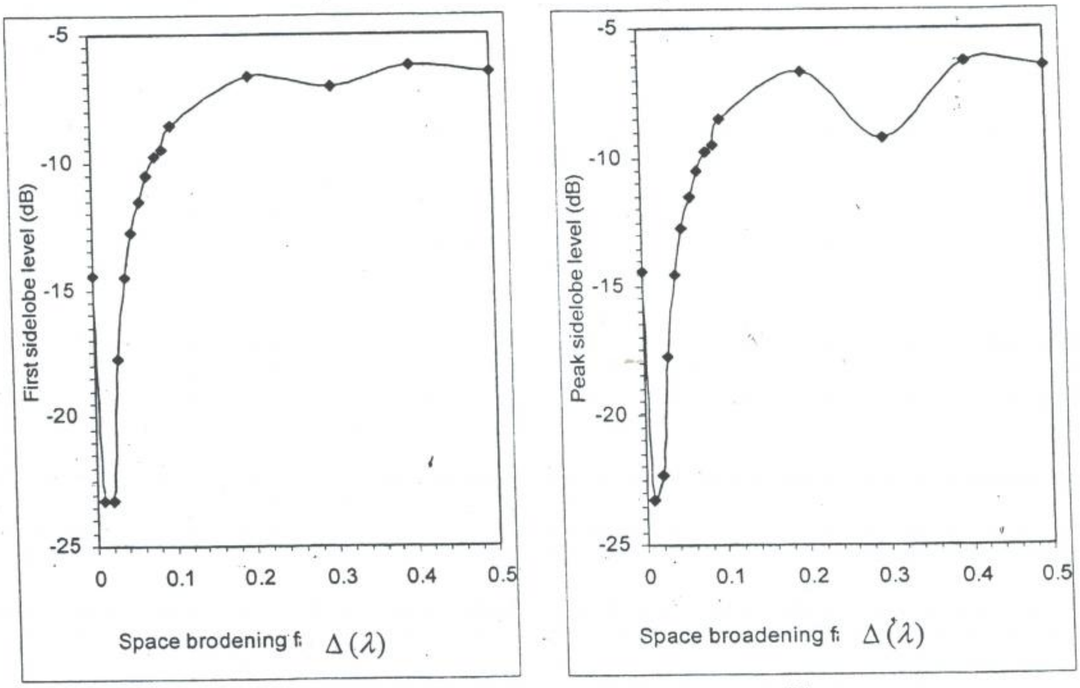
6) The co-polar radiation patterns have a similar behavior as H-plane patterns.

To achieve a narrow beamwidth and low sidelobes level performance for the radiation pattern, the antennas array is distributed as in Fig.(5) where the value of the space broadening factor ranges from 0.01λ to 0.02λ.

The directivity of uniformly excited unequally spaced arrays, for space broadening factor ranges from 0.01 λ to 0.1 λ, is shown in Fig.(7), the corresponding directivity value is 28.03 dB to 33.00 dB, respectively.

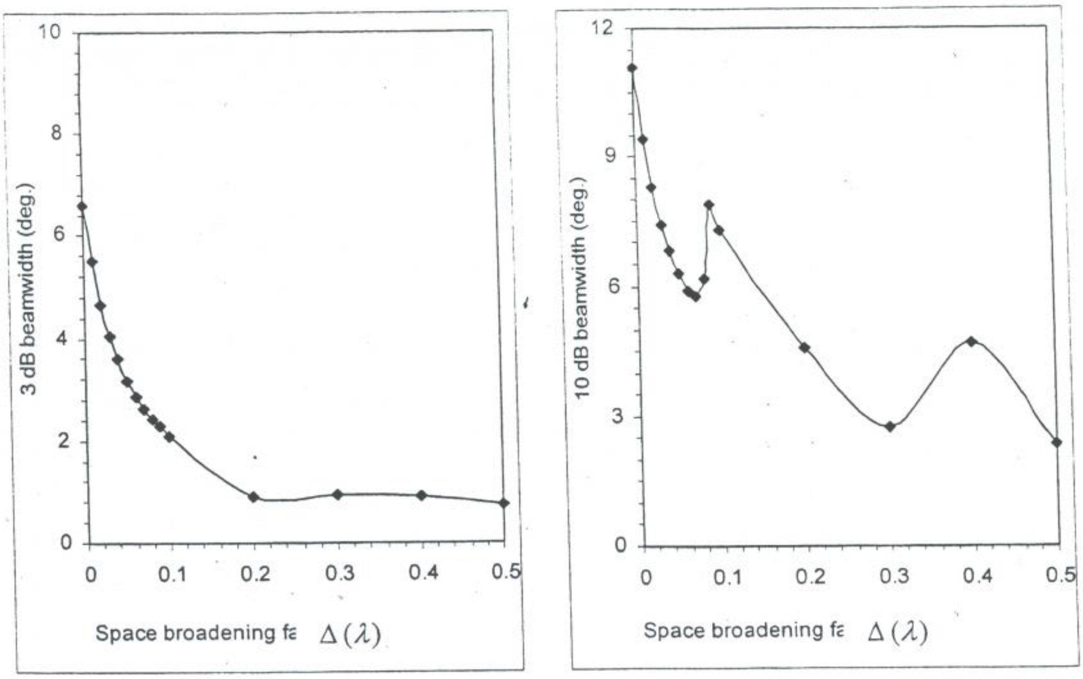
Table (1): Array element positions (in wavelength)

m	Non-uniform array		uniform array
	$\Delta = 0.01\ \lambda$	$\Delta = 0.02\ \lambda$	$\Delta = 0.00\ \lambda$
-7	4.78	6.06	3.50
-6	3.64	4.28	3.00
-5	2.82	3.14	2.50
-4	2.16	2.32	2.00
-3	1.58	1.66	1.50
-2	1.04	1.08	1.00
-1	0.52	0.54	0.50
0	0.00	0.00	0.00
1	0.52	0.54	0.50
2	1.04	1.08	1.00
3.	1.58	1.66	1.50
4	2.16	2.32	2.00
5	2.82	3.14	2.50
6	3.64	4.28	3.00
7	4.78	6.06	3.50



(a) (b)

Fig.(3): Sidelobe level of uniformly exited unequally spaced arrays variation with Δ (N=15), (a) FSLL and (b) PSLL.



(a) (b)

Fig.(4): Beamwidth of uniformly exited unequally spaced arrays variation with Δ (N=15), (a) 3 dB and (b) 10 dB.

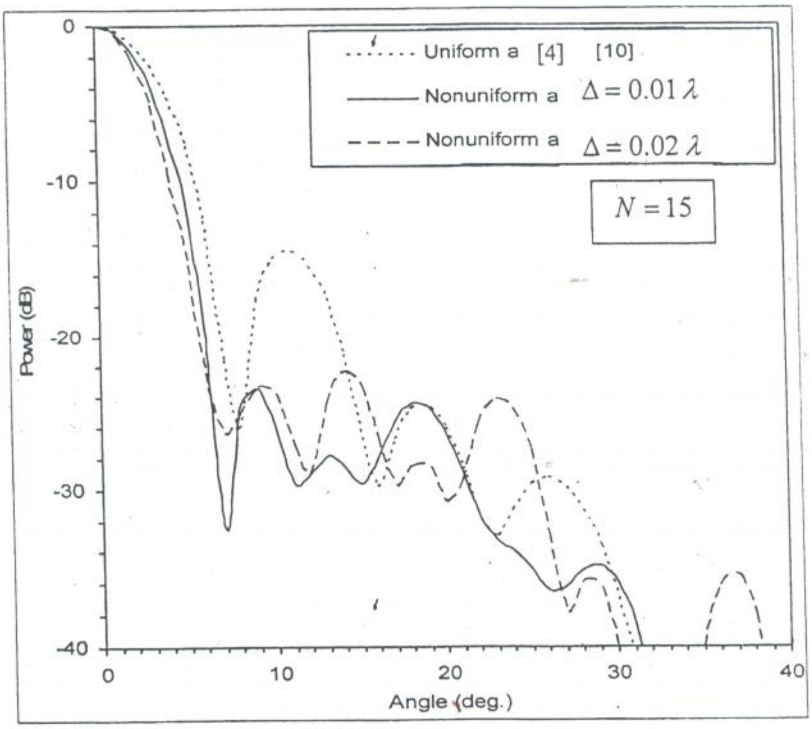


Fig.(5): Radiation patterns of uniformly exited equally and unequally spaced arrays for different Δ.

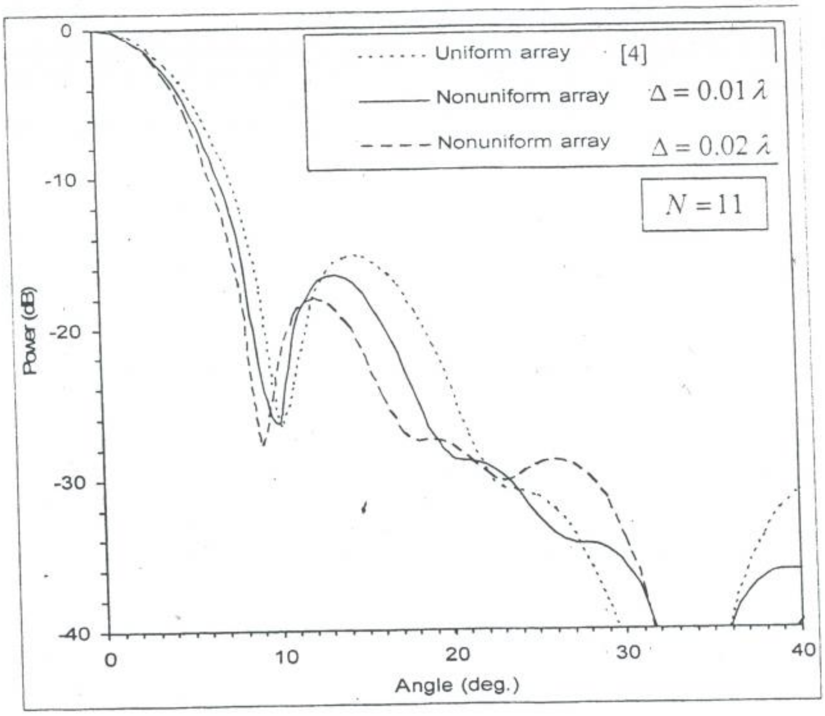


Fig.(6): Radiation patterns of uniformly exited equally and unequally spaced arrays for different Δ.

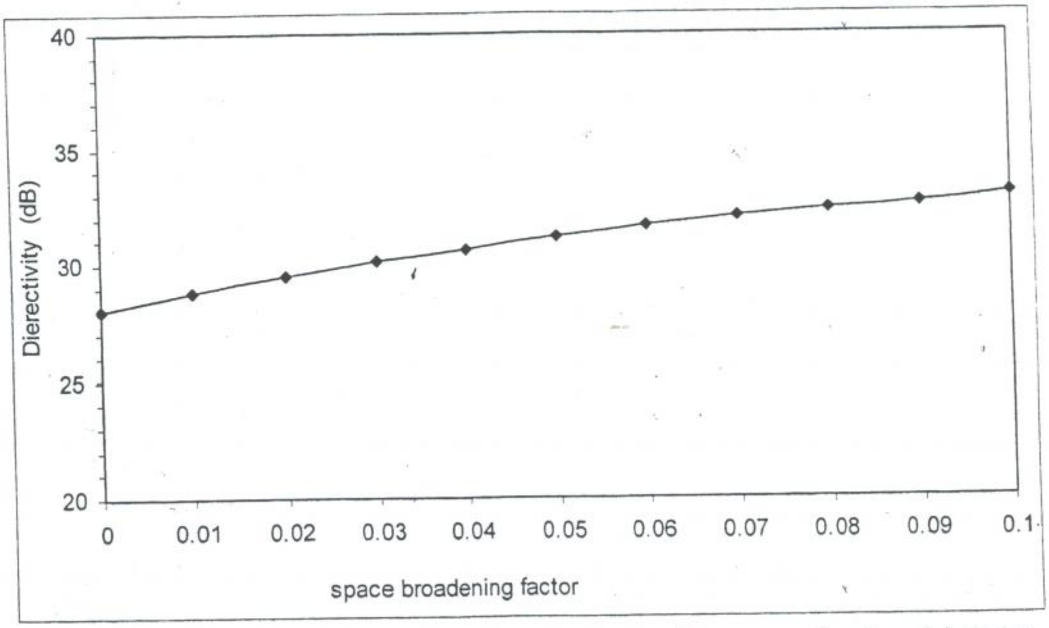


Fig.(7): The directivity of the uniformly exited unequally spaced arrays as a function of Δ (N=15).

4-Conclusions

This paper presents a new empirical relation to the synthesis of unequally spaced antenna arrays. The synthesis technique is practical in that it enables a designer to determine the pattern of the appropriate element spacings for a prescribed pattern. Numerical results have been presented for the radiation pattern arrays. An improvement in radiation characteristics has been obtained. The effects of unequal spacing, as reflected by the sidelobe levels and 3-dB beamwidth, are studied in detail. The results show that considerable improvement in array performance can be obtained in comparison with uniformly spaced arrays having the same number of elements and identical current distribution.

From the theory and the illustrations presented in this paper ,it is clear that a considerable reduction in the sidelobe level and narrow beamwidth obtained for optimum interelement spacings.

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المستخلص

تمت دراسة التحليل الرياضي للمصفوفات الخطية التي تتكون عناصرها من هوائي عكسي قصير مغذى بدليل موجة محوري ومثار بالنمط TE_{11} بطريقة البصريات الفيزيائية لتقليل مستوى الفلق الجانبية وتضييق الفلقة الرئيسة لهياكل إشعاعها من خلال تغيير عدد العناصر والمسافات التي تفصلها.

كان واضحاً من النتائج التي حصلنا عليها بأن مستوى الفلقة الجانبية الأولى قلّ بمقدار 8.85 dB بالنسبة إلى المصفوفة التي تكون المسافات بين عناصرها غير متساوية مقارنة بنفس مستوى الفلقة للمصفوفة التي تكون المسافات بين عناصرها متساوية وللعدد $N = 15$. تم في هذا العمل التوصل إلى علاقة تجريبية جديدة لاختيار المسافة الملائمة بين العناصر للحصول على أفضل النتائج المطلوبة من خلال تثبيت عامل توسيع المسافة بين العناصر (Δ).