Reconstruction of Digital Holograms by Iterative Phase Retrieval Algorithm: Improvements and Analysis

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Abstract: Iterative phase retrieval usually requires a good initial guess for the phase. In this paper, we consider two improvements to a recently proposed scheme (Opt. Eng. 55 (3), 033106 (2016)) to acquire the initial guess from a digital holographic system. This set-up is simpler than phase-shifting interferometry, and suppresses the twin image and dc terms without the resolution trade-off involved in off-axis holography. In this paper, we show the benefits of ensuring the propagation algorithm is unitary. We also use a recently proposed iterative phase retrieval algorithm which uses a combination of the Gerschberg-Saxton and hybrid input-output algorithms, and demonstrate the benefit of this algorithm for avoiding stagnation of the algorithm.

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1. Introduction

Digital holography (DH) is a technique includes an interference pattern recording and its object wave-filed numerical reconstruction [2-6]. Typically, two methods can be employed to extract DH information from either single or multi- intensity measurement: (i) The Off-axis Fourier Spatial Filtering (OFSF) [3, 6]; (ii) Phase Shifting Interferometry (PSI) [5, 6]. In [1] we proposed a new method for DH reconstruction from three intensity measurements by means of iterative phase retrieval algorithms, such as Gerchberg-Saxton (GS) algorithm, error reduction (ER) algorithm, and hybrid input output (HIO) algorithm [7]. The proposed measurement setup is shown in Fig. 1. The resulting reconstructed holograms are comparable with OFSF and PSI in both simulation and experiment [1].

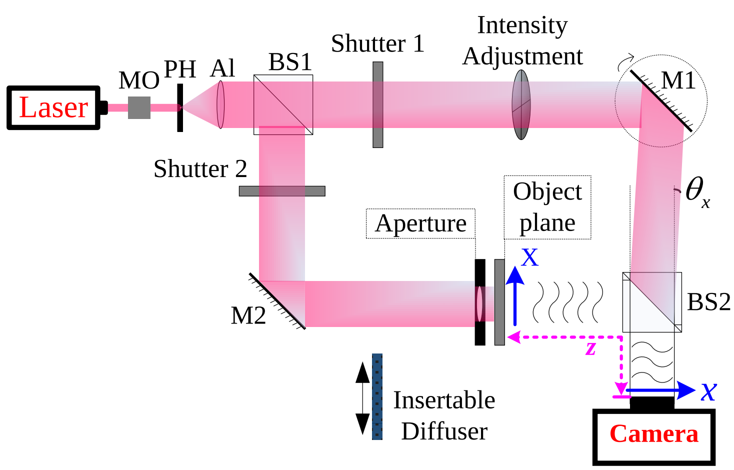

Fig. 1 Schematic depicting the measurement setup. MO, microscope objective X20; PH, pinhole; Al, achromatic lens; BS, beam-splitter; Mirror: M1 can be rotated to create both an in-line reference, or an off-axis reference, and M2 is fixed; z is the distance between the object plane and camera plane; Note insertion of the diffuser generates a diffuse object field.

Compared with [1], two improvements are made in this paper:

① Unitary Numerical Transforms;

② Combined Iterative Phase retrieval Algorithm;

in order to gain a better performance of phase retrieval algorithms, which can improve the quality of the resulting DH reconstruction. In Section 2, first the core concepts of those two improvements are presented, their effects on the final performance of iterative phase retrieval algorithm are analyzed. Second, how the improved proposed method works is discussed. Finally in Section 3 a brief conclusion is given.

2. Improvements and Discussion

2.1 Unitary Numerical Transforms

Unitary transforms conserve power and can be perfectly inverted. In [8] we demonstrate that superior GS iterative phase retrieval results can be obtained using the unitary algorithms.

We begin by intruding the Fresnel Transform (FST), which can model the free space propagation of a coherent object wave-field *U*(*X*) through a distance z, see Fig. 1:

$$u(x) = \mathrm{FST}_z\{U(X)\} = \sqrt{\frac{1}{j\lambda z}} \int_{-\infty}^{\infty} U(X) \exp\left[\frac{j\pi}{\lambda z}(X-x)^2\right] dX \tag{1}$$

Two different decompositions have been proposed in literatures [9-11] to numerically calculate the above FST: *Spectral method* (SM) and *Direct method* (DM). It should be noted that the discrete Fresnel transform (DFST) performed using DM and SM can both be *unitary*. This indicates that when applying either DM or SM unitary numerical calculation within the iterative phase retrieval loop, both calculation error and energy loss can be eliminated, which can help the iterative algorithm converge. We note that the resolution limit obtained by using DM and SM are different [9-11], therefore the scheme choice about whether unitary DM or SM should be preferred is dependent on various aspects The availability of this choice makes iterative phase retrieval algorithms more flexible in terms of sampling rates. Compared with a non-unitary numerical calculation, significantly better phase retrieval results can be obtained when the unitary SM/DM DFST is employed.

2.2 Combined Iterative Phase Retrieval *Algorithm*

In 1982 a wide variety of iterative phase retrieval algorithms, such as GS, ER, HIO, and so on, were summarized and compared by Fienup [7]. To the best of our knowledge, in [7] this is the first time that the combined phase retrieval algorithm is proposed and simulated. Combined phase retrieval algorithm means rather than using one single algorithm, either GS or HIO, during the iterative phase retrieval loop two algorithms are combined together. Take the combined GS+HIO algorithm for example, HIO is implemented after a few iterations of GS, see Fig. 2. In [12] the combined GS/ER+HIO algorithm is examined for input and output planes related by a Fourier transform (FT-GS+HIO or FT-ER+HIO). In that paper, the improved convergence and lower rates of stagnation of this combined phase retrieval algorithm over algorithms such as GS, HIO, and ER are demonstrated. Compared with one single algorithm, the combined phase retrieval algorithm is better at avoiding stagnation during phase retrieval loop, which allows the algorithm to successfully locate superior solutions. We note that since it is well-known how to obtain a unitary *fft* algorithm, neither calculation error nor energy loss is introduced into the phase loop by digital calculations in the FT-GS+HIO. However, it is not necessarily easy to find a unitary propagation algorithm when the input and output planes are related by a 2D non-separable linear canonical transform (2D-NS-LCT) or even a Fresnel transform (FST). In [13] we proposed a unitary fast numerical algorithm for the 2D-NS-LCT calculation; we demonstrated its use in a GS+HIO phase retrieval algorithm. We demonstrated that the combined phase retrieval algorithm works well, and a high quality reconstruction can be achieved with mean squared error of 10^{-30}.

2.3 The Improved Proposed Method

We note that in the proposed method [1], (i) the DFST is performed by the SM, which may be non-unitary; (ii) Only one single phase retrieval algorithm, either GS or HIO, is employed to extract DH information. According to the above discussions, the performance of the proposed method can be further improved by employing (1) *unitary* DFST based on the DM; and (2) Combined phase retrieval algorithm. The flow-chart of the improved proposed method is given in Fig. 2.

3. Conclusions and Future Work

In this paper, two further improvements are made to the former proposed method in order to gain high quality DH reconstruction by iterative phase retrieval algorithms. The value of our contribution has been demonstrated by simulation. Future work includes experimental verification of these simulation results.

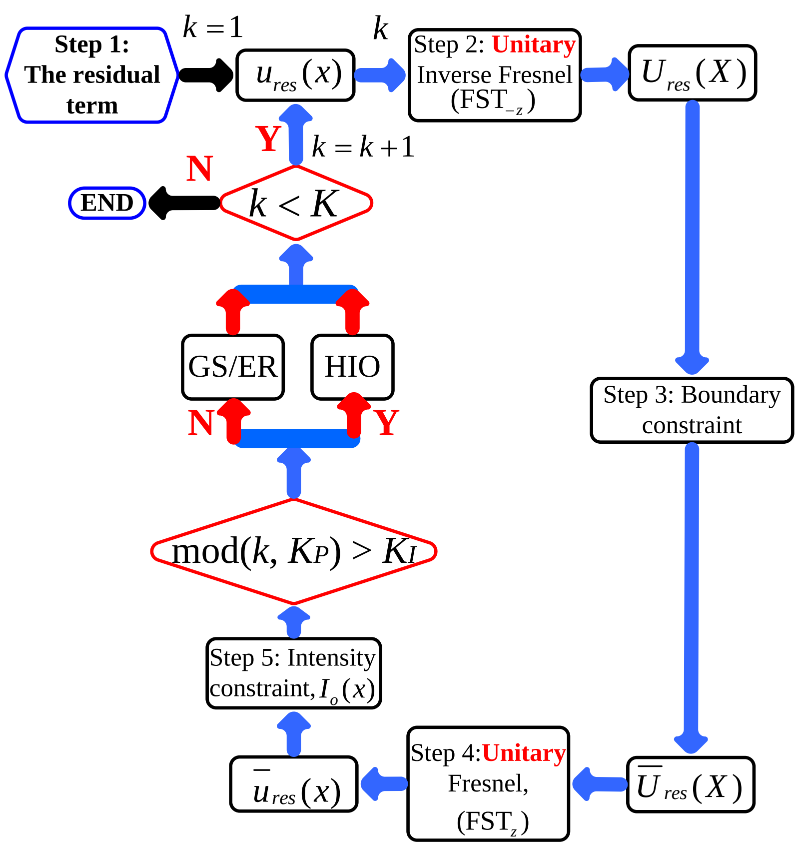

Fig. 2 Iterative phase retrieval algorithm flow chart. The residual term is calculated using the three intensity measurements as shown in [1]. The parameter K is the maximum number of iterations the combined GS/ER+HIO algorithm is performed. k represents the k^{th} iteration. K_P is the number of iterations in each cycle of combined phase retrieval algorithm. K_I is the number of iterations the GS/ER is performed in each cycle, while K_P-K_I is the number of iterations of the HIO performed in each cycle.

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