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Revealing the origin of the strong second harmonic generation of Li_2CdXS_4 and Li_2CdXS_4 ($X = \text{Ge}$ or Sn)

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Second harmonic generation (SHG) and the first hyperpolarizability (β_{ijk}) of two novel quaternary diamond-like semiconductors, $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$, are investigated based on the band structure calculations. Calculations show that these materials possess wide and direct energy gaps of about 3.10 eV ($\text{Li}_2\text{CdGeS}_4$) and 3.23 eV ($\text{Li}_2\text{CdSnS}_4$) in close agreement with the measured gaps (3.15 eV and 3.26 eV). The energy gap values confirm that these materials exhibit exceptional laser damage thresholds. The presence of polarizable M-S bonds tunes these compounds to exhibit strong SHG. The calculated linear optical properties exhibit considerable anisotropy, which favors the enhanced phase matching conditions necessary for observation of SHG and optical parametric oscillation. It has been found that $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ exhibit negative uniaxial anisotropy and positive birefringence. The calculated SHG of the dominant component is about 18.64 pm/V for $\text{Li}_2\text{CdGeS}_4$, which is larger than that obtained from $\text{Li}_2\text{CdSnS}_4$ (12.75 pm/V). These results are in concordance with the experimental value of the well known nonlinear crystal KTiOPO_4 . The calculated first hyperpolarizability (β_{333}) at $\lambda = 1064$ nm is about 13.015×10^{-30} esu for $\text{Li}_2\text{CdGeS}_4$ and 9.704×10^{-30} esu for $\text{Li}_2\text{CdSnS}_4$. © 2016 AIP Publishing LLC.

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I. INTRODUCTION

In recent years, ultra-violet (UV), visible, and infra-red (IR) laser radiation at wavelengths that are presently inaccessible via conventional sources have been in demand for many industrial, medical, biological, and entertainment applications.¹⁻³ However, it is still desirable to find materials with promising nonlinear optical (NLO) properties. The quest for the development of novel more efficient and compact quantum electronic devices requires a search for design and fabrication of new materials which should be less expensive and more technological, with novel applications.^{4,5} These materials can be used as converters, deflectors, and modulators of electromagnetic radiation, for the design and production of light emitting diodes, for more sensitive detectors, and as nonlinear optical materials.⁶⁻⁸ Recently, two novel quaternary diamond-like semiconductors, $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$,⁹⁻¹³ were synthesized by Lekse *et al.*¹⁴ It has been found that these materials have a wide band gap, which causes them to have a high laser damage threshold.^{14,15} In these materials, the orientation of the tetrahedral building blocks causes these materials to have a non-centrosymmetric structure, resulting in the loss of inversion symmetry which in turn gives a non-zero second harmonic generation (SHG).¹⁶ The presence of polarizable M-S bonds in these materials results in a strong SHG. Therefore, having a high laser damage threshold and strong SHG makes these materials promising candidates for nonlinear applications in the IR region.^{14,15}

Much attention has been given to the investigation of the $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ compounds. Lekse *et al.*¹⁴ have synthesized $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$, investigated their structures and the SHG. Jang *et al.*¹⁷ have reported that $\text{Li}_2\text{CdGeS}_4$ is an excellent NLO material with a wide transparency range. Recently, Brant *et al.*¹⁸ have investigated the SHG and the laser damage threshold of $\text{Li}_2\text{CdGeS}_4$ and found that the $\text{Li}_2\text{CdGeS}_4$ produces strong SHG in the IR region and possesses an exceptional laser damage threshold. Few theoretical calculations using non-full potential methods to calculate the electronic structure and the linear optical properties were reported.¹⁹⁻²¹ None of these calculations have reported the SHG of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ compounds and the relation between the structure and the resulting NLO properties. Toward this, we have addressed ourselves to performing comprehensive theoretical calculations using the full potential method within different exchange-correlation potentials to reach the maximum accuracy in calculating the band structure and hence the linear and nonlinear optical properties. It is well-known that the accuracy of the results is very sensitive to the selection of the exchange-correlation potential. The theoretical methods of studying the relationship between the structure and the nonlinear optical susceptibilities may be a better solution before venturing into the physical synthesizing of a new material. Unfortunately, the reported SHG measurements for $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ lack detailed descriptions about the complex second-order nonlinear optical susceptibility tensors; moreover, no information regarding the microscopic first hyperpolarizability (β_{ijk}) for these compounds is available in the literature. The microscopic first

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hyperpolarizability term cumulatively yields a bulk observable second order susceptibility term, $\chi_{ijk}^{(2)}(\omega)$, which in turn is responsible for the high SHG response.^{22,23} Due to the recent improvement in computational technologies, it has been proven that the first-principles calculation is a strong and useful tool to predict the crystal structure and the properties related to the electron configuration of a material before its synthesis.^{24–27}

II. COMPUTATIONAL DETAILS

The influence of replacing Sn by Ge on the electronic band structures and the linear and nonlinear optical susceptibilities were investigated by means of density functional theory (DFT). The all-electron full-potential linearized augmented plane wave plus the local orbitals (FP-LAPW + lo) method as implemented in the WIEN2k package²⁸ was used to solve the Kohn Sham equations. We have used the minimum radius of the muffin-tin spheres (R_{MT}) values as 2.0 a.u. for all atoms. This value was chosen to insure that there is no charge leakage out of the atomic sphere cores. The crystal structures of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ were taken from the x-ray diffraction data (XRD) reported by Lekse *et al.*¹⁴ Using Perdew-Burke-Ernzerhof generalized gradient approximation

(PBE-GGA),²⁹ the crystal structures of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ were optimized by minimizing the forces acting on each atom. The relaxed geometries of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ compounds were used to calculate the electronic band structures, the density of states, the electronic charge density, and the linear and nonlinear optical susceptibilities using the PBE-GGA and the recently modified Becke-Johnson potential (mBJ).³⁰ It has been found that the mBJ succeeds by a large amount in bringing the calculated energy gap closer to the experimental one. Therefore, we chose to present the results obtained by mBJ. The mBJ allows the calculation of the band gap with an accuracy similar to very expensive GW calculations. It is a semilocal approximation to an atomic “exact-exchange” potential and a screening term.³⁰ The following states are considered as valence electrons: ($2s^1$), ($4s^2 4p^2$), ($5s^2 5p^2$), ($4p^6 4d^{10} 5s^2$), and ($3s^2 3p^4$) for Li, Ge, Sn, Cd, and S, respectively. The crystal structures of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ are shown in Fig. 1; from the unit cell, it is clear that during substituting Ge by Sn atoms, the Sn atoms occupied the Cd atoms’ positions. To achieve the total energy convergence, the basis functions in the IR are expanded up to $R_{MT} \times K_{max} = 7.0$ and inside the atomic spheres for the wave function. The maximum value of l is taken as $l_{max} = 10$, while the charge density is Fourier

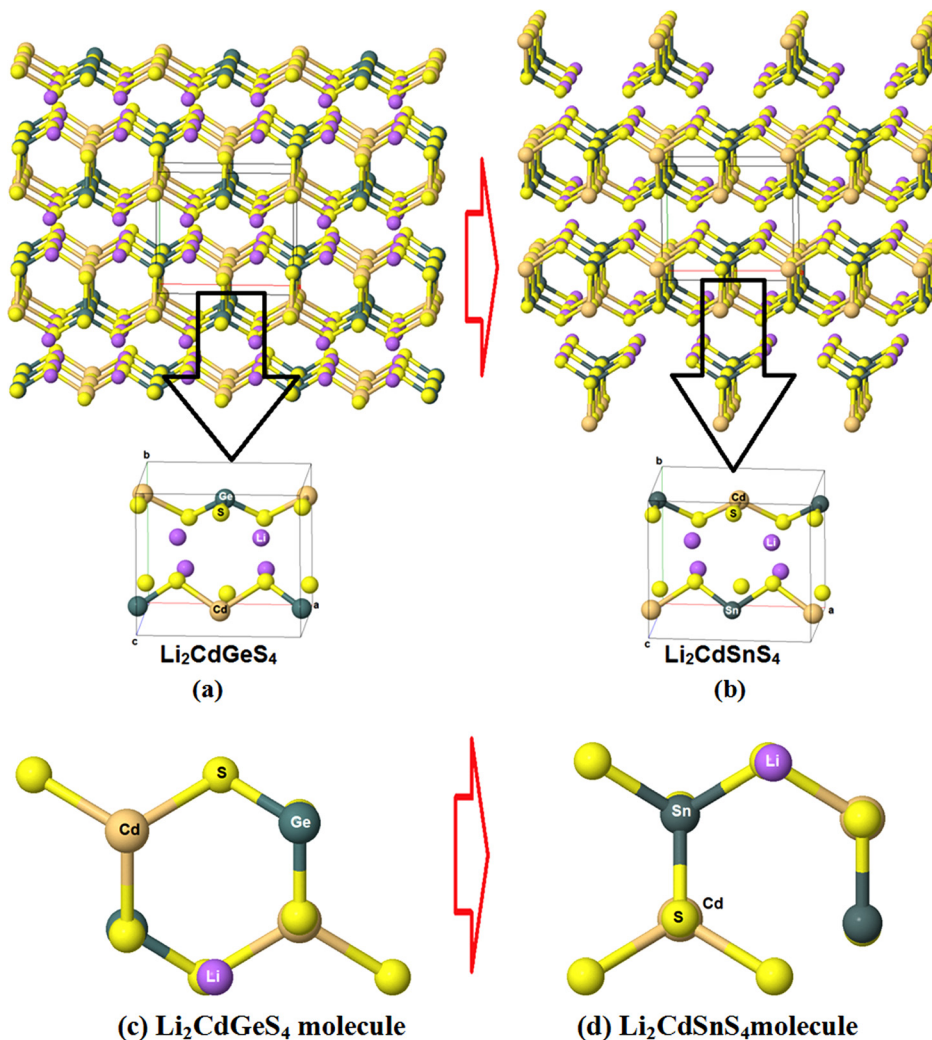


FIG. 1. (a) The crystal structure of $\text{Li}_2\text{CdGeS}_4$ along the unit cell. (b) The crystal structure of $\text{Li}_2\text{CdSnS}_4$ along the unit cell. (c) Molecule of $\text{Li}_2\text{CdGeS}_4$. (d) Molecule of $\text{Li}_2\text{CdSnS}_4$.

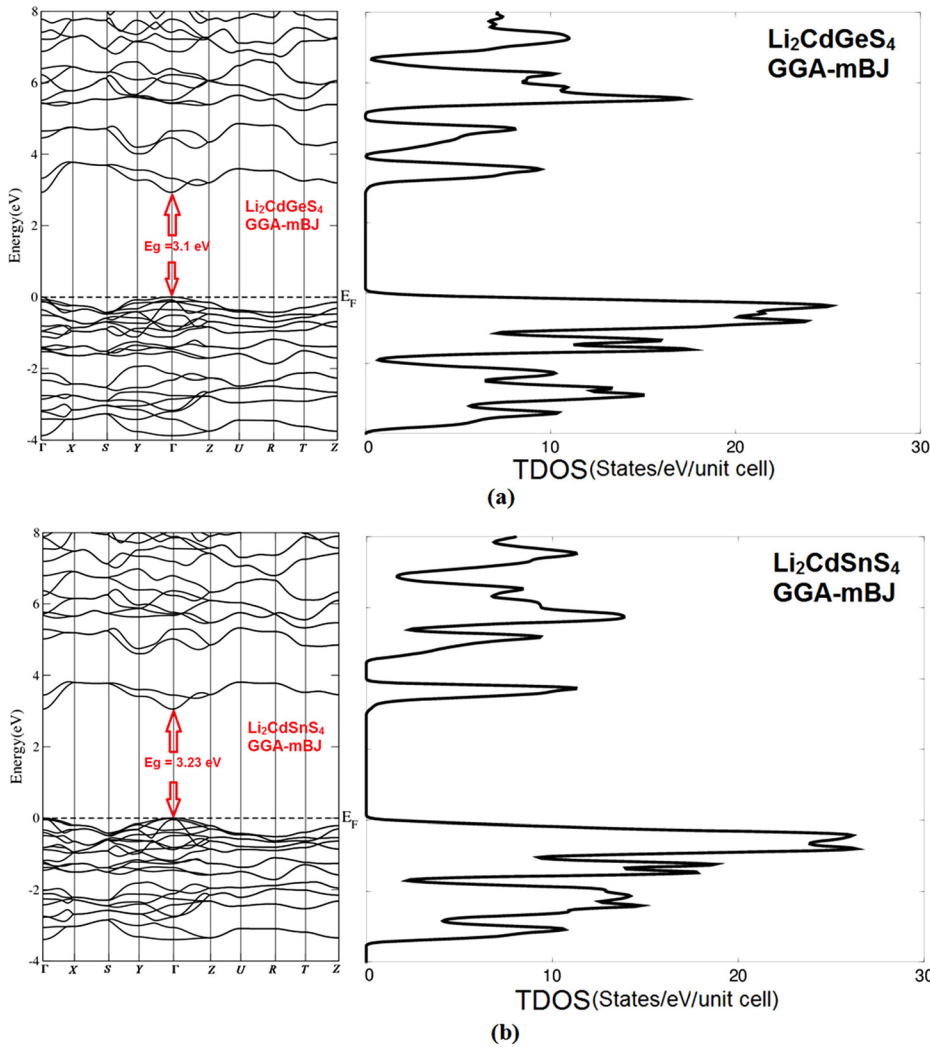


FIG. 2. (a) Calculated electronic band structure along with the calculated total density of states for $\text{Li}_2\text{CdGeS}_4$. (b) Calculated electronic band structure along with the calculated total density of states for $\text{Li}_2\text{CdSnS}_4$.

expanded up to $G_{\max} = 12.0(\text{a.u.})^{-1}$. A self-consistency is obtained using 300 k points in the irreducible Brillouin zone (*IBZ*). Since the total energy of the system is stable within 0.00001 Ry, the self-consistent calculations are converged. The calculations of the electronic band structures, the density of states, and the electronic charge density are performed within 1000 k points in the *IBZ*, while the linear and nonlinear optical susceptibilities are obtained by using 2000 k points in the *IBZ*.

III. RESULTS AND DISCUSSION

A. Electronic band structure and density of states

The calculated electronic band structure along the total density of states (TDOS) for $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ using mBJ is presented in Figs. 2(a) and 2(b). It has been found that both compounds exhibit a direct band gap ($\Gamma^{\text{VBM}} - \Gamma^{\text{CBM}}$). The calculated band gaps using mBJ are about 3.10 eV ($\text{Li}_2\text{CdGeS}_4$) and 3.23 eV ($\text{Li}_2\text{CdSnS}_4$), in good agreement with the measured one (3.15 eV for $\text{Li}_2\text{CdGeS}_4$)¹⁴ and (3.26 eV for $\text{Li}_2\text{CdSnS}_4$).¹⁴ Therefore, a material with such an energy band gap value is expected to possess a high laser damage threshold.^{14,18} The upper valence band and the lower conduction band around Fermi level show a very high k -dispersion. This reflects the high mobility of the carriers.

The conduction band minimum (CBM) of $\text{Li}_2\text{CdGeS}_4$ ($\text{Li}_2\text{CdSnS}_4$) is formed by S-3p and Ge-4s (Sn-5s) orbitals, whereas the valence band maximum (VBM) of both compounds is formed by S-3p orbitals. It is clear that the Cd orbitals exhibit an insignificant contribution to CBM and VBM; thus, the variations in divalent elements can be systematically explored to assess the effects on SHG efficiencies, as well as other attractive properties, while maintaining a wide band gap.¹⁸

The calculated TDOS confirms the existence of the wide energy gaps and the energy gap values. Furthermore, we have calculated the partial density of states (PDOS) to explore the orbital dispersions and the orbital roles with Ge replaced by Sn, as shown in Figs. 3(a)–3(n). It has been noticed that Ge-4s, Cd-4d, and S1,2,3–3s/3p orbitals exhibit insignificant influence in the matter of peak heights whilst replacing Ge by Sn, as shown in Figs. 3(a) and 3(b). Whereas a significant influence occurs in the matter of peak heights and locations for Ge-4p/4d, Cd-3s/4p, and Li-2s orbitals, as shown in Figs. 3(c) and 3(d). It is interesting to see that the contribution of S1,2,3-atoms is significantly influenced by substituting Ge by Sn (Figs. 3(i) and 3(j)), while Ge-, Cd-, and Li-atoms show only minor changes (Figs. 3(k)–3(n)). Therefore, the sum of all changes exhibits a clear

influence on the bonding character and the optical properties of the $\text{Li}_2\text{CdSnS}_4$ compound.

For deep investigation, to explore the influence of substituting Ge by Sn on the electronic structure and hence the resulting properties, we have taken a careful look into the valence electronic charge density distribution, taking into account the Slater covalent radii and the Pauling electronegativity of Li, Cd, Ge, Sn, and S. In Figs. 4(a) and 4(b), we have illustrated the valence electron charge density distribution

along the (1 0 1) crystallographic plane. The electronegativity of Li, Cd, Ge, Sn, and S is 0.98, 1.69, 2.01, 1.96, and 2.58, respectively. As we have mentioned, during substituting Ge by Sn, the Sn atoms occupied the Cd atoms' positions; therefore, the electronegativity difference between S and Ge is less than that between S and Cd atoms; thus, the S atom forms a covalent bonding with the Ge atom in $\text{Li}_2\text{CdGeS}_4$ (Fig. 4(a)), whereas in $\text{Li}_2\text{CdSnS}_4$ the S atom forms mostly an ionic and partially covalent bond with Cd and mostly covalent with the

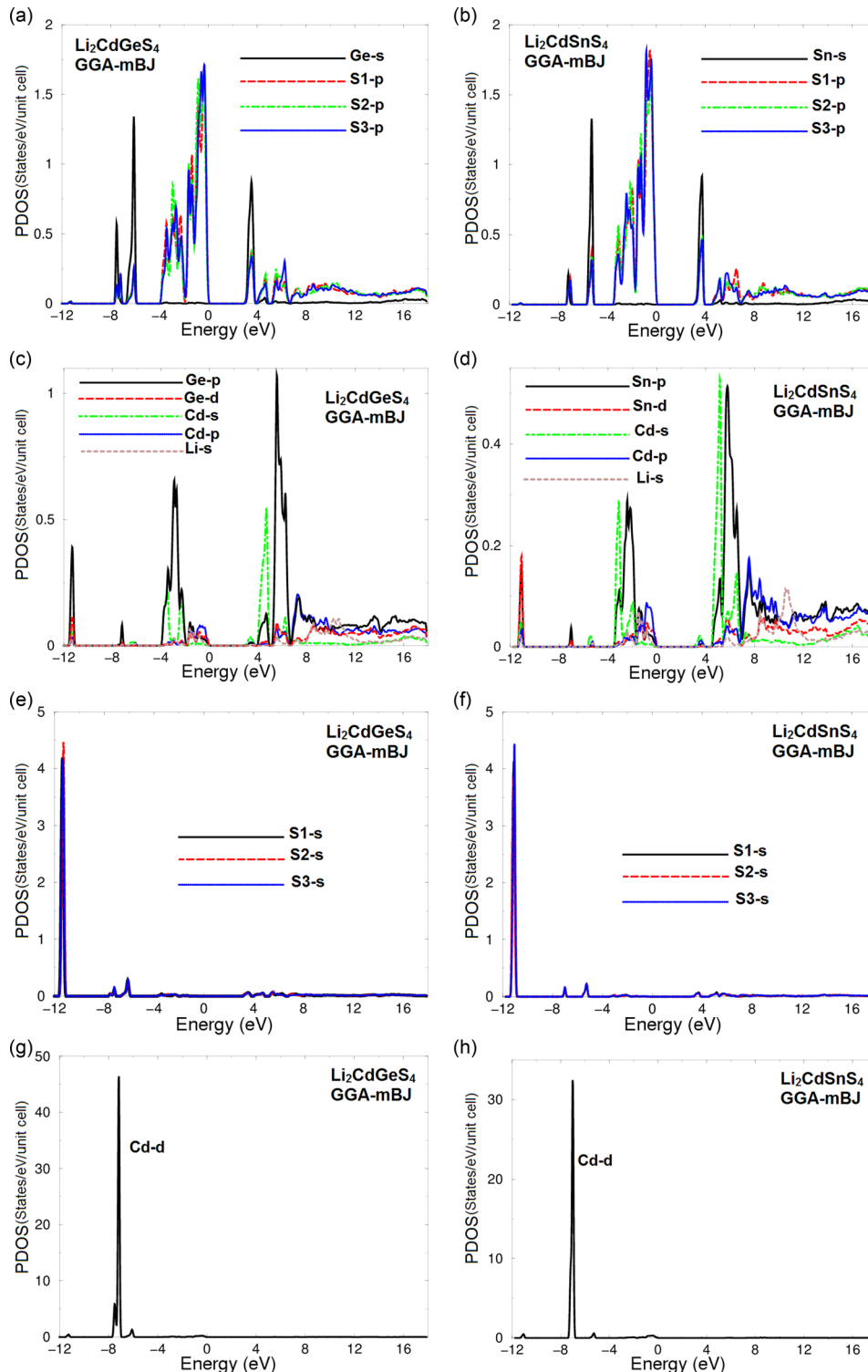


FIG. 3. (a)–(h) Calculated partial density of states for $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$.

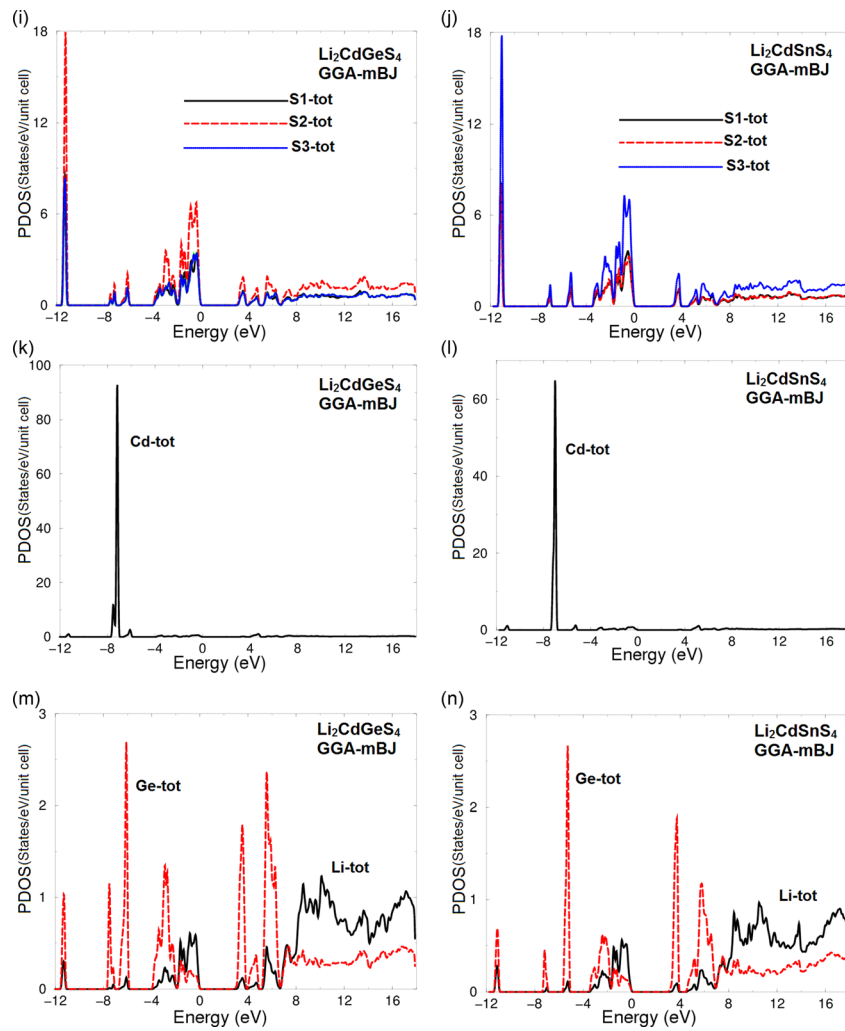


FIG. 3. (Continued.)

Sn atom. It has been noticed that the bond between Ge and S atoms leads to perturbation of the contours surrounding S atoms (Fig. 4(a)), while in Cd-S bonding the contours around S and Cd atoms remain uniform (Fig. 4(b)), confirming the

dominant ionic nature. These figures bring a clear visualization to understand the origin of the differences in the electronic properties of the two compounds. The covalent bonding is more favorable for the transport of the carriers than the

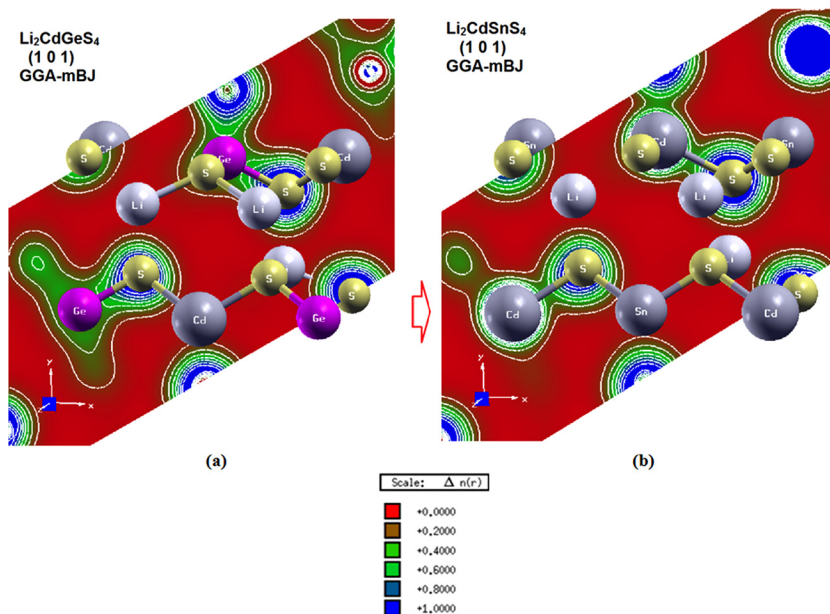


FIG. 4. (a) Calculated valence electronic charge density distribution of $\text{Li}_2\text{CdGeS}_4$. (b) Calculated valence electronic charge density distribution of $\text{Li}_2\text{CdSnS}_4$.

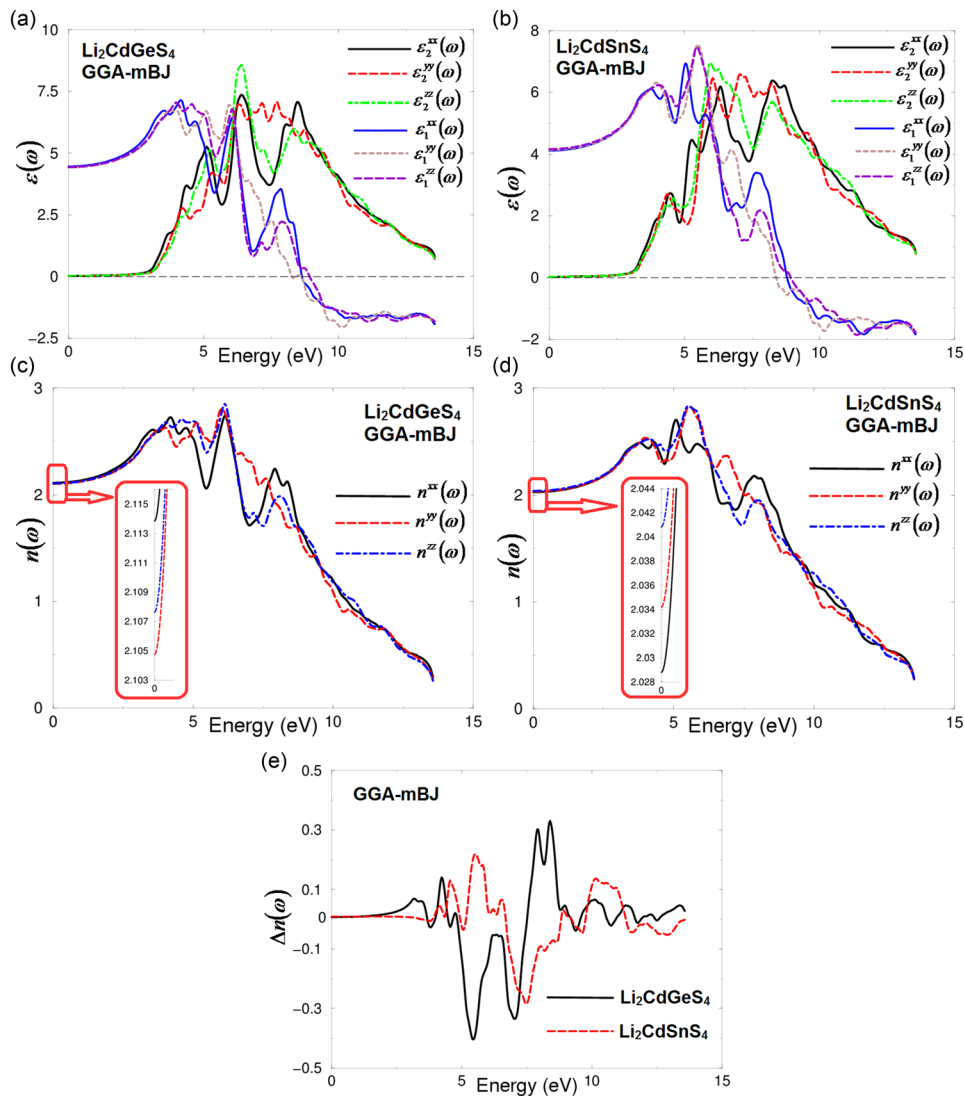


FIG. 5. (a) Calculated $\epsilon_2^{xy}(\omega)$ (dark solid curve-black), $\epsilon_2^{yy}(\omega)$ (light long dashed curve-red), and $\epsilon_2^{zz}(\omega)$ (light dotted dashed curve-green) along with calculated $\epsilon_1^{xx}(\omega)$ (dark solid curve-blue), $\epsilon_1^{yy}(\omega)$ (light dashed curve-brown), and $\epsilon_1^{zz}(\omega)$ (light solid curve-violet) for $\text{Li}_2\text{CdGeS}_4$. (b) Calculated $\epsilon_2^{xy}(\omega)$ (dark solid curve-black), $\epsilon_2^{yy}(\omega)$ (light long dashed curve-red), and $\epsilon_2^{zz}(\omega)$ (light dotted dashed curve-green) along with calculated $\epsilon_1^{xx}(\omega)$ (dark solid curve-blue), $\epsilon_1^{yy}(\omega)$ (light dashed curve-brown), and $\epsilon_1^{zz}(\omega)$ (light solid curve-violet) for $\text{Li}_2\text{CdSnS}_4$. (c) Calculated refractive indices $n^{xx}(\omega)$ (dark solid curve-black), $n^{yy}(\omega)$ (light dashed curve-red), and $n^{zz}(\omega)$ (light dotted dashed curve-blue) spectrum $\text{Li}_2\text{CdGeS}_4$. (d) Calculated refractive indices $n^{xx}(\omega)$ (dark solid curve-black), $n^{yy}(\omega)$ (light dashed curve-red), and $n^{zz}(\omega)$ (light dotted dashed curve-blue) spectrum $\text{Li}_2\text{CdSnS}_4$. (e) Calculated birefringence $\Delta n(\omega)$ for $\text{Li}_2\text{CdGeS}_4$ (dark solid curve-black) and $\text{Li}_2\text{CdSnS}_4$ (light long dashed curve-red).

ionic one.³¹ This may be one of the reasons for the strong linear and nonlinear optical properties in $\text{Li}_2\text{CdGeS}_4$.

B. Complex first-order linear optical dispersion

Based on the calculated band structure, we can obtain the linear optical properties. The linear optical properties are calculated using the optical code implemented in the WIEN2k package;²⁸ for more details, please see the user guide³² and Ref. 33. The optical properties can provide more detailed information about the electronic structure of the materials. It is well-known that the optical properties are very sensitive to the energy band gap. In the DFT, by solving the Kohn-Sham equations, we map an interacting many-body system to a non-interacting hypothetical system which has the same electron

density. The price that we will pay is the definition of a new functional that is called the exchange-correlation functional. Unfortunately, the exact form of exchange-correlation functional is unknown. Therefore, the accuracy of our results will be sensitive to selection of the exchange-correlation functional, and it can play a major role in the accuracy of our results, and this is one of the main questions in DFT. Thus, we would like to mention that in our previous works³⁴⁻³⁶ we have calculated the energy band gap and the linear and nonlinear optical susceptibilities using the FPLAPW method within mBJ on several systems whose energy band gap, and linear and nonlinear optical susceptibilities are known experimentally; in those previous calculations, we found very good agreement with the experimental data. Thus, we believe that our calculations reported in this paper would produce very

TABLE I. The calculated energy band gap in comparison with the experimental value, $\varepsilon_1^{xx}(0)$, $\varepsilon_1^{yy}(0)$, $\varepsilon_1^{zz}(0)$, $\varepsilon_1^{average}(0)$, $\delta\varepsilon$, ω_p^{xx} , ω_p^{yy} , ω_p^{zz} , $n^{xx}(0)$, $n^{yy}(0)$, $n^{zz}(0)$, and $\Delta n(0)$.

	Li ₂ CdGeS ₄		Li ₂ CdSnS ₄	
	This work	Expt.	This work	Expt.
Eg (eV)	3.10, 2.78 ^a	3.15 ^b	3.26, 2.50 ^a	3.23 ^b
$\varepsilon_1^{xx}(0)$	4.468		4.116	
$\varepsilon_1^{yy}(0)$	4.429		4.137	
$\varepsilon_1^{zz}(0)$	4.442		4.164	
$\varepsilon_1^{average}(0)$	4.446		4.139	
$\delta\varepsilon$	-0.00742		-0.00918	
ω_p^{xx}	8.666		8.902	
ω_p^{yy}	8.285		8.421	
ω_p^{zz}	8.938		8.938	
$n^{xx}(0)$	2.113		2.028	
$n^{yy}(0)$	2.104		2.034	
$n^{zz}(0)$	2.107		2.040	
$\Delta n(0)$	+0.009		+0.009	

^aReference 19 (other theoretical work).

^bReference 14 (experimental work).

accurate and reliable results. Therefore, we have used the mBJ to calculate the electronic band structures and hence the optical properties of Li₂CdGeS₄ and Li₂CdSnS₄. The imaginary part of the linear optical properties $\varepsilon_2^{xx}(\omega)$, $\varepsilon_2^{yy}(\omega)$, and $\varepsilon_2^{zz}(\omega)$ for the orthorhombic system (Li₂CdGeS₄ and Li₂CdSnS₄) is calculated and presented in Figs. 5(a) and 5(b). It has been noticed that substituting Ge by Sn leads to shift the whole spectral structure towards higher energies and lowers the magnitudes of the optical components. The spectral structure of Li₂CdGeS₄ exhibits three main peaks for $\varepsilon_2^{xx}(\omega)$ and $\varepsilon_2^{zz}(\omega)$ components with small humps, while $\varepsilon_2^{yy}(\omega)$ shows one broad peak with two small humps situated on the left shoulder. Whereas Li₂CdSnS₄ exhibits two main peaks for $\varepsilon_2^{xx}(\omega)$, $\varepsilon_2^{yy}(\omega)$, and $\varepsilon_2^{zz}(\omega)$ with several humps situated on the left and right shoulders. The absorption edges occur at 3.10 eV (Li₂CdGeS₄) and 3.23 eV (Li₂CdSnS₄), as shown in Figs. 5(a) and 5(b) and the supplementary material Fig. S1.³⁷ These absorption edges are originated from the optical transitions between S-3p and Ge-4s/4p (Sn-5s/5p) and Cd-5s/4p states. The optical transitions are calculated from the momentum matrix elements between the occupied and unoccupied bands, giving rise to the selection rules as shown in the supplementary material Fig. S2.³⁷ Furthermore, the calculated real part of the optical dielectric function can give information about the energy gap, since the calculated static electronic dielectric constant by $\varepsilon_\infty = \varepsilon_1(0)$ is inversely related to the energy gap, this could be explained on the basis of the Penn model.³⁸ As it is clear that the values of $\varepsilon_1^{xx}(0)$, $\varepsilon_1^{yy}(0)$, and $\varepsilon_1^{zz}(0)$ for Li₂CdGeS₄ are higher than those obtained from Li₂CdSnS₄, hence higher $\varepsilon_1(0)$ exhibits a lower energy gap. This supports our finding that substituting Ge by Sn leads to enlargement of the energy gap. It has been noticed that for the Li₂CdGeS₄ compound $\varepsilon_1^{yy}(0) \approx \varepsilon_1^{zz}(0)$; it represents the parallel component. While for the Li₂CdSnS₄ compound, $\varepsilon_1^{xx}(0) \approx \varepsilon_1^{yy}(0)$ represents the parallel component. The calculated values of $\varepsilon_1^{xx}(0)$, $\varepsilon_1^{yy}(0)$, and $\varepsilon_1^{zz}(0)$ along with the

calculated plasmon oscillations ω_p^{xx} , ω_p^{yy} , and ω_p^{zz} for both compounds are presented in Table I. The plasmon oscillations are associated with inter-band transitions that occur at energy where optical spectra of the real part cross zero. The optical components exhibit a considerable anisotropy; this is one of the important features of the optical spectra. The other important feature is the uniaxial anisotropy ($\delta\varepsilon$) which can be obtained from $\varepsilon_1^{xx}(0)$, $\varepsilon_1^{yy}(0)$, $\varepsilon_1^{zz}(0)$, and $\varepsilon_1^{average}(0)$. Following Figs. 5(a) and 5(b), we found that both compounds possess negative $\delta\varepsilon$, as shown in Table I.

The calculated refractive indices of Li₂CdGeS₄ and Li₂CdSnS₄ as derived from the complex dielectric function are presented in Figs. 5(c) and 5(d). It has been noticed that for Li₂CdGeS₄ compounds, $n^{yy}(0) \approx n^{zz}(0) = n_o(0)$ represents the parallel component ($n^{\parallel}(0)$), whereas $n^{xx}(0)$ is the perpendicular one ($n^{\perp}(0) = n_e(0)$). While Li₂CdSnS₄ compounds show that $n^{xx}(0) \approx n^{yy}(0)$ as $n_o(0)$ and $n^{zz}(0) = n_e(0) = n^{\perp}(0)$. Thus, the birefringence can be estimated from $\Delta n(\omega) = n_e(\omega) - n_o(\omega)$. It has been found that both compounds exhibit positive birefringence at the static limit, as shown in Table I. We found that the $\Delta n(0)$ for Li₂CdGeS₄ > Li₂CdSnS₄; hence, Li₂CdSnS₄ exhibits weaker anisotropy. The birefringence is necessary to fulfil the phase-matching condition. Therefore, we expected Li₂CdGeS₄ to produce higher second harmonic generation than Li₂CdSnS₄. The dispersion of the birefringence of the two compounds is illustrated in Fig. 5(e).

C. Complex second-order non-linear optical dispersion

To achieve accurate results for the calculated nonlinear optical properties, an accurate energy band gap is required, since the band gap value comes in the denominator of the complex nonlinear optical formulas which are used to calculate the complex nonlinear optical. These formulas are presented elsewhere.³⁹⁻⁴² The nonlinear optical properties are calculated using the NLO code, which is compatible with the WIEN2k package, see Refs. 39 and 40. Therefore, to achieve accurate results, we have applied the mBJ, which allows the calculation of the band gap with accuracy similar to the very expensive GW calculations.³⁰ Moreover, a quasi-particle self-energy correction at the level of scissors operators is applied to avoid the DFT drawback, since the calculated energy gap is still less than the measured one by around 0.05 eV for Li₂CdGeS₄ and 0.06 eV for Li₂CdSnS₄. In the scissors operators, the energy bands are rigidly shifted to merely bring the calculated energy gap to the exact experimental value. The orthorhombic symmetry allows seven non-zero tensor components $\chi_{113}^{(2)}(-2\omega; \omega; \omega) = \chi_{131}^{(2)}(-2\omega; \omega; \omega)$, $\chi_{223}^{(2)}(-2\omega; \omega; \omega) = \chi_{232}^{(2)}(-2\omega; \omega; \omega)$, $\chi_{311}^{(2)}(-2\omega; \omega; \omega)$, $\chi_{322}^{(2)}(-2\omega; \omega; \omega)$, and $\chi_{333}^{(2)}(-2\omega; \omega; \omega)$; two of them are equal, thus only five non-zero tensor components can be considered. One of those five tensors is considered to be the dominant one. Figs. 6(a) and 6(b) show the calculated absolute value $|\chi_{ijk}^{(2)}(-2\omega; \omega; \omega)|$ of those five tensors. It has been noticed that $|\chi_{333}^{(2)}(-2\omega; \omega; \omega)|$ is the dominant one, since it shows the highest value among the others, as shown in Tables II and III. Following these tables, we can see that Li₂CdGeS₄

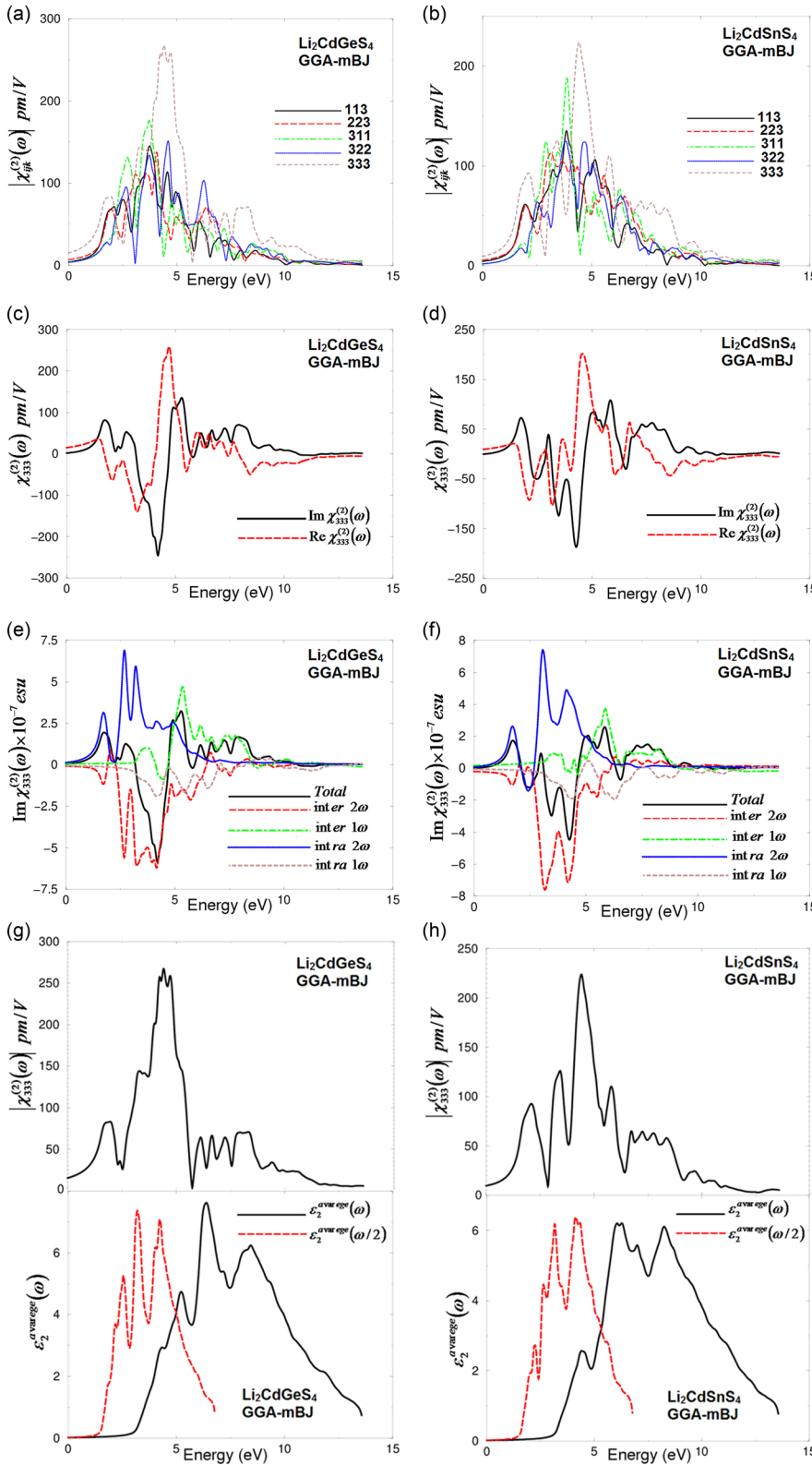


FIG. 6. (a) Calculated $|\chi_{ijk}^{(2)}(\omega)|$ for the five tensor components of $\text{Li}_2\text{CdGeS}_4$. (b) Calculated $|\chi_{ijk}^{(2)}(\omega)|$ for the five tensor components of $\text{Li}_2\text{CdSnS}_4$. (c) Calculated imaginary $\chi_{333}^{(2)}(\omega)$ (dark solid curve-black) and real $\chi_{333}^{(2)}(\omega)$ (light dashed curve-red) spectra of $\text{Li}_2\text{CdGeS}_4$. (d) Calculated imaginary $\chi_{333}^{(2)}(\omega)$ (dark solid curve-black) and real $\chi_{333}^{(2)}(\omega)$ (light dashed curve-red) spectra of $\text{Li}_2\text{CdSnS}_4$. (e) Calculated total $\text{Im } \chi_{333}^{(2)}(\omega)$ spectrum (dark solid curve-black) along with the intra $(2\omega)/(1\omega)$ (light solid curve-blue)/(light dashed dotted curve-cyan) and inter $(2\omega)/(1\omega)$ (light long dashed curve-red)/(light dotted curve-green)-band contributions of $\text{Li}_2\text{CdGeS}_4$, here all $\text{Im } \chi_{222}^{(2)}(\omega)$ are multiplied by 10^{-7} , in esu units. (f) Calculated total $\text{Im } \chi_{333}^{(2)}(\omega)$ spectrum (dark solid curve-black) along with the intra $(2\omega)/(1\omega)$ (light solid curve-blue)/(light dashed dotted curve-cyan) and inter $(2\omega)/(1\omega)$ (light long dashed curve-red)/(light dotted curve-green)-band contributions of $\text{Li}_2\text{CdSnS}_4$, here all $\text{Im } \chi_{222}^{(2)}(\omega)$ are multiplied by 10^{-7} , in esu units. (g) Upper panel—calculated $|\chi_{333}^{(2)}(\omega)|$ (dark solid curve-black); lower panel—calculated $\epsilon_2^{xy}(\omega)$ (dark solid curve-black) and calculated $\epsilon_2^{xy}(\omega/2)$ (dark dashed curve-red) of $\text{Li}_2\text{CdGeS}_4$. (h) Upper panel—calculated $|\chi_{333}^{(2)}(\omega)|$ (dark solid curve-black); lower panel—calculated $\epsilon_2^{xy}(\omega)$ (dark solid curve-black) and calculated $\epsilon_2^{xy}(\omega/2)$ (dark dashed curve-red) of $\text{Li}_2\text{CdSnS}_4$.

exhibits larger SHG than that of $\text{Li}_2\text{CdSnS}_4$ and very close to the experimental value of the well-known KTiOPO_4 (KTP) single crystals, which exhibits a SHG value of about 16.9,⁴³ 13.7,⁴⁴ 15.4 ± 0.2 ,⁴⁵ 14.6 ± 1.0 ,⁴⁶ 17.4 ± 1.7 ,⁴⁷ 16.9 ± 3.3 ,⁴⁸ 16.9 ± 1.7 ,⁴⁹ 10.6 ± 7.5 ,⁵⁰ 16.75,⁵¹ and 16.65.⁵¹

The real and imaginary parts of $\chi_{333}^{(2)}(-2\omega; \omega; \omega)$ are presented in Figs. 6(c) and 6(d) for $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$. These figures show that the $\text{Im } \chi_{333}^{(2)}(-2\omega; \omega; \omega)$ value rises at

a half value of the energy band gap due to 2ω terms oscillation, then at the exact value of the fundamental gap the ω terms start to oscillate to be added to 2ω terms. While in the higher energies, only ω terms will contribute. To analyze the spectral features of the $\text{Im } \chi_{333}^{(2)}(-2\omega; \omega; \omega)$, we have calculated the $2\omega/\omega$ inter-/intra-band contributions in order to understand the origin of the strong second harmonic generation of the dominant component. These are shown in Figs. 6(e) and

TABLE II. Calculated $|\chi_{ijk}^{(2)}(\omega)|$ and β_{ijk} of $\text{Li}_2\text{CdGeS}_4$, in pm/V at static limit and at $\lambda = 1064$ nm, in comparison with the experimental value of the well known KTiOPO_4 (KTP) single crystals, which exhibits a SHG value of about 16.9,⁴³ 13.7,⁴⁴ 15.4 ± 0.2 ,⁴⁵ 14.6 ± 1.0 ,⁴⁶ 17.4 ± 1.7 ,⁴⁷ 16.9 ± 3.3 ,⁴⁸ 16.9 ± 1.7 ,⁴⁹ 10.6 ± 7.5 ,⁵⁰ 16.75,⁵¹ and 16.65 (Ref. 51) at $\lambda = 1064$. Where 1 pm/V = 2.387×10^{-9} esu. The bold are the values of the dominate component.

$\text{Li}_2\text{CdGeS}_4$				
Tensor components	$\chi_{ijk}^{(2)}(0)$	Theory $d_{ijk} = 0.5 \chi_{ijk}^{(2)}(\omega)$	$\chi_{ijk}^{(2)}(\omega)$ at $\lambda = 1064$	Theory $d_{ijk} = 0.5 \chi_{ijk}^{(2)}(\omega)$
$ \chi_{113}^{(2)}(\omega) $	4.538	$d_{15} = 2.269$	15.409	$d_{15} = 7.704$
$ \chi_{223}^{(2)}(\omega) $	7.144	$d_{24} = 3.572$	17.527	$d_{24} = 8.763$
$ \chi_{311}^{(2)}(\omega) $	4.269	$d_{31} = 2.134$	12.583	$d_{31} = 6.291$
$ \chi_{322}^{(2)}(\omega) $	3.932	$d_{32} = 1.966$	11.879	$d_{32} = 5.939$
$ \chi_{333}^{(2)}(\omega) $	15.288	$d_{33} = 7.644$	37.297	$d_{33} = 18.648$
β_{333}	5.601×10^{-30} esu		13.015×10^{-30} esu	

TABLE III. Calculated $|\chi_{ijk}^{(2)}(\omega)|$ and β_{ijk} of $\text{Li}_2\text{CdSnS}_4$, in pm/V at static limit and at $\lambda = 1064$ nm, in comparison with the experimental value of the well known KTiOPO_4 (KTP) single crystals, which exhibits a SHG value of about 16.9,⁴³ 13.7,⁴⁴ 15.4 ± 0.2 ,⁴⁵ 14.6 ± 1.0 ,⁴⁶ 17.4 ± 1.7 ,⁴⁷ 16.9 ± 3.3 ,⁴⁸ 16.9 ± 1.7 ,⁴⁹ 10.6 ± 7.5 ,⁵⁰ 16.75,⁵¹ and 16.65 (Ref. 51) at $\lambda = 1064$. Where 1 pm/V = 2.387×10^{-9} esu. The bold are the values of the dominate component.

$\text{Li}_2\text{CdSnS}_4$				
Tensor components	$\chi_{ijk}^{(2)}(0)$	Theory $d_{ijk} = 0.5 \chi_{ijk}^{(2)}(\omega)$	$\chi_{ijk}^{(2)}(\omega)$ at $\lambda = 1064$	Theory $d_{ijk} = 0.5 \chi_{ijk}^{(2)}(\omega)$
$ \chi_{113}^{(2)}(\omega) $	4.442	$d_{15} = 2.221$	13.269	$d_{15} = 6.634$
$ \chi_{223}^{(2)}(\omega) $	5.700	$d_{24} = 2.850$	13.928	$d_{24} = 6.964$
$ \chi_{311}^{(2)}(\omega) $	1.918	$d_{31} = 0.959$	7.714	$d_{31} = 3.857$
$ \chi_{322}^{(2)}(\omega) $	1.933	$d_{32} = 0.966$	7.338	$d_{32} = 3.669$
$ \chi_{333}^{(2)}(\omega) $	9.569	$d_{33} = 4.784$	25.507	$d_{33} = 12.753$
β_{333}	3.737×10^{-30} esu		9.704×10^{-30} esu	

6(f). Furthermore, the spectral structure of the dominant component can be associated with the absorptive part of the corresponding dielectric function $\varepsilon_2(\omega)$ as a function of both $\omega/2$ and ω , to identify the origin of the spectral peaks as caused by $2\omega/\omega$ inter-/intra-band contributions, as shown in Figs. 6(g) and 6(h). The first structure in $|\chi_{333}^{(2)}(\omega)|$ between 1.57 and 3.15 eV (1.63 and 3.26 eV) for $\text{Li}_2\text{CdGeS}_4$ ($\text{Li}_2\text{CdSnS}_4$) is mainly originated from 2ω resonance [see $\varepsilon_2(\omega/2)$, Figs. 6(g) and 6(h)—lower panel]. The second structure between 3.15 and 5.0 eV (3.26 and 5.0 eV) is associated with interference between 2ω and ω resonances (the threshold of $\varepsilon_2(\omega)$) [see $\varepsilon_2(\omega/2)$ and $\varepsilon_2(\omega)$, Figs. 6(g) and 6(h)—lower panel]. The last spectral structure from 5.0 and above is mainly due to ω resonance and is associated with the second structure in $\varepsilon_2(\omega)$. With the aid of the existing values of the SHG for the dominant components of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$, we obtained the values of the microscopic first hyperpolarizability, β_{333} ,⁵⁰ the vector component along the dipole moment direction, at the static limit and at $\lambda = 1064$ nm. These values are listed in Tables II and III. In general, the microscopic first hyperpolarizability term, β_{ijk} , cumulatively yields a bulk observable second order susceptibility term, $\chi_{ijk}^{(2)}(\omega)$, which in turn is responsible for the high SHG response.^{22,23}

IV. CONCLUSIONS

The influence of replacing Ge by Sn on the electronic band structures and the linear and nonlinear optical susceptibilities were investigated by means of DFT. The all-electron FP-LAPW + lo method was used. Using PBE-GGA, the experimental geometries of $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ were

optimized by minimizing the forces acting on the atoms. The resulting geometries were used to calculate the electronic band structure, the density of states, the electronic charge density, and the linear and nonlinear optical susceptibilities using the PBE-GGA and the mBJ. It has been found that the mBJ succeeds by a large amount in bringing the calculated energy gap closer to the experimental one. The electronic band structure shows the direct band nature of the investigated compounds, and the PDOS reveals the roles of the orbitals. The valence electronic charge density exhibits a clear visualization to understand the origin of bonding characters. The considerable anisotropy in the linear optical properties, the presence of polarizable M-S bonding, and the wide energy gap causes in $\text{Li}_2\text{CdGeS}_4$ and $\text{Li}_2\text{CdSnS}_4$ compounds to exhibit strong SHG and exceptional laser damage thresholds. It has been found that $\text{Li}_2\text{CdGeS}_4$ exhibits higher SHG than that of $\text{Li}_2\text{CdSeS}_4$. The resulting SHG is comparable to that of the well-known KTP single crystals.

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- ³⁷See supplementary material at <http://dx.doi.org/10.1063/1.4943100> for Fig. S1: (a) The calculated electronic band structure of Li₂CdGeS₄ compound in the energy region between the Fermi level and 4.0 eV in comparison with the measured absorption edges taken from Ref. 14, we have used special software to extract the data from Figure 4 of Ref. 14, then we re-plot the absorption vs electronic band structure. (b) The calculated electronic band structure of Li₂CdSnS₄ compound in the energy region between the Fermi level and 4.0 eV in comparison with the measured absorption edges taken from Ref. 14, we have used special software to extract the data from Figure 4 of Ref. 14, then we re-plot the absorption vs electronic band structure. Fig. S2: (a) The optical transitions depicted on a generic band structure of Li₂CdGeS₄ compound. For simplicity, we have labeled the optical transitions as A, B, and C. The transitions (A) are responsible for the structures for $\epsilon_2^{xy}(\omega)$, $\epsilon_2^{yy}(\omega)$, and $\epsilon_2^{zz}(\omega)$ in the spectral range 0.0–5.0 eV; the transitions (B) 5.0–10.0 eV, and the transitions (C) 10.0–14.0 eV. (b) The optical transitions depicted on a generic band structure of Li₂CdSnS₄ compound. For simplicity, we have labeled the optical transitions as A, B, and C. The transitions (A) are responsible for the structures for $\epsilon_2^{xy}(\omega)$, $\epsilon_2^{yy}(\omega)$, and $\epsilon_2^{zz}(\omega)$ in the spectral range 0.0–5.0 eV; the transitions (B) 5.0–10.0 eV, and the transitions (C) 10.0–14.0 eV.
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