

1. Surface Drainage Systems

1.1 Introduction

Surface drainage is the process of removing excess water from the surface of the land to prevent ponding, erosion, and damage to crops, structures, and transportation systems. It is an essential component of agricultural, urban, and transportation infrastructure, ensuring that precipitation and irrigation water are efficiently conveyed away from areas where they may cause harm. A properly designed surface drainage system promotes soil stability, maintains trafficability, enhances crop productivity, and reduces the risk of flooding and waterlogging.

The fundamental objective of surface drainage is to collect and dispose of runoff through an interconnected network of surface features such as land grading, field drains, ditches, and main collectors so that water moves rapidly and safely to designated outlets. In agricultural settings, surface drainage helps maintain an optimal soil moisture regime for crop growth, particularly in fine-textured soils with low infiltration rates. In urban environments, it supports flood control, prevents structural damage, and safeguards public health and safety.

The design of surface drainage systems requires a comprehensive understanding of hydrologic processes, topography, soil properties, and rainfall characteristics. Engineers must integrate these factors to determine the required drainage capacity, layout, and geometric features that achieve both hydraulic efficiency and economic feasibility. The system's performance also depends on proper maintenance and adaptation to local environmental conditions.

This chapter provides a comprehensive overview of surface drainage systems, beginning with the need and objectives of surface drainage and its role in preventing waterlogging and enhancing soil productivity. It then classifies the main types of surface drains field drains and collector drains and explains their layout and functional interrelationships within a complete drainage network. The chapter also presents the fundamental principles of channel design, emphasizing hydraulic considerations and practical design procedures.

1.2 Need and Objectives of Surface Drainage

The primary objective of surface drainage is to remove excess surface water from agricultural fields, construction sites, and urban areas in order to maintain favorable soil and environmental conditions. Surface drainage systems are designed to intercept, collect, and safely convey runoff generated from rainfall or irrigation before it can accumulate on the land surface. Excess water, if not removed promptly, can lead to reduced soil aeration, delayed field operations, and long-term degradation of soil structure and fertility.

In agricultural settings, efficient surface drainage ensures that the soil remains in a condition suitable for timely planting, cultivation, and harvesting. In addition, it helps to prevent erosion, protect infrastructure, and sustain crop yields by maintaining optimal moisture levels near the surface. In engineering applications, such as roadways, airports, and residential developments, surface drainage is essential for protecting pavements, foundations, and other structures from damage caused by ponding or prolonged saturation.

The objectives of surface drainage can therefore be summarized as follows:

- To prevent or reduce waterlogging in low-lying areas.
- To maintain or improve soil productivity by ensuring proper soil aeration and workability.
- To minimize erosion and protect soil and structural integrity.
- To promote safe and efficient removal of runoff from agricultural and urban catchments.

1.2.1 Prevention of Waterlogging

Waterlogging occurs when the soil becomes saturated with water, filling its pores and excluding air. This condition severely restricts root respiration and nutrient uptake, resulting in stunted plant growth and yield reduction. Prolonged waterlogging can also lead to the accumulation of harmful gases such as carbon dioxide and methane in the root zone, further impairing plant development.

Surface drainage systems help prevent waterlogging by facilitating the rapid removal of excess surface water immediately after rainfall or irrigation. By maintaining the water table below the root zone and minimizing the duration of surface ponding, these systems preserve soil aeration and promote healthy root conditions. Effective drainage is particularly important in heavy clay soils and flat topographic areas where infiltration is slow and natural runoff is limited.

1.2.2 Improvement of Soil Productivity

Proper surface drainage contributes directly to improved soil productivity by maintaining an optimal balance between soil moisture and aeration. Excessive water at or near the surface reduces soil strength, delays field operations, and can destroy soil structure through compaction and puddling. Conversely, well-drained soils promote microbial activity, organic matter decomposition, and nutrient availability all of which are essential for sustaining high crop yields.

Additionally, surface drainage enhances the efficiency of other land management practices such as fertilization, tillage, and irrigation. When the soil remains well-drained and friable, root systems can penetrate deeper, resulting in better water and nutrient uptake. Over time, these benefits lead to increased agricultural productivity, reduced soil degradation, and improved environmental sustainability.

1.3 Types of Surface Drains

The efficiency of a surface drainage system depends on the correct selection, design, and integration of its components. The system generally consists of field drains, collector drains, and a well-planned layout that ensures functional connectivity between them and the natural or artificial outlets.

1.3.1 Field Drains

Field drains represent the primary component of surface drainage systems, functioning as the first line of defense against excess surface water accumulation. Their principal role is to intercept, collect, and convey rainfall or irrigation runoff from the field surface before it causes waterlogging, erosion, or crop damage. By maintaining a balance between infiltration and surface flow, field drains protect the soil structure, promote aeration in the root zone, and ensure optimum moisture conditions for plant growth.

Field drains are generally constructed as shallow open ditches or furrows, following the natural slope of the terrain. Their geometry usually trapezoidal or parabolic is determined by land slope, soil texture, and expected discharge. Typical depths range between 0.3 and 0.6 meters, and longitudinal slopes are kept mild (0.05–0.2%) to sustain self-cleansing velocities while avoiding erosion. On agricultural lands, drains are often spaced systematically across the field, oriented along the natural or graded slope to facilitate uniform removal of excess water.

Depending on permanence, field drains may be:

- Temporary drains, reshaped each season for short-term water removal in annual cropping systems.
- Permanent drains, stabilized by vegetation or shallow lining, integrated into long-term farm management plans.

In low-gradient areas, grassed waterways (Fig. 1.1) or broad shallow channels are often preferred to open ditches, as they reduce erosion, enhance infiltration, and allow agricultural machinery to pass without obstruction.

In many poorly drained soils particularly clayey or silty profiles surface drains alone are insufficient to maintain suitable field moisture conditions. To enhance drainage efficiency, they are often combined with subsurface (tile) drainage systems using perforated pipes buried below the soil surface (Fig. 1.2).

These perforated plastic pipes (commonly made from PVC or HDPE) collect infiltrated water from the root zone and convey it to collector drains or sumps. The pipes are slotted or perforated along their length and are often enclosed in filter envelopes made of gravel or synthetic geotextile to prevent soil ingress and clogging. Installation depths typically range from 0.8 to 1.5 meters, with lateral spacing determined by soil permeability closer in fine-textured soils and wider in coarse soils.



Fig. 1.1 Grassed waterways

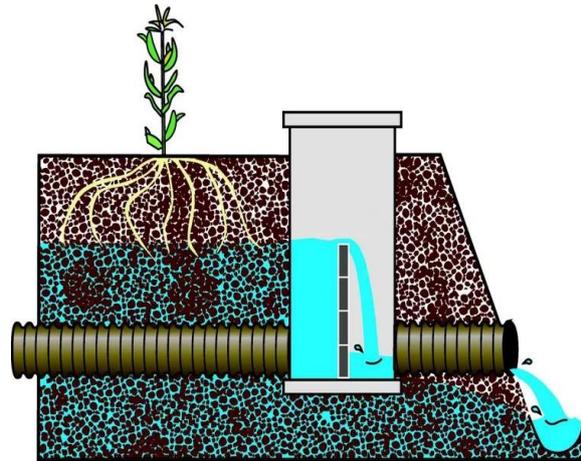


Fig. 1.2 Subsurface drainage using perforated pipes)

Field drains and subsurface laterals are hydraulically interconnected through surface inlets or riser pipes, enabling ponded surface water to enter the underground system when surface conveyance becomes limited. This integrated drainage approach improves the reliability of water removal, reduces salinity risks, and extends the productive season in high-water-table regions.

Sustained performance of field drains depends heavily on maintenance. Open channels must be periodically desilted, reshaped, and kept free of vegetation or obstructions, while perforated pipes require routine flushing and inspection through access pits to prevent sediment or biological clogging. Proper maintenance ensures that the hydraulic capacity of the drainage network remains intact and that both surface and subsurface components continue to function as an integrated system.

1.3.2 Collector Drains

Collector drains form the secondary network in a surface drainage system, functioning as the main conduits that receive discharge from multiple field drains and transport it efficiently to the main or outfall drain. Their role is pivotal in ensuring that the water intercepted at the field level is safely and continuously conveyed toward a natural watercourse, reservoir, or pump discharge point.

Collector drains are typically deeper and wider than field drains, designed to carry a larger cumulative discharge. They are usually aligned along natural depressions or low-lying areas, ensuring gravity flow from the upper fields. The cross-section of collector drains is determined through hydraulic design principles commonly using Manning's equation to provide adequate capacity for peak design flows while

maintaining permissible flow velocities (typically 0.3–1.0 m/s, depending on soil stability).

To prevent scouring or sidewall collapse, collector drains are stabilized using vegetative lining, riprap, or concrete depending on local soil and flow conditions. Where they intersect farm roads or pathways, culverts and cross-drainage structures are installed to maintain accessibility without impeding flow. In irrigated regions, collector drains may also include control gates or check structures to regulate flow, retain moisture when desired, or divert water to storage ponds.

In modern agricultural drainage systems, collector drains often work in combination with buried subsurface collector pipes, which receive discharge from several subsurface laterals (perforated field drains). These buried collector lines are larger in diameter (100–250 mm) and are laid at depths sufficient to maintain gravity flow to the outlet. The connection between surface and subsurface collectors is achieved through inspection pits or junction boxes, which allow monitoring, flushing, and maintenance access. This hybrid configuration ensures effective removal of both surface runoff and excess groundwater, offering a more reliable drainage solution in areas with variable rainfall or irrigation practices.

Within the overall drainage hierarchy, collector drains act as the intermediate link between the numerous field drains and the main outfall system (Fig. 1.3). Their hydraulic performance determines the efficiency of the entire network: if collector drains are undersized or poorly graded, upstream field drains will quickly lose function due to backwater effects. Thus, careful coordination between field drain layout, collector alignment, and outfall design is essential to achieve a self-sustaining drainage system that minimizes flooding, erosion, and maintenance costs.

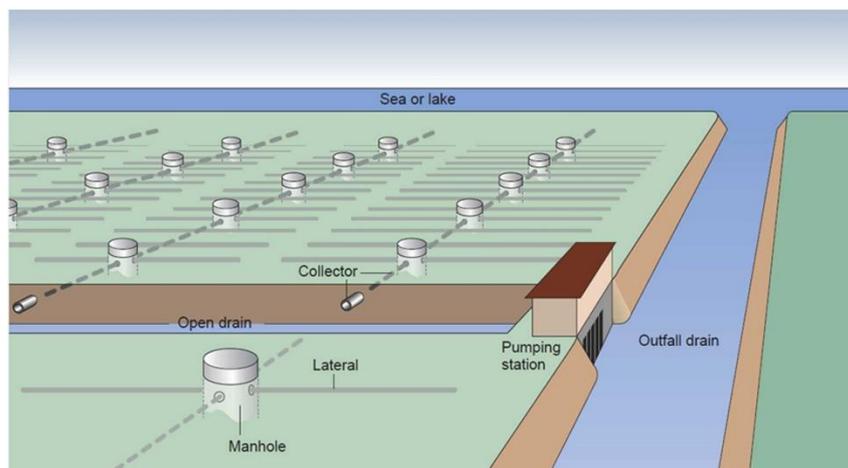


Fig. 1.3 Integrated surface and subsurface drainage system layout

Fig. 1.3 illustrates a typical integrated surface and subsurface drainage system commonly used in irrigated agricultural areas, especially in low-lying or coastal regions. The system combines both open drains and buried perforated pipes to ensure the efficient removal of excess surface water and groundwater. The laterals, which are small-diameter perforated pipes made of PVC or HDPE, are installed below the soil

surface at regular spacing and gentle slopes to collect percolated water from the root zone. These laterals discharge into collector drains, which are larger subsurface pipes designed to convey the accumulated flow toward the open drains. At junctions along the system, manholes or inspection wells are provided to facilitate maintenance, flushing, and performance monitoring.

The open drain shown in the figure functions as the main surface channel that receives water from both surface runoff and the subsurface collectors, safely transporting it toward the outlet. In regions where the land lies below the outlet level, a pumping station is required to lift the collected water into the outfall drain, which finally discharges it into a natural body such as a river, lake, or sea. This integrated arrangement of laterals, collectors, open drains, and pumping facilities ensures effective control of both the water table and surface ponding, preventing waterlogging and soil salinization. Overall, the system represents a coordinated approach to drainage design, maintaining favorable soil moisture conditions, protecting crops, and sustaining agricultural productivity in irrigated landscapes.

1.3.3 Layout and Functional Relationships

The overall performance of a drainage system depends not only on the proper design of its individual components but also on their layout and functional interrelationships. The layout defines how the field drains, collector drains, and outfall drains are spatially arranged and hydraulically connected to ensure efficient and uniform water removal from the entire area. A well-designed layout facilitates the natural movement of water by gravity, minimizes construction and maintenance costs, and maintains compatibility with agricultural practices and land use.

In most drainage projects, the layout is developed based on topographic, soil, and hydrologic characteristics of the area. The first step involves a topographic survey to identify natural slopes, depressions, and potential flow paths. This is followed by hydrologic analysis to estimate runoff volumes, infiltration rates, and the volume of water to be removed per unit area per day. These parameters form the basis for deciding the spacing, depth, and alignment of field drains and their hydraulic connection to collector and main drains.

There are generally three common types of drainage layouts used in agricultural lands:

1. Random Layout:

Employed in irregular or gently undulating areas where water accumulates in natural depressions. Drains are aligned along these low-lying zones to intercept runoff efficiently. This type is often used for improving existing fields with uneven topography.

2. Parallel Layout:

Adopted in flat or uniformly sloping lands, where drains are placed parallel to each other and at regular spacing. It is most suitable for mechanized farming since it provides systematic drainage coverage and minimal interference with agricultural operations.

3. Composite Layout:

A combination of random and parallel arrangements, used in areas with complex topography. It provides flexibility by following natural depressions while maintaining systematic alignment in flatter portions of the field.

1.4 Channel Design

1.4.1 Introduction

The design of open drainage channels represents a critical stage in the development of any surface drainage system. Channels serve as the primary conduits for conveying excess water collected from fields and collector drains toward the main outlet or natural watercourse. An efficient design ensures that this water is safely removed without causing erosion of the channel bed or banks, and without leading to sediment deposition that may obstruct flow or reduce capacity.

The principal objectives of channel design are therefore to achieve adequate hydraulic capacity, structural stability, and self-cleansing ability. To meet these objectives, the designer must determine the channel dimensions, shape, and slope that will convey the design discharge economically and safely under existing soil and flow conditions. The design must also ensure that the velocity of flow remains within permissible limits high enough to prevent silting but low enough to avoid erosion of the channel surface.

Several interrelated factors influence the performance and stability of a drainage channel. These include:

- Design discharge and flow regime, which determine the hydraulic requirements of the section.
- Soil type and erodibility, affecting permissible velocity and side slope stability.
- Sediment load and particle size, which influence whether the channel will deposit or transport sediments.
- Channel roughness and lining material, which govern the resistance to flow.
- Longitudinal slope and energy gradient, which control the overall flow velocity and sediment transport capacity.

Based on the interaction between flow and channel boundary material, two broad categories of drainage channels are recognized:

1. Rigid boundary (clear-water) channels, where the bed and sides are stable and do not deform under flow. These include lined channels or those excavated in cohesive soils.
2. Alluvial (mobile boundary) channels, where the bed and banks consist of non-cohesive materials such as sand or silt that can be eroded or deposited by flowing water.

Each category requires a distinct design approach. For rigid boundary channels, the problem is primarily hydraulic concerned with flow capacity and permissible velocity. For alluvial channels, the problem extends to sediment transport and the establishment of regime or stable conditions, in which the channel maintains its form over time. Thus, the study of channel design in drainage engineering involves not only the application of hydraulic equations but also an understanding of sediment behavior, soil mechanics, and long-term stability. The following sections discuss the principles and methods applicable to the design of clear-water and sediment-laden channels in greater detail.

1.4.2 Channel Carrying Clear Water

Channels carrying clear water are those in which the transported flow contains little or no sediment load, and the boundaries whether natural or artificial are sufficiently resistant to erosion. These channels are typically excavated in cohesive soils or constructed with lining materials such as concrete, brick, or riprap, which form rigid boundaries. Because the boundary does not significantly deform under the flow action, the design of such channels is primarily governed by hydraulic capacity and stability against erosion.

The objective of the design is to determine a cross-section, slope, and lining type that can safely convey the design discharge without exceeding the permissible tractive force or velocity for the boundary material. Two complementary approaches are commonly employed in practice:

- (a) Hydraulic design using Manning's equation, and
- (b) Stability check using tractive force method.

(a) Hydraulic Design Using Manning's Equation

The hydraulic characteristics of uniform flow in open channels are expressed by Manning's equation:

$$Q = \frac{1}{n} AR^{2/3} S^{1/2} \quad (1.1)$$

Where

- Q : Discharge (m³/s),
- n : Manning's roughness coefficient (Table 1.1),
- A : Flow area (m²),

R : Hydraulic radius (m), and
 S : Bed slope.

Table 1.1 Manning’s roughness coefficient (n) for different soil and channel surface types

| Surface Type | Description / Condition | Manning’s n |
|--|--|---------------|
| Smooth Earth | Uniform, well-shaped channel | 0.020 – 0.025 |
| Fine Gravel / Sandy Bed | Slightly irregular surface | 0.022 – 0.030 |
| Coarse Gravel | Moderately rough, non-uniform bed | 0.025 – 0.035 |
| Firm Clay | Smooth, compacted surface | 0.018 – 0.024 |
| Unlined Channel (average soil) | Irregular section, minor vegetation | 0.025 – 0.035 |
| Natural Channel (clean, straight) | Little vegetation, regular cross-section | 0.025 – 0.033 |
| Natural Channel (weedy / irregular) | Moderate vegetation, uneven bed | 0.035 – 0.050 |
| Lined Channel (concrete, smooth) | Finished surface, well-constructed | 0.012 – 0.016 |
| Lined Channel (rough concrete / masonry) | Slightly irregular joints, rough texture | 0.015 – 0.022 |
| Vegetated Earth Channel | Grass or light vegetation cover | 0.030 – 0.050 |
| Dense Vegetation Channel | Heavy grass or weeds, uneven flow area | 0.050 – 0.100 |

This equation provides the relationship between discharge, slope, and channel geometry for a given roughness condition. The design process involves assuming a suitable cross-sectional shape most commonly trapezoidal and determining its dimensions (bottom width b , depth y , and side slope z) such that the required discharge is conveyed at the given slope.

The value of the roughness coefficient n depends on the channel surface material and flow conditions. For example, smooth concrete linings have $n = 0.013 – 0.015$, while unlined earth channels may range from $n = 0.020$ to 0.035 .

After computing the flow depth and velocity, the resulting mean velocity $V = Q/A$ must be compared with permissible velocity limits for the channel material to ensure stability and prevent erosion. If the computed velocity exceeds the safe value, adjustments can be made by flattening the slope, enlarging the section, or introducing lining.

A variety of channel shapes and sizes can be employed in drainage systems, including triangular, rectangular, trapezoidal, and circular sections. Among these, the trapezoidal cross-section is the most widely used due to its adaptability to different flow conditions and construction methods. It is suitable for all types of channels whether lined or unlined, and whether the surface is earthen, partially lined, or fully waterproofed. Rectangular and circular sections are also commonly adopted,

particularly for lined or concrete channels, where construction precision and structural stability are required. The geometric parameters corresponding to various channel shapes are summarized in Table 1.2.

(b) Stability Check Using Tractive Force Method

Although the velocity approach is simple and practical, it does not always provide a complete picture of boundary stability especially where erosion is controlled more directly by the shear stress exerted by flowing water. The tractive force (or shear stress) method is therefore used as a complementary check.

The tractive force acting on the channel bed is given by:

$$\tau_0 = \gamma RS \tag{1.2}$$

Where

τ_0 : Boundary shear stress (N/m²),

γ : Unit weight of water (≈ 9810 N/m³),

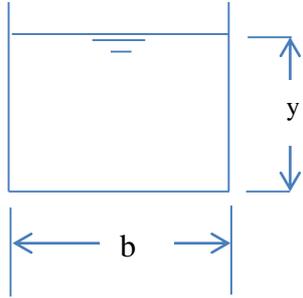
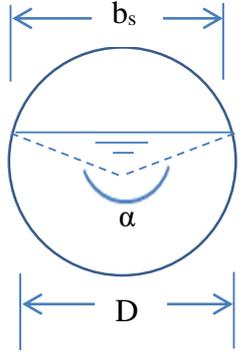
R : Hydraulic radius (m), the hydraulic radius is defined as the ratio of the cross-sectional area of flow (A) to the wetted perimeter (P). It is expressed as:

$$R = \frac{A}{P}$$

S : Bed slope.

Table 1.2 Geometric variables for various channel shapes

| Shape | Section | Flow Area, A | Wetted Perimeter, P | Hydraulic Radius, R |
|-------------|---------|----------------|-----------------------|--|
| Trapezoidal | | $y(b + ycota)$ | $b + \frac{2y}{sina}$ | $\frac{y(b + ycota)}{b + \frac{2y}{sina}}$ |
| Triangular | | $y^2 cota$ | $\frac{2y}{sina}$ | $\frac{ycosa}{2}$ |

| Shape | Section | Flow Area, A | Wetted Perimeter, P | Hydraulic Radius, R |
|-------------|--|---------------------------------------|-----------------------|--|
| Rectangular |  | by | $b + 2y$ | $\frac{by}{b + 2y}$ |
| Circular |  | $(\alpha - \sin\alpha) \frac{D^2}{8}$ | $\frac{\alpha D}{2}$ | $\frac{D}{4} \left(1 - \frac{\sin\alpha}{\alpha}\right)$ |

The shear stress exerted by flowing water is not distributed uniformly along the wetted perimeter of a channel. As illustrated in Fig. 1.4, the maximum shear stress occurs at different locations on the bed and the side slopes of the section. The peak value acting on the side slopes, denoted as τ_{sm} , is generally lower than that acting on the bed, τ_{bm} . For trapezoidal channels, experimental and analytical investigations have shown that the maximum tractive shear on the sides is approximately 0.76 times that on the channel bed.

This non-uniform distribution of shear stress is an important consideration in the tractive force method of channel design. When evaluating channel stability, both bed and side stresses must be checked against the permissible tractive forces of the boundary material. Neglecting this variation may lead to side-slope erosion or bank failure, even when the bed remains stable. Therefore, accurate estimation of the shear stress distribution is essential for ensuring a hydraulically efficient and structurally stable channel section. This distribution determines the zones of maximum erosion potential and guides the design of protective measures such as lining or vegetation. It also influences sediment transport patterns and the long-term evolution of the channel cross-section. By accounting for the non-uniformity of shear stress, designers can optimize channel geometry to minimize maintenance and enhance flow efficiency.

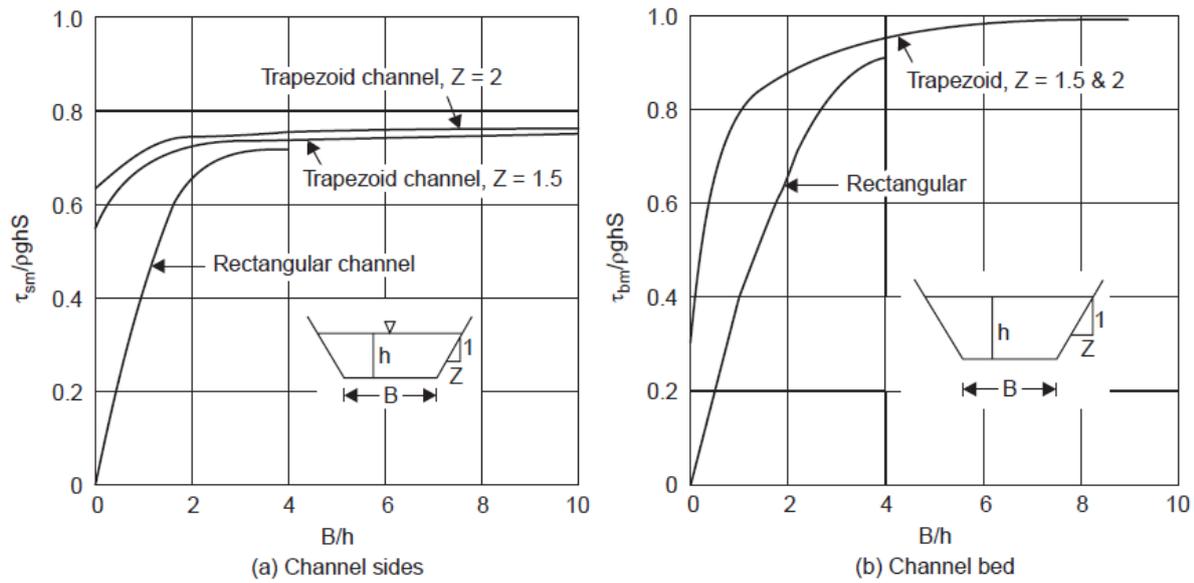


Fig. 1.4 Maximum shear stress on (a) sides and (b) bed of smooth channels in uniform flow

For a particle resting on a level or gently sloping bed, the condition for incipient motion that is the point at which the particle just begins to move, can be expressed by the following relationship:

$$\tau_{bl}a = W_s \tan\theta \quad (1.3)$$

Where

τ_{bl} : Limiting (or critical) bed shear stress. This represents the maximum shear stress that the bed material can resist before particle motion begins. It defines the threshold condition for incipient movement of sediment particles resting on the bed. When the applied bed shear stress equals or exceeds τ_{bl} , erosion or particle entrainment is initiated, a is the effective area of the particle.

W_s : Submerged weight of the particle per unit area of bed. This is the effective weight of the sediment particle acting normal to the bed, after accounting for buoyant forces. It is given by

$$W_s = (\gamma_s - \gamma)V \quad (1.4)$$

Where

γ_s is the unit weight of the solid particle, γ is the unit weight of water, and V is the volume of the particle (often represented per unit bed area in simplified form).

θ : Angle of repose (or angle of internal friction) of the bed material. This is the maximum angle that the surface of the granular material can maintain without sliding

or slumping. It reflects the frictional resistance between particles and typically ranges from 25° to 40° depending on particle size, shape, and packing.

For a particle resting on the sloping side of a channel (Fig. 1.5), the condition for incipient motion, when the particle is on the verge of movement under the action of flow-induced forces, can be expressed as follows:

$$(W_s \sin \alpha)^2 + (\tau_{sl} a)^2 = (W_s \cos \alpha \tan \theta)^2$$

$$\tau_{sl} = \frac{W_s}{a} \cos \alpha \cdot \tan \theta \sqrt{1 - \frac{\tan^2 \alpha}{\tan^2 \theta}} \quad (1.5)$$

By combining Eqs. (1.3) and (1.5), the ratio of the limiting shear on the side to that on the bed is expressed as:

$$k = \frac{\tau_{sl}}{\tau_{bl}} = \cos \alpha \sqrt{1 - \frac{\tan^2 \alpha}{\tan^2 \theta}} \quad (1.6)$$

Simplifying

$$k = \sqrt{1 - \frac{\sin^2 \alpha}{\sin^2 \theta}} \quad (1.7)$$

To ensure that neither the bed nor the side slopes are subjected to erosive forces, the following conditions must be satisfied:

$$\tau_{bm} \leq \tau_{bl}$$

$$\tau_{sm} \leq \tau_{sl}$$

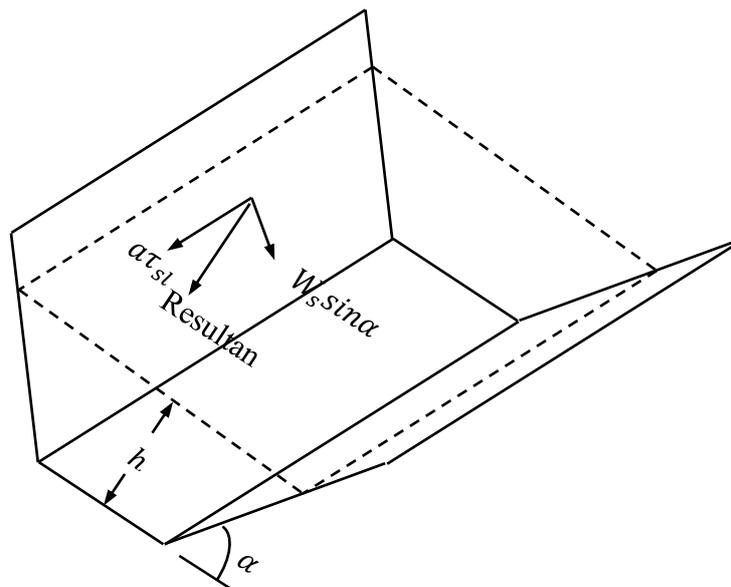


Fig. 1.5 Forces causing movement of a particle resting on a channel bank

It is evident that, for a bank to be stable, the angle of the bank α , must be less than of the angle of repose θ , or in other words, for stability reasons, $\theta > \alpha$.

The critical shear stress τ_c can be directly computed using the following empirical equation derived from the Shields curve:

$$\frac{\tau_c}{\Delta\rho_s g \left(\frac{\rho v^2}{\Delta\rho_s g}\right)^{1/3}} = 0.243 + \frac{0.06d_*^2}{(3600+d_*^2)^{1/2}} \quad (1.8)$$

Where

τ_c : Critical shear stress at the threshold of particle motion (the minimum bed shear stress required to initiate sediment movement) (N/m²),

ρ : Density of the fluid (usually water) (kg/m³),

ρ_s : Density of sediment particles (kg/m³),

$\Delta\rho_s$: Submerged specific density difference, defined as ($\rho_s - \rho$). It represents the effective density contrast driving particle settling (kg/m³),

g : Acceleration due to gravity (m/s²),

ν : Kinematic viscosity of the fluid (m²/s),

d_* : Dimensionless particle parameter (or dimensionless grain size), defined as

$$d_* = \frac{d}{\left(\frac{\rho v^2}{\Delta\rho_s g}\right)^{1/3}}$$

For the specific case of water at 20°C and sediment with a specific gravity of 2.65, Eq. (1.9) may be simplified to express the critical shear stress τ_c directly in terms of the particle diameter d as follows:

$$\tau_c = 0.155 + \frac{0.409d^2}{(1+0.177d^2)^{1/2}} \quad (1.9)$$

Where τ_c is given in N/m² and d is expressed in millimeters.

Yalin and Karahan (1979) developed a similar relationship between the dimensionless critical shear stress τ_c^* and the R_c^* , based on an extensive compilation of experimental data (Fig. 1.6). Their results show that for high values of R_c^* (i.e., $R_c^* > 70$), the constant value of τ_c^* approaches 0.045, which is slightly different from the conventional Shields curve.

The Yalin–Karahan relationship is generally considered to provide a better overall fit to modern experimental observations, especially at high Reynolds numbers, compared with the original Shields formulation.

Dimensionless critical shear stress (Shields parameter)

$$\tau_c^* = \frac{\tau_c}{(\rho_s - \rho)gd} \quad (1.10)$$

Particle Reynolds number

$$R_c^* = \frac{u_{*c} \cdot d}{\nu} = \sqrt{\frac{\tau_c \cdot d}{\rho \cdot \nu}} \quad (1.11)$$

Where

u_{*c} : Critical shear velocity (m/s)

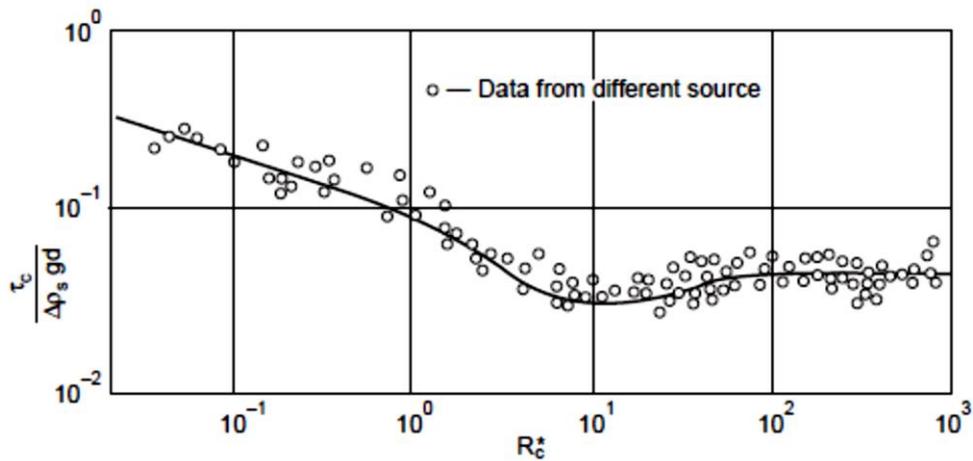


Fig. 1.6 Relationship between dimensionless critical shear stress (τ_c) and particle Reynolds number (R_c^*). The solid curve represents the empirical relationship, while the open circles denote experimental data from various sources.

The relationship between R_0^* , as illustrated in Fig. 1.7, can be employed to directly determine the value of the critical shear stress τ_c^* for specified particle diameter d , sediment density ρ_s , fluid density ρ , and kinematic viscosity ν .

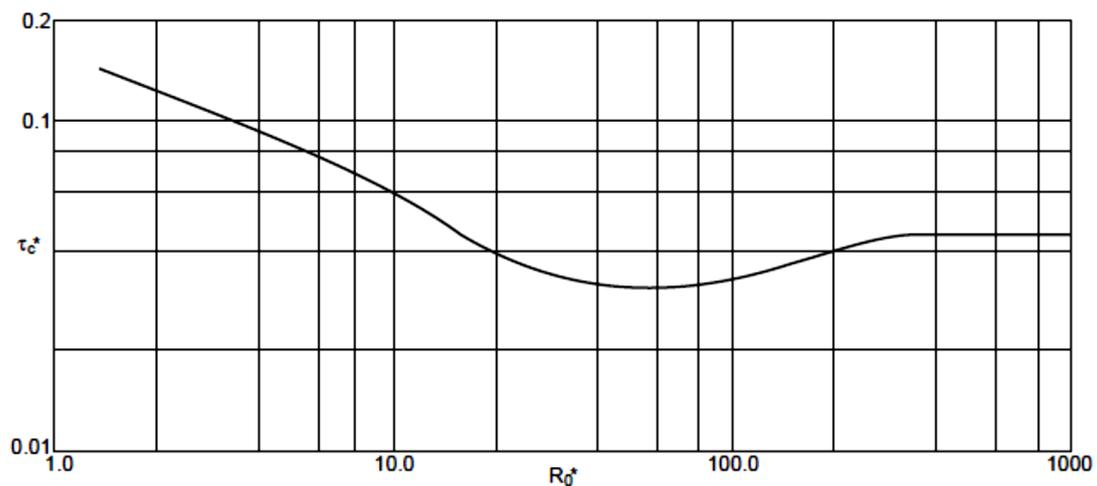


Fig. 1.7 Relationship between critical shear stress (τ_c^*) and (R_0^*)

$$R_0^* = \left(\frac{\Delta\rho_s g d^3}{\rho v^2} \right)^{1/2} \quad (1.12)$$

Example 1.1

Water flows at a depth of 0.50 m in a wide stream with bed slope $S = 5 \times 10^{-4}$. The median sand size on the bed is $d = 0.60$ mm. Determine whether the grains are stationary or moving. Use water at 20 °C with $\nu = 1.0 \times 10^{-6}$ m²/s, $\rho = 1000$ kg/m³, $\rho_s = 2650$ kg/m³, and $g = 9.81$ m/s².

Solution

1. Bed shear stress (wide channel: $R \approx y$),

For a wide channel, the flow width is much greater than the depth ($b \gg y$), so the wetted perimeter is approximately equal to the channel width. Under this condition, the hydraulic radius

$$R = \frac{A}{P} \approx \frac{by}{b} \approx y$$

Hence, the hydraulic radius and the flow depth are nearly identical.

$$\tau_0 = \gamma R S = \rho g y S = 1000(9.81)(0.5)(5 \times 10^{-4}) = 2.45 \text{ N/m}^2$$

2. Critical shear stress from the simplified Shields-based fit (Eq. 1.9)

For water at 20 °C and $S = 2.65$, with d in mm:

$$\tau_c = 0.155 + \frac{0.409d^2}{(1+0.177d^2)^{1/2}} = 0.155 + \frac{0.409(0.6)^2}{(1+0.177 \times 0.6^2)^{1/2}} = 0.298 \text{ N/m}^2$$

3. Compare τ_0 vs τ_c

Since $\tau_0 \gg \tau_c$, the grains are in motion.

(Optional checks)

Compute the dimensionless grain size d_*

$$\ell_v = \left(\frac{\rho v^2}{\Delta\rho_s g} \right)^{1/3} = \left(\frac{1000 (10^{-6})^2}{1650 \times 9.81} \right)^{1/3} = 3.953 \times 10^{-5} \text{ m}$$

$$d_* = \frac{d}{\left(\frac{\rho v^2}{\Delta\rho_s g} \right)^{1/3}} = \frac{6 \times 10^{-4}}{3.953 \times 10^{-5}} = 15.178$$

Use Eq. (1.8) to get τ_c

$$\frac{\tau_c}{\Delta\rho_s g \left(\frac{\rho v^2}{\Delta\rho_s g} \right)^{1/3}} = 0.243 + \frac{0.06d_*^2}{(3600+d_*^2)^{1/2}}$$

$$\text{RHS: } 0.243 + \frac{0.06(15.178)^2}{(3600+(15.178)^2)^{1/2}} = 0.4663$$

Hence

$$\tau_c = 0.4663(1650)(9.81)(3.953 \times 10^{-5}) = \frac{0.298 \text{ N}}{\text{m}^2}$$

$$\text{Comparison: } \tau_0 = 2.45 \gg \tau_c = 0.298 \text{ N/m}^2$$

Verdict: Bed grains are moving (initiated into motion)

Example 1.2

Water flows at a depth of 0.50 m in a wide channel having a bed slope of $S = 5 \times 10^{-4}$. The temperature of the water is 20°C, giving a kinematic viscosity of $\nu = 1.0 \times 10^{-6}$ m²/s. The sediment has a specific gravity of 2.65 ($\rho_s = 2650$ kg/m³) and the fluid density is $\rho = 1000$ kg/m³. Determine the largest particle diameter (in mm) that will not be initiated into motion under these flow conditions.

Solution

1. Bed shear stress (τ_0):

$$\tau_0 = \rho g y S = 1000 \times 9.81 \times 0.50 \times 5 \times 10^{-4} = 2.4525 \text{ N/m}^2$$

2. Viscosity–density length scale (ℓ_v):

$$\ell_v = \left(\frac{\rho \nu^2}{\Delta \rho_s g} \right)^{1/3} = \left(\frac{1000 (10^{-6})^2}{1650 \times 9.81} \right)^{1/3} = 3.9532 \times 10^{-5} \text{ m}$$

3. Substitute into Eq. (1.8):

$$\frac{\tau_c}{\Delta \rho_s g \left(\frac{\rho \nu^2}{\Delta \rho_s g} \right)^{1/3}} = 0.243 + \frac{0.06 d_*^2}{(3600 + d_*^2)^{1/2}}$$

At the threshold of motion (incipient condition), these two stresses are equal:

$$\tau_c = \tau_0$$

$$\tau_0 / (\Delta \rho_s g \ell_v) = 0.243 + (0.06 d_*^2) / (3600 + d_*^2)^{1/2}$$

$$\text{Left-hand side} = 2.4525 / (1650 \times 9.81 \times 3.9532 \times 10^{-5}) = 3.8327$$

Solve for $d_* \rightarrow d_* \approx 76.16$

4. Convert to actual particle diameter:

$$d_* = \frac{d}{\left(\frac{\rho \nu^2}{\Delta \rho_s g} \right)^{1/3}}$$

$$d = d_* \ell_v = 76.16 \times 3.9532 \times 10^{-5} = 3.01 \times 10^{-3} \text{ m} = 3.01 \text{ mm}$$

Therefore, the particle diameter that will not be initiated into motion is approximately 3.0 mm. Grains smaller than 3.0 mm will move, while grains of 3.0 mm or larger remain stationary.

Example 1.3

Design a trapezoidal channel (side slopes 2H:1V) to carry 1.5 m³/s of clear water with a bed slope of 2×10^{-4} . The bed and banks consist of coarse sand with a mean particle size of 2.0 mm and an angle of repose of 33°. The kinematic viscosity of water may be taken as $\nu = 1.0 \times 10^{-6}$ m²/s. Check the stability of both the bed and the side slopes using the tractive force method.

Solution

1. From Eq. (1.12):

$$R_0^* = \left(\frac{\Delta \rho_s g d^3}{\rho \nu^2} \right)^{1/2} = \left(\frac{(1650)(9.81)(2.0 \times 10^{-3})^3}{1000(10^{-6})^2} \right)^{1/2} = 359.85$$

2. Determine the Critical Shear Stress

From Fig. 1.7, $\tau_c^* = 0.045$

$$\tau_c = 0.045(1650)(9.81)(2.0 \times 10^{-3}) = 1.46 \text{ N/m}^2$$

Taking allowable shear stress:

$$\tau_{bl} = 0.9\tau_c = 1.31N/m^2 = \tau_{bm}$$

3. Compute Side-Slope Shear Factor

4. For side slope $z = 2H:1V$:

$$\alpha = \tan^{-1} \frac{1}{2} = 26.6^\circ, \theta = 33^\circ$$

5. Using Eq. (4.6):

$$k = \cos \alpha \sqrt{1 - \frac{\tan^2 \alpha}{\tan^2 \theta}} = 0.894 \sqrt{1 - \frac{0.5^2}{0.649^2}} = 0.57$$

$$\tau_{sl} = k \cdot \tau_{bl} = 0.57 \times 1.31 = 0.747N/m^2 = \tau_{sm}$$

6. Assume $(B/h) = 10$

$$\text{From Fig. 1.4, } \frac{\tau_{sm}}{\rho ghS} = 0.78, h = \frac{0.747}{9810 \times 2 \times 10^{-4} \times 0.78} = 0.488m$$

$$\text{Also, from Fig. 1.4, } \frac{\tau_{bm}}{\rho ghS} = 0.99, h = \frac{1.31}{9810 \times 2 \times 10^{-4} \times 0.99} = 0.674m$$

Choosing the lesser of the two values of h, 0.488 m

$$B = 10h = 4.88m$$

7. Using Manning's equation

$$\text{Geometry: } A = Bh + zh^2 = 4.88(0.488) + 2(0.488)^2 = 2.86m^2$$

$$P = B + 2h\sqrt{1 + z^2} = 4.88 + 2(0.488)\sqrt{1 + 2^2} = 7.06m$$

$$R = \frac{A}{P} = \frac{2.86}{7.06} = 0.4m$$

$$Q = \frac{1}{n} AR^{2/3} S^{1/2}, n = \frac{d^{1/6}}{25.6} = \frac{(2 \times 10^{-3})^{1/6}}{25.6} = 0.0139$$

$$Q = \frac{1}{0.0139} (2.86)(0.4^{2/3})(2 \times 10^{-4})^{1/2} = 1.59 m^3/sec$$

When the computed discharge by Manning's equation is smaller than the required design discharge (1.5 m³/s), indicating that the selected cross section is hydraulically inadequate to convey the desired flow under the given slope and roughness conditions. To meet the design discharge safely, the channel dimensions should be increased typically by enlarging the bed width B, increasing the flow depth h, or adopting a steeper bed slope (if topography allows). However, any adjustment must still satisfy the tractive-force stability criteria to ensure that both the bed and banks remain resistant to erosion.

The computed discharge by Manning's equation is greater than the required design discharge (1.5 m³/s), which indicates that the selected channel section is hydraulically oversized for the intended flow. Although the section is hydraulically efficient and easily conveys the design discharge, the larger dimensions may not be economical in terms of excavation and lining costs. To improve efficiency, the channel size can be slightly reduced (by decreasing bed width or depth) while ensuring that the resulting shear stresses remain below the permissible tractive limits to maintain bed and bank stability.

1.4.3 Channel Carrying Sediment-Laden Flow

Channels that convey sediment-laden flow such as irrigation canals, drainage ditches, and natural streams must be designed to maintain a non-silting and non-scouring condition. This means that the flow velocity should be sufficient to transport the sediment load without depositing it on the bed, yet not so high as to erode the channel surface. Designing such channels requires balancing hydraulic efficiency, stability, and sediment transport dynamics.

1. Fundamental Considerations

Sediment-laden flow involves complex interactions between flow hydraulics, particle size, channel geometry, and bed material characteristics. Key parameters include:

- Sediment concentration (C)
- Median particle size (d_{50})
- Flow velocity (V)
- Depth of flow (y)
- Bed slope (S)

An ideal design ensures the “regime condition”, where the channel shape, slope, and hydraulic parameters remain stable without progressive silting or scouring.

2. Empirical Regime Theories

Several empirical methods have been developed based on extensive observations of stable alluvial channels. The most notable among them are the Kennedy, Lindley, and Lacey methods.

(a) Kennedy’s Theory (1895)

Kennedy’s work was based on observations of canals in the Upper Bari Doab system in India. He found that critical velocity (V_0) depends mainly on the flow depth (y) and the characteristics of suspended sediment.

$$V_0 = 0.55h^{0.64} \quad (1.12)$$

Where

h : Depth of flow (m)

The constant (0.55) varies with the sediment type and bed roughness. Kennedy introduced a critical velocity ratio (m) to modify the relation for different sediment sizes:

$$V_0 = 0.55mh^{0.64} \quad (1.13)$$

Typical values of m range from 0.8 to 1.2, depending on sediment coarseness. Kennedy’s theory does not explicitly include slope or sediment size, so it is best suited for coarse, uniform sand channels where empirical calibration is possible.

The design procedure derived from Kennedy's theory is an iterative process. For known values of discharge Q , Manning's roughness coefficient n , and channel slope S , an initial trial value of flow depth h is assumed. From this, the corresponding mean velocity V can be determined using Kennedy's velocity equation (Eq. 1.13).

Applying the continuity equation ($A.V$), the cross-sectional area A can be obtained, allowing the channel width B to be computed for the assumed depth h . With these values of B and h , the mean velocity can also be evaluated using Manning's equation (Eq. 1.1).

If the computed velocity from Manning's formula agrees with the critical velocity derived from Kennedy's equation, the assumed value of h is satisfactory, and the corresponding values of B and A define the stable channel dimensions. If the two velocities do not match, another value of h must be assumed, and the procedure repeated until satisfactory agreement is achieved.

Ranga Raju and Misri proposed a simplified method to eliminate the need for iterative calculations. Their approach is based on the final side slope of 1H:2V typically attained by an alluvial channel under equilibrium conditions.

During the construction phase, the side slopes of a newly excavated channel are generally flatter than the natural angle of repose of the sediment. However, after a period of flow, sediment deposition and adjustment cause the channel to develop steeper side slopes, eventually approaching the stable configuration of 1H:2V.

For a channel with this final cross-sectional shape (trapezoidal section), the following relationships apply:

$$A = Bh + 0.5h^2 = h^2(p + 0.5) \quad (1.14)$$

$$P = B + 2.236h = h(p + 2.236) \quad (1.15)$$

Where

$$p = \frac{B}{h}$$

The hydraulic radius is then given by:

$$R = \frac{h(p+0.5)}{p+2.236} \quad (1.16)$$

and the mean velocity of flow can be expressed as:

$$V = \frac{Q}{h^2(p+0.5)} \quad (1.17)$$

These expressions form the basis for simplified computation of the channel geometry and hydraulic parameters without trial and error, making the method practical for the preliminary design of stable alluvial and drainage channels.

By substituting the expressions for velocity V and hydraulic radius R into Manning's equation, the following relationship is obtained:

$$S = \frac{Q^2 n^2 (p+2.236)^{4/3}}{h^{16/3} (p+0.5)^{10/3}} \quad (1.18)$$

Similarly, substituting the expression for V from Kennedy's equation yields the following relationship for the flow depth:

$$h = \left[\frac{1.818Q}{(p+0.5)m} \right]^{0.378} \quad (1.19)$$

On substituting the value of h from Eq. (1.19) into Eq. (1.18), the final form of the combined relationship between flow, geometry, and slope can be expressed as:

$$\frac{SQ^{0.002}}{n^2m^2} = 0.299 \left(\frac{\frac{B}{h} + 2.236}{\frac{B}{h} + 0.5} \right)^{1.393} \quad (1.20)$$

The graphical representation of Eq. (1.20) is shown in Fig. 1.8, illustrating the variation of B/h with respect to the combined parameter $\frac{SQ^{0.002}}{n^2m^2}$

For a given discharge Q, Manning's roughness coefficient n, sediment parameter m, and a suitably assumed bed slope S, the following design procedure is recommended:

1. Compute the value of $\frac{SQ^{0.002}}{n^2m^2}$
2. From Fig. 1.8, determine the corresponding ratio B/h.
3. Check from Table 1.3 whether the obtained value of B/h is satisfactory for the given discharge and sediment conditions.
4. If the value of B/h requires adjustment, select another slope and repeat the computation.
5. Once an acceptable B/h ratio is obtained, calculate h using Eq. (1.19).
6. Finally, determine the channel width B from the relationship $B = p.h$.

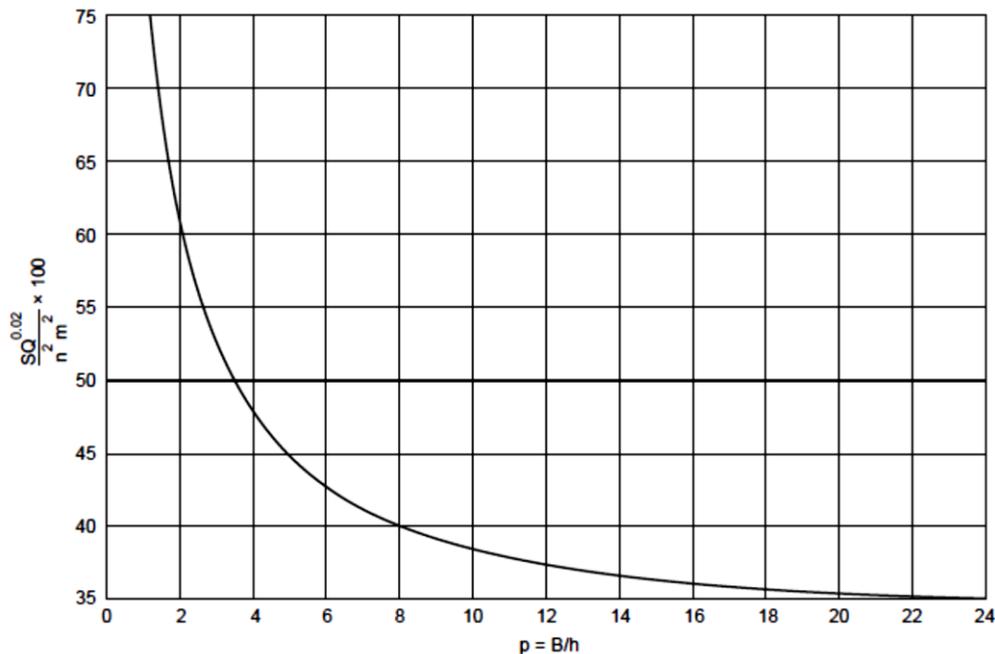


Fig. 1.8 Relationship between $\frac{SQ^{0.002}}{n^2m^2}$ and $p = \frac{B}{h}$ for regime channel design

Table 1.3 Recommended $\frac{B}{h}$ ratios for preliminary design

| | | | | | | | |
|-------------------------|-----|------|------|------|-------|-------|-------|
| (Q) (m ³ /s) | 5.0 | 10.0 | 15.0 | 50.0 | 100.0 | 200.0 | 300.0 |
| (B/h) | 4.5 | 5.0 | 6.0 | 9.0 | 12.0 | 15.0 | 18.0 |

Example 1.4

Design a stable trapezoidal channel to carry a discharge of 25 m³/s in an alluvial soil. The channel has a side slope of 1.5H: 1.0V, a bed slope of 1 in 6000, a critical velocity ratio (m) = 1.1, and Manning's $n = 0.020$. Determine the channel dimensions using Kennedy's method.

Solution

1. Assume a trial depth, $h = 1.8$ m

From Kennedy's equation: $V_0 = 0.55mh^{0.64} = 0.55(1.1)(1.8)^{0.64} = 0.88$ m/s

$$A = \frac{Q}{V} = \frac{25}{0.88} = 28.4 \text{ m}^2$$

2. Express A in terms of B

For a trapezoidal channel with side slope 1.5H: 1.0V:

$$A = (B + h.z)h = 1.8B + 1.5(1.8)^2 = 1.8B + 4.86 = 28.4 \Rightarrow B = 13.07 \text{ m}$$

3. Compute the hydraulic radius R

$$P = B + 2h\sqrt{1 + z^2} = 13.07 + 2(1.8)\sqrt{1 + 1.5^2} = 19.56 \text{ m}$$

$$R = \frac{A}{P} = \frac{28.4}{19.56} = 1.45 \text{ m}$$

4. Check velocity from Manning's equation

$$V = \frac{1}{n}R^{2/3}S^{1/2} = \frac{1}{0.02}(1.45)^{2/3}\left(\frac{1}{6000}\right)^{1/2} = 0.828 \text{ m/s}$$

Since $V_{\text{Kennedy}} = 0.88$ m/s and $V_{\text{Manning}} = 0.828$ m/s differ slightly, decrease h and repeat. Take $h = 1.7$ m,

$$V_0 = 0.849 \frac{\text{m}}{\text{s}}, A = 29.42 \text{ m}^2, B = 14.76 \text{ m}, R = 1.409 \text{ m}, V_{\text{Manning}} = 0.811 \text{ m/s}$$

The following table presents the results of successive trials carried out to determine the most suitable depth for a stable channel section. In this process, the flow depth (h) was varied several times, and for each trial the corresponding flow area, bottom width, wetted perimeter, hydraulic radius, and average velocity were recalculated.

As shown in the table, changing the flow depth affects all other geometric and hydraulic characteristics of the channel. When the assumed depth increases, the channel becomes narrower and the hydraulic radius generally increases, while smaller depths require a wider channel to convey the same discharge.

The main objective of this iterative procedure is to obtain a condition where the mean velocity obtained from the empirical stability relationship is nearly equal to the velocity determined from the hydraulic resistance relationship. From the table, this agreement occurs when the flow depth is approximately 1.54 to 1.63 m, indicating that at these depths the channel is hydraulically stable and capable of carrying the design discharge without silting or scouring.

| h | V ₀ | A | B | P | R | V(Manning) |
|------|----------------|----------|----------|----------|------------|------------|
| 1.8 | 0.881312 | 28.36679 | 13.05933 | 19.53933 | 1.45177924 | 0.827615 |
| 1.7 | 0.849655 | 29.4237 | 14.75806 | 20.87806 | 1.40931208 | 0.811395 |
| 1.65 | 0.833576 | 29.99127 | 15.70153 | 21.64153 | 1.38582043 | 0.802353 |
| 1.53 | 0.794251 | 31.47619 | 18.27767 | 23.78567 | 1.32332557 | 0.778046 |
| 1.54 | 0.79757 | 31.34522 | 18.04404 | 23.58804 | 1.32886081 | 0.780215 |

Example 1.5

Design a stable alluvial channel using Ranga Raju and Misri's method based on Kennedy's theory. The channel is required to carry a discharge of $Q = 50 \text{ m}^3/\text{s}$ with a bed slope of $S = 1/5000$. Assume Manning's roughness coefficient $n = 0.020$ and sediment parameter $m = 1.1$ (representing fine-to-medium sand).

Solution

$$\frac{SQ^{0.002}}{n^2m^2} = \frac{\frac{1}{5000}(50)^{0.002}}{0.02^2 \cdot 1.1^2} = 0.416$$

$$\text{From Fig. 1.8, } p = \frac{B}{h} = 7.8$$

$$h = \left[\frac{1.818Q}{(p+0.5)m} \right]^{0.378} = \left[\frac{1.818 \times 50}{(7.8+0.5) \times 1.1} \right]^{0.378} = 2.38 \text{ m}$$

$$\therefore B = 7.8 \times h = 18.56 \text{ m}$$

$$V = \frac{Q}{h^2(p+0.5)} = \frac{50}{2.38^2(7.8+0.5)} = 1.06 \text{ m/s}$$

This method eliminates the need for iterative trial-and-error calculations used in the original Kennedy approach. Instead, it combines the velocity–depth relationship from Kennedy's theory with Manning's equation and an assumed stable side slope of 1H:2V to give direct formulas for channel geometry.

(b) Lindley's Method (1919)

Lindley analyzed the observed characteristics of the Lower Chenab Canal system in India, assuming a Manning's roughness coefficient of $n = 0.025$ and side slopes of 0.5H:1V. Based on extensive field data, he derived empirical relationships expressing the mean velocity (V), flow depth (h), and channel width (B) as follows:

$$V_0 = 0.57h^{0.57} \quad (1.21)$$

$$V_0 = 0.27B^{0.355} \quad (1.22)$$

By combining Equations (1.21) and (1.22), Lindley obtained the following relationship between channel width and depth:

$$B = 7.86h^{1.606} \quad (1.23)$$

These equations describe the regime conditions for alluvial channels in equilibrium and are useful for the preliminary design of stable drainage or irrigation canals. However, it should be noted that the derived coefficients do not explicitly account for

sediment size or gradation, which can influence the stability and dimensions of the channel. Later researchers, such as Woods (1927), proposed equations of similar form to those of Lindley, with adjustments to improve their applicability to channels carrying different sediment types.

Example 1.6

A canal is to carry a discharge of 10 m³/s through an alluvial reach similar to those studied by Lindley on the Lower Chenab Canal. Assuming side slopes of 0.5H : 1V and Manning's n = 0.025, use Lindley's regime equations to determine:

1. The regime depth h, bed width B, and mean velocity V₀.
2. The hydraulic radius, wetted perimeter, and bed slope S required for uniform flow under these conditions.

Solution

Trapezoidal geometry: $A = h (B + z h)$, $P = B + 2h\sqrt{1 + z^2}$, $R = A / P$, and Manning's equation: $V = (1/n) R^{(2/3)} S^{(1/2)}$.

From Lindley's relations:

$$V_0 = 0.57 h^{0.57}$$

$$V_0 = 0.27 B^{0.355}$$

$$B = 7.86 h^{1.606}$$

Using continuity $Q = A V_0 = h (B + z h) (0.57 h^{0.57})$ with $z = 0.5$, solving numerically: $h = 1.266$ m, $B = 11.48$ m, $V_0 = 0.652$ m/s.

Check: $A = h (B + z h) = 15.34$ m², $Q = 15.34 \times 0.652 \approx 10.0$ m³/s ✓

Using Manning's equation: $S = (V n / R^{(2/3)})^2$

$$S = (0.652 \times 0.025 / 1.072^{(2/3)})^2 = 2.42 \times 10^{-4} \approx 0.000242 \text{ (0.242 m/km)}.$$

(c) Lacey's Method (1930)

Lacey developed his regime theory based on observations of numerous stable canals in the alluvial soils. Unlike Lindley's purely empirical relationships, Lacey's method was derived from both field data and the assumption of a dynamic equilibrium between the flow, sediment load, and channel form. In such a regime channel, the bed and banks are stable neither silting nor scouring occurs and the channel dimensions adjust naturally to the discharge and the properties of the transported sediment.

Lacey expressed the relationships between discharge, velocity, hydraulic radius, slope, and sediment size through the following regime equations:

$$P = 4.75\sqrt{Q} \tag{1.24}$$

$$R = 0.48(Q/f)^{1/3} \tag{1.25}$$

$$S = 0.0003 \frac{f^{5/3}}{Q^{1/6}} \tag{1.26}$$

$$V = 10.8R^{2/3}S^{1/3} \tag{1.27}$$

Where

V : Mean velocity (m/s)

R : Hydraulic radius (m)

Q : Discharge (m³/s)

P : Wetted perimeter (m)

S : Longitudinal bed slope

f : Silt factor, representing sediment size and properties, given by $f = 1.76\sqrt{d_{50}}$

d_{50} : Mean sediment particle diameter (mm).

Lacey's equations are widely used for the design of stable irrigation and drainage canals in alluvial soils. Once the design discharge and representative sediment size are known, the regime dimensions and slope can be computed directly. The method provides a rational, physics-based foundation for predicting canal geometry under natural equilibrium conditions.

Although earlier methods notably Kennedy (1895) and Lindley (1919) introduced the concept of regime flow, Lacey (1930) developed a more comprehensive and physically consistent framework that remains fundamental in canal design. Later researchers, such as Blench (1951) and Simons & Albertson (1960s), refined Lacey's regime concepts to account for sediment gradation and variable flow, but Lacey's equations continue to serve as a cornerstone in the theory and practice of stable channel design.

Example 1.7

A canal is to be designed to carry a discharge of 15 m³/s through an alluvial reach where the mean sediment size is 0.25 mm. The channel is to be unlined and self-formed under regime (stable) conditions. Using Lacey's regime equations, determine the regime depth (h), bed width (B), mean velocity (V), hydraulic radius (R), wetted perimeter (P), and the bed slope (S) required for stable uniform flow. Assume the canal has side slopes of 0.5H : 1V and a Manning's roughness coefficient $n = 0.025$ for verification.

Solution

$$f = 1.76\sqrt{d_{50}} = 1.76\sqrt{0.25} = 0.88$$

1. Wetted perimeter

$$P = 4.75\sqrt{Q} = 4.75\sqrt{15} = 18.397m$$

2. Hydraulic radius

$$R = 0.48(Q/f)^{1/3} = 0.48\left(\frac{15}{0.88}\right)^{1/3} = 1.235m$$

3. Bed slope

$$S = 0.0003\frac{f^{5/3}}{Q^{1/6}} = 0.0003\frac{0.88^{5/3}}{15^{1/6}} = 1.54 \times 10^{-4}$$

4. Mean velocity

$$V = 10.8R^{2/3}S^{1/3} = 10.8(1.235)^{2/3}(1.54 \times 10^{-4})^{1/3} = 0.667\text{m/s}$$

5. Flow area (from $Q = AV$)

$$A = \frac{Q}{V} = \frac{15}{0.667} = 22.5\text{m}^2$$

6. Check consistency

$$A \approx RP = 1.235 \times 18.397 = 22.72 \text{ m}^2$$

7. Trapezoidal geometry (with $z = 0.5$)

For a trapezoid:

$$P = B + 2h\sqrt{1 + z^2} \Rightarrow B = P - 2.236h$$

$$A = (B + h.z)h = (P - 2.236h + 0.5h)h = h(P - 1.736h)$$

Solve the quadratic, $h = 1.428\text{m}$, $B = 15.2 \text{ m}$

2. Subsurface Drainage

2.1 Introduction

Subsurface drainage is an essential component of modern agricultural and environmental engineering, designed to control and manage the movement of excess water within the soil profile. Unlike surface drainage systems, which remove standing water from the land surface, subsurface drainage focuses on regulating the water table and improving the internal aeration of the root zone through a network of buried conduits or permeable materials.

The primary objective of subsurface drainage is to maintain the water table at an optimum depth that ensures favorable conditions for plant growth, soil strength, and efficient land use. Excess groundwater can cause waterlogging, reduce soil oxygen, hinder root development, and decrease crop productivity. By allowing surplus water to percolate into subsurface drains typically made of perforated pipes, tiles, or gravel envelopes such systems promote natural leaching of salts, prevent salinity buildup, and enhance soil structure over time.

In agricultural contexts, subsurface drainage is particularly critical in irrigated lands, clayey soils with poor natural permeability, and areas with shallow water tables. Beyond agriculture, it also plays a vital role in urban development, transportation infrastructure, and environmental protection helping stabilize foundations, prevent seepage damage, and reduce soil erosion caused by groundwater emergence.

Designing an efficient subsurface drainage system requires an understanding of soil hydraulics, groundwater flow, and crop water requirements. Key factors include soil permeability, drainage depth, spacing, and outlet conditions, all of which influence the system's hydraulic performance and long-term sustainability. Analytical models and empirical equations such as Hooghoudt's and Ernst's formulas are commonly employed to estimate drain spacing and discharge rates based on field conditions.

In essence, subsurface drainage serves as a vital engineering intervention that harmonizes soil-water relationships, ensuring optimal productivity and environmental balance. It exemplifies the integration of hydrological science, soil physics, and agricultural management for the sustainable use of land and water resources.

2.2 Principles of Subsurface Drainage

The principles of subsurface drainage are based on understanding how water moves through the soil and how it can be effectively controlled to maintain desirable moisture and aeration conditions within the root zone. The system functions by intercepting excess groundwater and safely conveying it away from the field through buried conduits, thus preventing waterlogging and salinity problems.

1. Hydraulic Behavior of Water in Soil

Water movement in the soil occurs through interconnected pores under the influence of gravity and pressure differences. The rate and direction of this flow depend on the

soil's permeability and the existing hydraulic gradient. Coarse-textured soils, such as sandy loams, allow rapid movement of water, whereas fine-textured soils, such as clays, restrict movement and are more prone to waterlogging. Subsurface drainage utilizes these natural flow paths by providing an outlet for groundwater to move laterally toward the drains.

2. Control of the Water Table

One of the main objectives of subsurface drainage is to control the level of the water table so that it remains below the crop root zone. When the water table rises too close to the surface, the soil becomes saturated, air is displaced, and root respiration is restricted. By maintaining the water table at a safe depth, subsurface drainage ensures a balance between moisture availability and aeration, promoting healthy root development and optimum crop yields.

3. Flow Toward the Drains

When the water table rises after rainfall or irrigation, the excess water moves laterally through the soil toward the drains. The drains act as collectors that intercept this groundwater and carry it to a main collector or open outlet. The shape of the groundwater surface between adjacent drains depends on soil type, drain spacing, and recharge rate. In coarse soils, the water table tends to flatten quickly after drainage, while in clayey soils, it takes longer for equilibrium to be restored.

4. Influence of Soil Characteristics

Soil properties greatly influence the design and performance of subsurface drainage systems. The most important factors are the hydraulic conductivity, depth to the impermeable layer, and the degree of anisotropy meaning the variation in permeability in different directions. In many soils, water moves more readily horizontally than vertically, which must be considered when determining the proper depth and spacing of drains.

5. Boundary and Outlet Conditions

The operation of a subsurface drainage system also depends on boundary conditions such as the presence of an impermeable layer beneath the drains, the topography of the land, and the hydraulic conditions at the outlet. The outlet should be at a lower elevation than the drains to allow free discharge, or it may include a control structure to regulate outflow when needed. Efficient outlet design prevents backflow and ensures continuous removal of excess water.

6. Environmental and Agronomic Considerations

Beyond controlling water levels, subsurface drainage also helps manage soil salinity, particularly in irrigated regions where salts can accumulate due to evaporation. By allowing periodic leaching, the system removes excess salts from the root zone, preserving soil fertility and preventing degradation. Moreover, proper drainage design

contributes to environmental protection by reducing surface runoff, minimizing erosion, and improving the stability of agricultural lands and infrastructure.

2.3 Flow to Drains: Darcy's Law Application

The flow of water through soil toward subsurface drains is governed by the fundamental principles of soil permeability and hydraulic head distribution. The concept originates from Darcy's Law, which defines the relationship between the rate of flow through a porous medium and the energy gradient driving the flow. Understanding this law is essential for predicting groundwater movement, estimating drain discharge, and designing efficient subsurface drainage systems.

1. Flow Through Porous Soil

Soil is a heterogeneous mass consisting of solid mineral particles, organic matter, air, and water. The spaces between these particles known as voids are interconnected, forming continuous channels that permit water movement. The size, shape, and arrangement of these voids determine the soil's permeability, which is the measure of its ability to transmit water.

When a difference in total energy (hydraulic head) exists between two points within the soil, water naturally moves from the zone of higher energy to the zone of lower energy. This movement occurs due to the combined effects of elevation head and pressure head, while velocity head is negligible because flow velocities in soil pores are very small.

2. Development of Darcy's Law

The experimental work of Henry Darcy (1856) established a linear relationship between the discharge rate of water and the hydraulic gradient in a saturated soil.

Consider a soil sample of length L and cross-sectional area A , as illustrated in Fig. 2.1. Water enters the soil at point A and exits at point B, producing a measurable head loss Δh between the two ends.

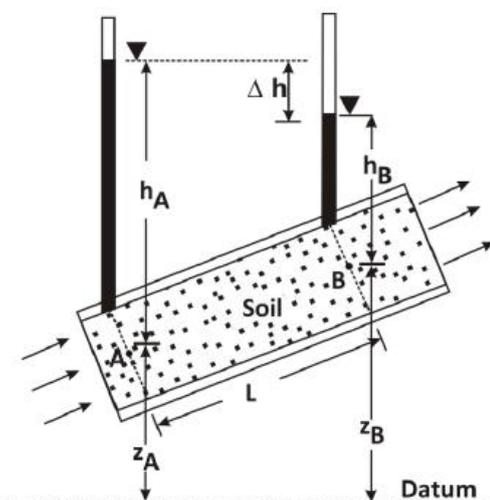


Fig. 2.1 Development of Darcy's law

The total head at any point in the soil is the sum of:

- Elevation head (z): the height of the point above a fixed reference or datum.
- Pressure head (h): the height of a water column producing the pressure at that point.

According to Bernoulli's theorem, the total heads at points A and B can be written as:

Total head at A = $z_A + h_A$, Total head at B = $z_B + h_B$

The head loss between the two points is therefore:

$$\Delta h = (z_A + h_A) - (z_B + h_B) \quad (2.1)$$

The hydraulic gradient, defined as the loss of head per unit length of flow, is given by:

$$i = \frac{\Delta h}{L} \quad (2.2)$$

Darcy demonstrated that the discharge velocity of water through the soil is proportional to this hydraulic gradient:

$$v = Ki \quad (2.3)$$

Where

v : Discharge velocity,

i : Hydraulic gradient, and

K : Coefficient of permeability (a property of the soil).

Multiplying by the cross-sectional area A gives the total rate of seepage through the soil:

$$q = KiA \quad (2.4)$$

3. Application to Flow Toward Subsurface Drains

When subsurface drains (such as perforated pipes or tile lines) are installed below the soil surface, groundwater flows through the saturated zone toward these drains under the influence of hydraulic head differences. The drains act as low-pressure boundaries that collect and remove the infiltrating water, thereby maintaining the desired groundwater level.

By applying Darcy's Law to the flow domain between drains, engineers can:

- Quantify the rate of groundwater flow reaching each drain.
- Determine drain spacing and depth to achieve the target water table.
- Evaluate system performance under various recharge and soil conditions.

These analyses are extended through analytical solutions such as **Hooghoudt's**, **Ernst's**, and **Glover–Dumm's equations**, which adapt Darcy's Law to layered soils, anisotropy, and unsteady flow situations.

4. Engineering Significance

The application of Darcy's Law to subsurface drainage is not merely theoretical, it provides a quantitative basis for practical design. It allows the prediction of:

- How quickly excess water will be removed from the root zone,
- How soil permeability influences drainage performance, and
- How different soil layers and drain depths affect flow patterns.

Understanding and correctly applying Darcy's Law ensures that drainage systems are both hydraulically efficient and economically feasible, contributing to long-term soil productivity, salinity control, and environmental protection.

Example 2.1

Consider an agricultural field where subsurface drains are installed at a spacing of 20 m and a depth of 1.5 m. During a period of high recharge, the water table rises to 0.5 m above the drain invert at the midpoint between two drains. The horizontal distance for water flow from the midpoint to the drain is therefore 10 m. The soil in the field has a saturated hydraulic conductivity of $K = 1.0$ m/day, and the average thickness of the saturated flow zone contributing to drainage is 1.2 m. Assuming steady, one-dimensional flow toward the drain and neglecting any entrance or exit losses, use Darcy's Law to calculate the discharge per meter length of drain.

Solution

1. Head loss

$$\Delta h = h_A - h_B = 0.5 - 0.0 = 0.5 \text{ m}$$

2. Hydraulic gradient

$$i = \frac{\Delta h}{L} = \frac{0.5}{10} = 0.05$$

3. Darcy discharge velocity

$$v = Ki = (1.0)(0.05) = 0.05 \text{ m/day}$$

4. Discharge per meter length of drain

$$Q = v.A = 0.05 \times 1.2 = 0.06 \text{ m}^3/\text{day}$$

2.4 Validity of Darcy's Law

Darcy's Law assumes laminar flow of water through the interconnected voids of a saturated soil. This condition implies that the velocity of flow is directly proportional to the hydraulic gradient. However, this relationship holds true only within a certain range of flow velocities and soil conditions. When the flow becomes turbulent, the linear relationship between velocity and gradient no longer applies, and Darcy's Law ceases to be valid.

The range within which Darcy's Law is valid can be examined using the Reynolds number, which represents the ratio of inertial to viscous forces within a moving fluid. For flow through soil pores, the Reynolds number is defined as:

$$R_n = \frac{vD_p\rho}{\mu} \quad (2.5)$$

Where

- v : Discharge (superficial) velocity (cm/s)
- D_p : Average diameter of soil particles (cm)
- ρ : Density of the fluid (g/cm³)
- μ : Dynamic viscosity of the fluid [g/(cm·s)]

Experimental studies have shown that Darcy's Law remains valid as long as the flow is laminar, that is, when: $R_n \leq 1$

For finer soils such as sands, silts, and clays, flow remains laminar and fully satisfies Darcy's relationship. However, as particle size increases (in coarse sands and gravels), flow velocity increases, and turbulent effects begin to appear. In such cases, the hydraulic gradient i is no longer linearly related to velocity v . Instead, the relationship can be expressed as:

$$i = av + bv^2 \quad (2.6)$$

Where a and b are empirical constants depending on soil texture and pore geometry (Forchheimer, 1902). The first term represents viscous (laminar) resistance, while the second term accounts for inertial (turbulent) effects.

Further investigations, summarized by Leps (1973), extended the analysis of flow through coarse gravel and fractured rocks. It was found that the average velocity of flow through such media can be expressed empirically as:

$$u_v = CR_H^{0.5}i^{0.54} \quad (2.7)$$

Where

- u_v : Average velocity of flow through voids
- C : A constant which is a function of shape and roughness of rock particles
- R_H : Hydraulic mean radius
- i : Hydraulic gradient

2.5 Steady-State Flow Equation

In the design and analysis of subsurface drainage systems, steady-state flow equations are essential for describing the long-term, equilibrium conditions of groundwater movement toward drains. Under steady-state conditions, the rate of recharge to the soil equals the rate of discharge through the drains, and the water table profile remains constant over time. Two of the most widely used analytical solutions for predicting the relationship between drain spacing, hydraulic conductivity, and water table depth are the Hooghoudt Equation and the Ernst Equation. These equations provide

practical means for estimating the required drain spacing to maintain desired water table levels, considering soil stratification, depth of impermeable layers, and drainage intensity.

2.5.1 The Hooghoudt Equation

Consider a condition of steady-state groundwater flow toward vertically-walled open drains that extend down to an impervious layer, as illustrated in Fig. 2.2. According to the Dupuit–Forchheimer theory, the groundwater flow in the horizontal direction can be described using Darcy’s Law. For a vertical plane at a horizontal distance x from the drain, the unit discharge per unit width of flow, q_x , is given by:

$$q_x = Ky \frac{dy}{dx} \quad (2.8)$$

Where

q_x : Unit discharge in the x-direction (m²/d)

K : Hydraulic conductivity of the soil (m/d)

y : Height of the water table above the impervious layer at distance x (m)

$\frac{dy}{dx}$: Hydraulic gradient at position x

The principle of continuity requires that the total volume of water entering the soil within the surface area between two adjacent drains must equal the volume of groundwater flowing laterally through any vertical plane at distance x . This condition ensures equilibrium between recharge and discharge under steady-state flow.

As shown in Fig. 2.2, water infiltrating the soil surface between drains percolates downward and moves laterally toward the drains under the influence of the hydraulic gradient. The flow path extends from the midpoint between drains (where $x = L/2$) toward the drain wall (where $x = 0$), through the saturated zone above the impervious layer.

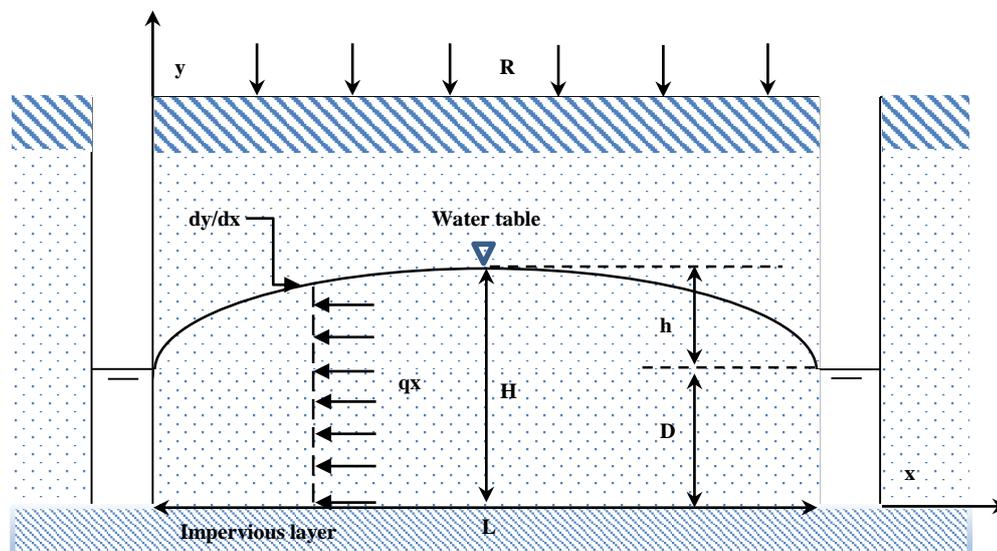


Fig. 2.2 Steady-state groundwater movement toward vertically-walled drains

As discussed earlier, all water infiltrating into the soil within the surface area between two adjacent drains must eventually flow laterally through a vertical plane located at a distance x from the drain. If R represents the rate of recharge (infiltration) per unit area of the soil surface, then the flow per unit width passing through this plane per unit time can be expressed as:

$$q_x = R \left(\frac{1}{2}L - x \right) \quad (2.9)$$

Where

R : Rate of recharge per unit surface area (m/d)

L : Drain spacing (m)

Under steady-state conditions, the flow described by Darcy's Law must equal the flow entering through the surface (Eq. 2.9). Thus, equating the two expressions gives:

$$Ky \frac{dy}{dx} = R \left(\frac{1}{2}L - x \right) \quad (2.10)$$

Rearranging:

$$Ky \cdot dy = R \left(\frac{1}{2}L - x \right) dx \quad (2.11)$$

To determine the relationship between the drain spacing, hydraulic conductivity, and water table elevations, we integrate this differential equation between the appropriate limits:

$$\text{For } x = 0 \rightarrow y = D, \text{ and for } x = \frac{1}{2}L \rightarrow y = H$$

Where

D : Elevation of the water level in the drain (m)

H : Elevation of the water table midway between the drains

Integrating and substituting the limits yields:

$$L^2 = \frac{4K(H^2 - D^2)}{R} \quad (2.12)$$

or equivalently, the discharge per unit width of drain is:

$$q = R = \frac{4K(H^2 - D^2)}{L^2} \quad (2.13)$$

Where

q : Drain discharge per unit width (m/d)

This relationship, originally derived by Hooghoudt (1936) and later referred to as the Donnan Equation (1946), provides the fundamental steady-state flow expression for subsurface drainage design in homogeneous soils.

Eq. (2.13) can also be expressed in an alternative but equivalent form:

$$q = \frac{4K(H+D)(H-D)}{L^2} \quad (2.14)$$

This equation forms the basis for determining the required spacing between drains to maintain a desired water table depth under steady recharge conditions.

From Fig. 2.2, it follows that $H-D = h$ and $H+D = 2D+h$, where h represents the height of the water table above the water level in the drain. Substituting these expressions into Eq. (2.14) gives:

$$q = \frac{8KDh+4Kh^2}{L^2} \quad (2.15)$$

If the water level within the drain is very low (i.e., $D \approx 0$), Equation (2.15) simplifies to:

$$q = \frac{4Kh^2}{L^2} \quad (2.16)$$

Conversely, if the impervious layer lies far below the drain level ($D \gg h$), the second term in the numerator of Equation (2.15) becomes negligible, yielding:

$$q = \frac{8KDh}{L^2} \quad (2.17)$$

The following sections discuss the principles governing flow in layered soil profiles, the development and determination of equivalent depth, and the derivation of the Hooghoudt Equation for drains installed above the impervious layer one of the most fundamental relationships in subsurface drainage engineering.

1. Flow in Layered Soil Profiles

In many practical cases, the soil profile consists of two distinct layers with different hydraulic conductivities. If the drain level coincides with the interface between these two layers, Equation (2.15) can be modified to account for the difference in permeability as follows:

$$q = \frac{8K_b Dh+4K_t h^2}{L^2} \quad (2.18)$$

Where

K_t : Hydraulic conductivity of the soil layer above the drain level (m/d)

K_b : Hydraulic conductivity of the soil layer below the drain level (m/d)

2. Equivalent Depth Concept for Drains Above the Impervious Layer

When the pipe or open drains do not extend down to the impervious layer, the flow lines of groundwater tend to converge toward the drains rather than remaining horizontal (Fig. 2.3A). As a result, the flow paths become longer, and additional head loss occurs due to the converging flow. To maintain the same rate of discharge into the drains, a higher water table is required, which leads to an increased hydraulic head above the drains.

To simplify this complex flow situation, Hooghoudt (1940) introduced the concept of equivalent depth, based on two primary assumptions (Fig. 2.3B):

1. An imaginary impervious layer is assumed to exist above the actual impervious layer. This reduces the effective thickness of the soil layer through which water flows toward the drains.
2. The actual drains are replaced by imaginary ditches whose bottoms lie on this imaginary impervious layer.

Under these assumptions, the same analytical approach used for horizontal flow can be applied, provided that the actual depth to the impervious layer (D) is replaced by a smaller equivalent depth (d). This equivalent depth represents the reduced soil thickness that would yield the same rate of flow toward the drains as in the real case, but without the head losses caused by flow convergence.

Accordingly, Equation (2.15) may be rewritten as:

$$q = \frac{8Kdh + 4Kh^2}{L^2} \quad (2.19)$$

where the term d accounts for the head loss associated with the radial (or converging) flow component near the drain.

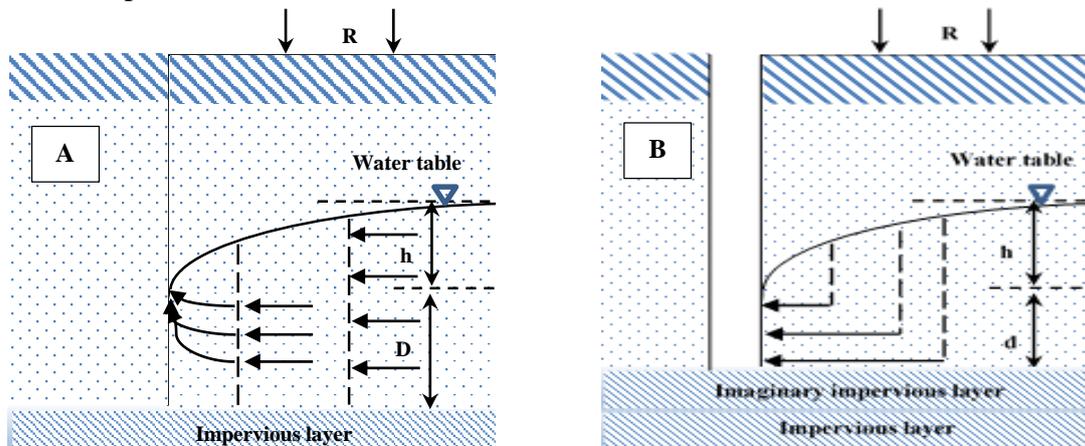


Fig. 2.3 The concept of equivalent depth (d) is used to convert a system that involves both horizontal and radial flow (A) into an equivalent system with purely horizontal flow (B).

3. Determination of the Equivalent Depth

The main challenge lies in determining the appropriate value of the equivalent depth d . Using the method of mirror images, Hooghoudt derived a relationship between d , the drain spacing L , the depth to the impervious layer D , and the radius of the drain r_0 . This relationship is expressed as an infinite series, which, although mathematically rigorous, is cumbersome to evaluate directly.

To facilitate practical design, Hooghoudt developed tables from which the equivalent depth d can be obtained for common combinations of drain radius and spacing. One such example is provided in Table 2.1, corresponding to $r_0 = 0.1$ m.

From this table, it can be observed that d increases with D until $D \approx 1/4L$. Beyond this point, further increases in the depth of the impervious layer have little effect on d , as the flow field becomes effectively independent of the actual depth.

Since drain spacing L itself depends on d , Equation (2.19) must often be solved iteratively. Manual use of Hooghoudt's tables can therefore be time-consuming. To address this, Van Beers (1979) developed a set of nomographs that allow rapid estimation of d without lengthy computation.

With the availability of modern computational tools, the equivalent depth can now be determined precisely through analytical or numerical solutions. Later researchers, such as Van der Molen and Wesseling (1991), extended Hooghoudt's work to produce exact solutions for d based on the mirror image method, improving both accuracy and ease of application in drainage design.

4. Derivation of the Equivalent Depth Formula

The derivation of the equivalent depth (d) is based on the method of mirror images, a mathematical approach used to model the radial component of groundwater flow toward a drain when the impervious layer lies below the drain level.

In the actual physical system, the flow lines near the drain are curved, consisting of both horizontal and vertical components. To simplify analysis, Hooghoudt assumed a series of imaginary drains (mirror images) placed symmetrically above and below the impervious layer. This arrangement ensures that the vertical components of flow are mirrored, producing a pattern equivalent to flow between two parallel, impervious planes.

The distance between the real drain and its first image is $2D$, where D is the depth from the drain center to the impervious layer. Using this configuration, Hooghoudt expressed the relationship between the equivalent depth (d), the drain spacing (L), the depth to the impervious layer (D), and the drain radius (r_0) as an infinite series:

$$d = \frac{\pi L}{8} \left[\frac{1}{\ln\left(\frac{L}{\pi r_0}\right) + F(x)} \right] \quad (2.20)$$

with $x = \frac{2\pi D}{L}$

and the series $F(x) = 2 \sum_{n=1}^{\infty} \ln[\coth(nx)]$

Table 2.1 Values for the equivalent depth of Hooghoudt for $r_0 = 0.1$ m, D and L in m

| L → | 5 m | 7.5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | L → | 50 | 75 | 80 | 85 | 90 | 100 | 150 | 200 | 250 | |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| D | | | | | | | | | | | | | | | | | | | | | | |
| 0.5 m | 0.47 | 0.48 | 0.49 | 0.49 | 0.49 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 |
| 0.75 | 0.60 | 0.65 | 0.69 | 0.71 | 0.73 | 0.74 | 0.75 | 0.75 | 0.75 | 0.76 | 0.76 | 0.76 | 0.76 | 0.96 | 0.97 | 0.97 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 | 0.99 |
| 1.00 | 0.67 | 0.75 | 0.80 | 0.86 | 0.89 | 0.91 | 0.93 | 0.94 | 0.96 | 0.96 | 0.96 | 0.96 | 2 | 1.72 | 1.80 | 1.82 | 1.83 | 1.85 | 1.00 | 1.92 | 1.94 | 1.94 |
| 1.25 | 0.70 | 0.82 | 0.89 | 1.00 | 1.05 | 1.09 | 1.12 | 1.13 | 1.14 | 1.14 | 1.15 | 3 | 2.29 | 2.49 | 2.52 | 2.54 | 2.56 | 2.60 | 2.72 | 2.70 | 2.83 | 2.83 |
| 1.50 | 0.70 | 0.88 | 0.97 | 1.11 | 1.19 | 1.25 | 1.28 | 1.31 | 1.34 | 1.35 | 1.36 | 4 | 2.71 | 3.04 | 3.08 | 3.12 | 3.16 | 3.24 | 3.46 | 3.58 | 3.66 | 3.66 |
| 1.75 | 0.70 | 0.91 | 1.02 | 1.20 | 1.30 | 1.39 | 1.45 | 1.49 | 1.52 | 1.55 | 1.57 | 5 | 3.02 | 3.49 | 3.55 | 3.61 | 3.67 | 3.78 | 4.12 | 4.31 | 4.43 | 4.43 |
| 2.00 | 0.70 | 0.91 | 1.08 | 1.28 | 1.41 | 1.5 | 1.57 | 1.62 | 1.66 | 1.70 | 1.72 | 6 | 3.23 | 3.85 | 3.93 | 4.00 | 4.08 | 4.23 | 4.70 | 4.97 | 5.15 | 5.15 |
| 2.25 | 0.70 | 0.91 | 1.13 | 1.34 | 1.50 | 1.69 | 1.69 | 1.76 | 1.81 | 1.84 | 1.86 | 7 | 3.43 | 4.14 | 4.23 | 4.33 | 4.42 | 4.62 | 5.22 | 5.57 | 5.81 | 5.81 |
| 2.50 | 0.70 | 0.91 | 1.13 | 1.38 | 1.57 | 1.69 | 1.79 | 1.87 | 1.94 | 1.99 | 2.02 | 8 | 3.56 | 4.38 | 4.49 | 4.61 | 4.72 | 4.95 | 5.68 | 6.13 | 6.43 | 6.43 |
| 2.75 | 0.70 | 0.91 | 1.13 | 1.42 | 1.63 | 1.76 | 1.88 | 1.98 | 2.05 | 2.12 | 2.18 | 9 | 3.66 | 4.57 | 4.70 | 4.82 | 4.95 | 5.23 | 6.09 | 6.63 | 7.00 | 7.00 |
| 3.00 | 0.70 | 0.91 | 1.13 | 1.45 | 1.67 | 1.83 | 1.97 | 2.08 | 2.16 | 2.23 | 2.29 | 10 | 3.74 | 4.74 | 4.89 | 5.04 | 5.18 | 5.47 | 6.45 | 7.09 | 7.53 | 7.53 |
| 3.25 | 0.70 | 0.91 | 1.13 | 1.48 | 1.71 | 1.88 | 2.04 | 2.16 | 2.26 | 2.35 | 2.42 | 12.5 | 3.74 | 5.02 | 5.20 | 5.38 | 5.56 | 5.92 | 7.20 | 8.06 | 8.68 | 8.68 |
| 3.50 | 0.70 | 0.91 | 1.13 | 1.50 | 1.75 | 1.93 | 2.11 | 2.24 | 2.35 | 2.45 | 2.54 | 15 | 3.74 | 5.20 | 5.40 | 5.60 | 5.80 | 6.25 | 7.77 | 8.84 | 9.64 | 9.64 |
| 3.75 | 0.70 | 0.91 | 1.13 | 1.52 | 1.78 | 1.97 | 2.17 | 2.31 | 2.44 | 2.54 | 2.64 | 17.5 | 3.74 | 5.30 | 5.53 | 5.76 | 5.99 | 6.44 | 8.20 | 9.47 | 10.4 | 10.4 |
| 4.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.81 | 2.02 | 2.22 | 2.37 | 2.51 | 2.62 | 2.71 | 20 | 3.74 | 5.30 | 5.62 | 5.87 | 6.12 | 6.60 | 8.54 | 9.97 | 11.1 | 11.1 |
| 4.50 | 0.70 | 0.91 | 1.13 | 1.52 | 1.85 | 2.08 | 2.31 | 2.50 | 2.63 | 2.76 | 2.87 | 25 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 8.99 | 10.7 | 12.1 | 12.1 |
| 5.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.15 | 2.38 | 2.58 | 2.75 | 2.89 | 3.02 | 30 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 9.27 | 11.3 | 12.9 | 12.9 |
| 5.50 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.20 | 2.43 | 2.65 | 2.84 | 3.00 | 3.15 | 35 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 9.44 | 11.6 | 13.4 | 13.4 |
| 6.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.20 | 2.48 | 2.70 | 2.92 | 3.09 | 3.26 | 40 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 9.44 | 11.8 | 13.8 | 13.8 |
| 7.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.20 | 2.54 | 2.81 | 3.03 | 3.24 | 3.43 | 45 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 9.44 | 12.0 | 13.8 | 13.8 |
| 8.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.20 | 2.57 | 2.85 | 3.13 | 3.35 | 3.56 | 50 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 9.44 | 12.1 | 14.3 | 14.3 |
| 9.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.20 | 2.57 | 2.89 | 3.18 | 3.43 | 3.66 | 60 | 3.74 | 5.30 | 5.74 | 5.96 | 6.20 | 6.79 | 9.44 | 12.1 | 14.6 | 14.6 |
| 10.00 | 0.70 | 0.91 | 1.13 | 1.52 | 1.88 | 2.20 | 2.57 | 2.89 | 3.23 | 3.48 | 3.74 | ∞ | 3.88 | 5.38 | 5.76 | 6.00 | 6.26 | 6.82 | 9.55 | 12.2 | 14.7 | 14.7 |
| ∞ | 0.71 | 0.93 | 1.14 | 1.53 | 1.89 | 2.24 | 2.58 | 2.91 | 3.24 | 3.56 | 3.88 | | | | | | | | | | | |

which is exactly equivalent to the rapidly convergent form

$$F(x) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4e^{-2nx}}{n(1-e^{-2nx})} \quad (2.21)$$

This form converges rapidly when $x > 1$, that is, when the depth to the impervious layer is relatively large compared to the drain spacing.

For small values of x (specifically $x \leq 0.5$), the convergence of the above series becomes slow. In this range, comparison with Dagan's formula yields an approximation for $F(x)$ that is highly accurate and computationally efficient:

$$F(x) = \frac{\pi^2}{4x} + \ln\left(\frac{x}{2\pi}\right) \quad (2.22)$$

This approximation provides reliable results for shallow impervious layers (small D/L) and is widely used in practical drainage design when the exact series evaluation is unnecessary.

These assumptions imply that the entrance perimeter u is equivalent to the wet perimeter of a semicircle (the term πr_0 in Equation 2.20). Accordingly,

$$r_0 = \frac{u}{\pi} \quad (2.23)$$

Where

r_0 : Radius of the drain (m)

u : Wet perimeter (m)

For open drains, the equivalent radius r_0 can be determined by substituting the wet perimeter of the open drain for u in Equation (2.23). For pipe drains installed in trenches, the wet perimeter is given by:

$$u = b + 2r_0 \quad (2.24)$$

where b is the width of the trench (m).

If an envelope material is placed around the pipe drain (Figure 2.4), Equation (2.24) is modified to:

$$u = b + 2(2r_0 + m) \quad (2.25)$$

Where

m : Height of the envelope above the drain (m).

5. Modern Computation of Equivalent Depth

With the advent of digital computation, the evaluation of d no longer requires tables or nomographs. The exact Hooghoudt equation can be easily solved using numerical methods. Modern drainage design software, such as DRAINMOD, and EnDrain,

routinely incorporate these formulations to compute d accurately for a wide range of soil and drain configurations.

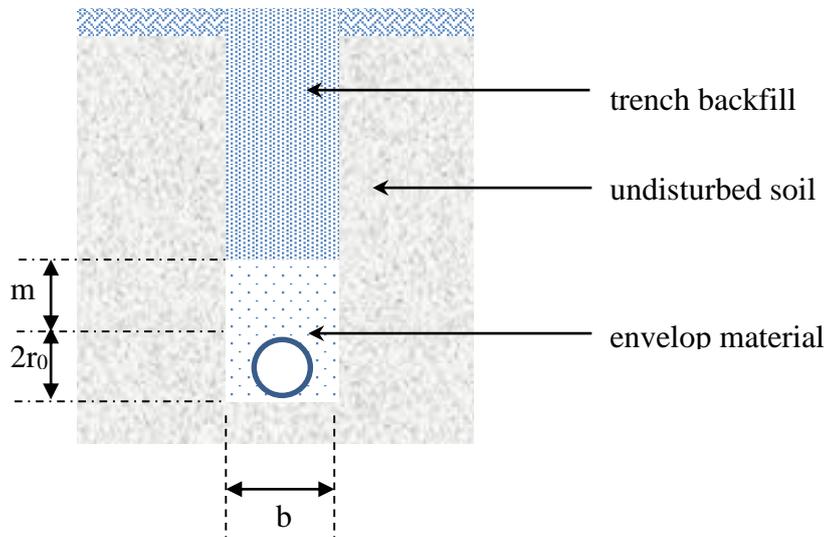


Fig. 2.4 Drain pipe with gravel envelop in drain trench

Example 2.2

A natural-grass soccer field is located in a sandy-loam area with poor natural drainage. To maintain playability after rainfall and prevent waterlogging of the turf, a subsurface drainage system is to be installed. The objective is to design the spacing between parallel pipe drains required to keep the water table approximately 1.5 m below the ground surface midway between the drains under steady recharge conditions.

The drain pipes are to be laid at a depth of 2.5 m and have a radius of $r_0 = 0.10$ m. The underlying soil has a hydraulic conductivity of $K = 1.0 \text{ m d}^{-1}$, and the design recharge rate is $q = 0.005 \text{ m d}^{-1}$. A low-permeability layer is encountered at a depth of 9.5 m, which represents the base of the flow region.

Using Hooghoudt's equation and the equivalent-depth concept, determine the required drain spacing, L , to satisfy the design conditions.

Solution

Using the Hooghoudt equation (for parallel drains above an impervious layer):

$$q = \frac{8Kdh + 4Kh^2}{L^2} \rightarrow L^2 = \frac{8Kdh + 4Kh^2}{q} = \frac{8(1.0)d(1.0) + 4(1.0)(1.0)^2}{0.005}$$

$$L^2 = 1600d + 800$$

Trial and Error Solution

Since d depends on L , we must find L by trial and error, using the Hooghoudt equivalent depth table for $r_0 = 0.1$ m.

First Estimate: $L = 90$ m, depth to impervious layer (D) = $9.5 - 2.5 = 7.0$ m (Fig. 2.5), from Table 2.1, $d = 4.42$ m $\Rightarrow L = 88.7$ m.

Second Estimate: $L = 88$ m, interpolate for $d = 4.386$ m $\Rightarrow L = 88.4$ m.

Hence, the correct drain spacing is approximately 88 m.

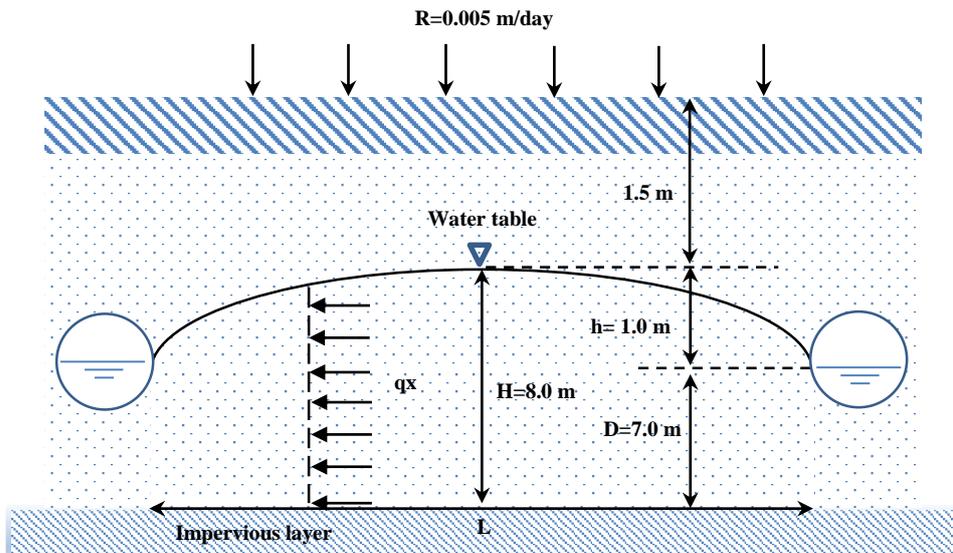


Fig. 2.5 Flow to subsurface drains and definition of drainage parameters

Example 2.3

The objective is to design the spacing between parallel open drains required to keep the water table approximately 1.5 m below the ground surface midway between the drains under steady recharge conditions. The underlying soil has a hydraulic conductivity of 1.0 m/day. The design recharge rate is 0.005 m/day, and a low-permeability layer is encountered at a depth of 9.5 m, which represents the base of the flow region. Using Hooghoudt's equation and the equivalent-depth concept, determine the required drain spacing, L , to satisfy the design conditions. The open drains have a depth of 3.0 m, a bottom width of 0.5 m, and side slopes of 1:1. The design water depth in the ditches is 0.5 m, meaning that the water level in the drains is 2.5 m below the soil surface.

Solution

For a trapezoid, the wetted perimeter u , will be

$$u = b + 2h\sqrt{1 + z^2} \Rightarrow u = 0.5 + 2(0.5)\sqrt{1 + 1^2} = 1.91m$$

and consequently the equivalent radius (Equation 2.23)

$$r_0 = \frac{u}{\pi} = \frac{1.91}{\pi} = 0.61m$$

Using the Hooghoudt equation (for parallel drains above an impervious layer):

$$q = \frac{8Kdh+4Kh^2}{L^2} \rightarrow L^2 = \frac{8Kdh+4Kh^2}{q} = \frac{8(1.0)d(1.0)+4(1.0)(1.0)^2}{0.005}$$

$$L^2 = 1600d + 800$$

$$\text{First estimate: } L = 90 \text{ m, } x = \frac{2\pi D}{L} = \frac{2\pi(7.0)}{90} = 0.49$$

$$F(x) = \frac{\pi^2}{4x} + \ln\left(\frac{x}{2\pi}\right) = \frac{\pi^2}{4(0.49)} + \ln\left(\frac{0.49}{2\pi}\right) = 2.48$$

$$d = \frac{\pi L}{8} \left[\frac{1}{\ln\left(\frac{L}{\pi r_0}\right) + F(x)} \right] = \frac{\pi(90)}{8} \left[\frac{1}{\ln\left(\frac{90}{\pi(0.61)}\right) + 2.48} \right] = 5.58$$

$$L^2 = 1600d + 800 = 1600(5.58) + 800 \Rightarrow L = 98.6 \text{ m}$$

Second estimate: $L = 95 \text{ m}$, $x = 0.46$, $F(x) = 2.76$, $d = 5.6$, $L = 98.8 \text{ m}$

Third estimate: $L = 98 \text{ m}$, $x = 0.45$, $F(x) = 2.84$, $d = 5.7$, $L = 99.4 \text{ m}$

(use $\sim 99 - 100 \text{ m}$ for design)

Example 2.4

The design objective is to determine the required spacing between parallel pipe drains so that the water table remains approximately 1.5 m below the ground surface midway between the drains under steady recharge conditions.

The drain pipes are to be laid at a depth of 2.5 m and have a radius of $r_0 = 0.10 \text{ m}$. The soil profile, however, is not homogeneous; it consists of two distinct layers. The upper layer, 2.5 m thick, has a hydraulic conductivity of $K_t = 0.06 \text{ m/day}$, while the underlying layer, extending 7.0 m to the impervious base, has a hydraulic conductivity of $K_b = 0.30 \text{ m/day}$. The design recharge rate is $q = 0.005 \text{ m/day}$, and a low-permeability layer is encountered at a depth of 9.5 m, representing the base of the flow region.

Using Hooghoudt's equation and the equivalent-depth concept for layered soils, determine the required drain spacing L that will maintain the desired water table level under the given steady-state drainage conditions.

Solution

Modified Hooghoudt (drain at layer interface):

$$q = \frac{8K_b dh + 4K_t h^2}{L^2} = \frac{8(0.3)d(1.0) + 4(0.06)(1.0)^2}{L^2}$$

$$L^2 = 480d + 48$$

From table (Hooghoudt equivalent depth for $r_0 = 0.10 \text{ m}$), read d at the given $D = 7 \text{ m}$ for the current trial L . (Interpolations are linear between tabulated L values.)

Trial $L = 40 \text{ m}$, $d = 3.03$

$$L^2 = 480d + 48 = 480(3.03) + 48 \Rightarrow L = 38 \text{ m}$$

Trial $L = 38 \text{ m}$,

Table (interpolate near $L = (35 - 40)$ at $D = 7 \text{ m} \rightarrow d = 2.942$

$$L^2 = 480d + 48 = 480(2.942) + 48 \Rightarrow L = 38 \text{ m}$$

The smaller spacing compared with the homogeneous. Because the top layer (with $K_t = 0.06 \text{ m/d}$) (Fig. 2.6) restricts the flow of water toward the drains, the term $\frac{4K_t h^2}{L^2}$ in the Hooghoudt equation becomes much smaller, reducing the total drainage capacity and therefore requiring the drains to be placed closer together to maintain the same water-table level.

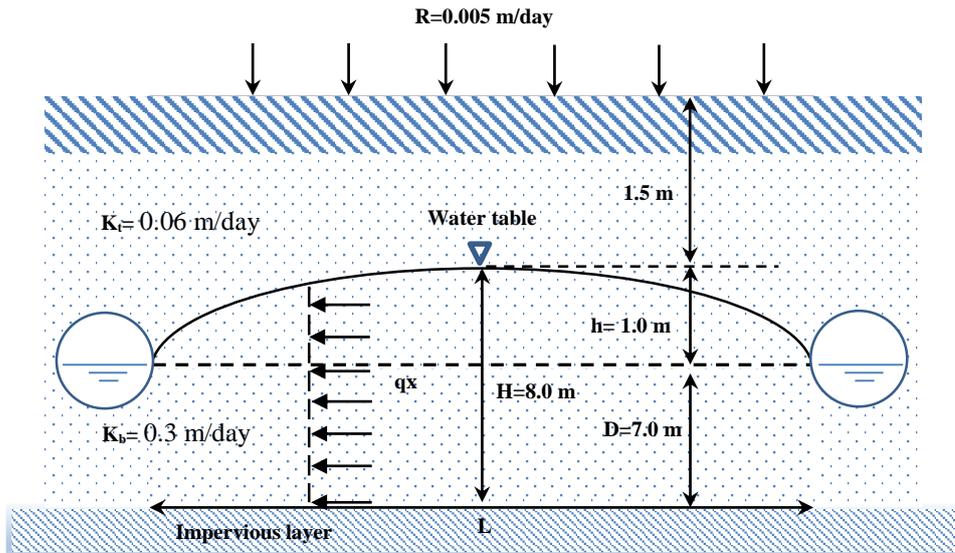


Fig. 2.6 Subsurface drainage between parallel drains in a two-layer soil profile

2.5.2 The Ernst Equation

In previous discussions, the presented drainage solutions were restricted to either homogeneous soil profiles or two-layered profiles where the interface between the two layers coincides with the drain level. The Ernst Equation, however, extends beyond these limitations, providing a versatile approach applicable to any configuration of a two-layered soil system.

A key advantage of the Ernst Equation over the Hooghoudt Equation lies in its flexibility: the interface between the soil layers may occur either above or below the drain level. This feature is particularly beneficial in cases where the upper layer exhibits a significantly lower hydraulic conductivity compared to the underlying layer, as it allows for a more accurate representation of flow conditions across stratified soils.

To develop a general solution for soil profiles comprising layers of differing hydraulic conductivities, Ernst (1956; 1962) conceptualized the total flow toward the drains as comprising three distinct components:

$$h = h_v + h_h + h_r \quad (2.26)$$

Where the total available head (h) is expressed as the sum of the head losses attributed to each flow component, the vertical head loss (h_v), the horizontal head loss (h_h), and the radial head loss (h_r).

The vertical flow is considered to occur within the soil layer bounded by the water table and the drain level (Figure 2.7). The corresponding head loss associated with this vertical movement of water can be determined by applying Darcy's Law.

$$q = K_v \frac{h_v}{D_v} \quad (2.27)$$

The head loss due to vertical flow can be expressed as:

$$h_v = q \frac{D_v}{K_v} \quad (2.28)$$

Where

D_v : Thickness of the layer through which vertical flow occurs (m)

K_v : Vertical hydraulic conductivity (m/d)

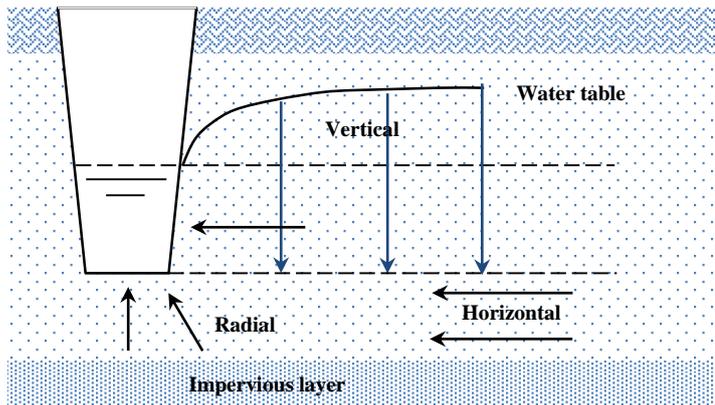


Fig. 2.7 Geometry of two-dimensional flow toward subsurface drains (after Ernst)

The horizontal flow is assumed to occur below the drain level (Figure 2.7). Analogous to Eq. 2.17, the head loss due to horizontal flow, h_h , can be expressed as:

$$h_h = q \frac{L^2}{8 \sum (KD)_h} \quad (2.29)$$

Where

$\sum (KD)_h$: Transmissivity of the soil layers through which water flows horizontally (m^2/d)

If the impervious layer lies at a considerable depth, the value of $\sum (KD)_h$ approaches infinity, causing the horizontal head loss to approach zero. To avoid this condition, the effective thickness of the soil layer below the drain level through which horizontal flow is considered is limited to a maximum of $\frac{1}{4} L$.

The radial flow is also assumed to take place below the drain level (Fig. 2.7). The head loss associated with radial flow can be determined from the following relationship:

$$h_r = q \frac{L}{\pi K_r} \ln \frac{aD_r}{u} \quad (2.30)$$

Where

K_r : Radial hydraulic conductivity (m/d)

- a : Geometry factor of radial resistance (-)
- D_r : Thickness of the layer in which radial flow occurs (m)
- u : Wet perimeter of the drain (m)

In layered soil profiles, the geometry factor a plays an important role in describing subsurface drainage behavior. This factor depends on both the soil stratification and the vertical position of the drain.

For a homogeneous soil profile, the geometry factor equals unity ($a = 1$). In a layered soil, however, a varies depending on whether the drains are located in the upper or lower layer. When the drains lie within the bottom layer, radial flow is primarily confined to that layer, and therefore $a = 1$. Conversely, when the drains are installed in the top layer, the geometry factor a depends on the relative hydraulic conductivities of the top (K_t) and bottom (K_b) layers.

Based on the relaxation method, Ernst (1962) proposed the following classification for determining the geometry factor a :

- For $\frac{K_b}{K_t} < 0.1$: The bottom layer can be regarded as impervious, and the problem simplifies to that of a homogeneous soil, where $a = 1$.
- For $0.1 < \frac{K_b}{K_t} < 50$: The geometry factor a depends on the ratios $\frac{K_b}{K_t}$ and $\frac{D_b}{D_t}$, as presented in Table 2.2.
- For $\frac{K_b}{K_t} > 50$: The geometry factor is taken as $a = 4$.

The total hydraulic head h at the drain level may be expressed as the sum of vertical, horizontal, and radial flow components. Substituting the respective expressions for these components (Equations 2.28, 2.29, and 2.30) into the general head equation (Equation 2.26), we obtain:

$$h = q \left(\frac{D_v}{K_v} + \frac{L^2}{8\Sigma(KD)_h} + \frac{L}{\pi K_r} \ln \frac{aD_r}{u} \right) \quad (2.31)$$

This relationship is commonly referred to as the Ernst Equation, and it provides a comprehensive means of estimating the total head loss in subsurface drainage systems, accounting for both layered soil conditions and variable hydraulic conductivities.

When both the drainage discharge rate (q) and the available total hydraulic head (h) are known, the Ernst Equation can be rearranged into a quadratic form with respect to the drain spacing (L). This allows the spacing between drains to be determined directly by solving the quadratic relationship, either analytically or through numerical computation. The resulting value of L represents the optimal spacing that satisfies the given hydraulic and soil conditions while maintaining the desired drainage performance.

Table 2.2 The geometry factor (a) obtained by the relaxation method (after Van Beers, 1979)

| $\frac{K_b}{K_t}$ | $\frac{D_b}{D_t}$ | | | | | |
|-------------------|-------------------|----------|----------|----------|-----------|-----------|
| | 1 | 2 | 4 | 8 | 16 | 32 |
| 1 | 2.0 | 3.0 | 5.0 | 9.0 | 15.0 | 30.0 |
| 2 | 2.4 | 3.2 | 4.6 | 6.2 | 8.0 | 10.0 |
| 3 | 2.6 | 3.3 | 4.5 | 5.5 | 6.8 | 8.0 |
| 5 | 2.8 | 3.5 | 4.4 | 4.8 | 5.6 | 6.2 |
| 10 | 3.2 | 3.6 | 4.2 | 4.5 | 4.8 | 5.0 |
| 20 | 3.6 | 3.7 | 4.0 | 4.2 | 4.4 | 4.6 |
| 50 | 3.8 | 4.0 | 4.0 | 4.0 | 4.2 | 4.6 |

If the drains are positioned at the interface between two soil layers, the Hooghoudt Equation is applied. This equation explicitly distinguishes between the hydraulic conductivities above and below the drain level, allowing accurate modeling of flow conditions across the interface.

When the drains are installed either above or below the interface, the hydraulic conductivities cannot be treated separately in this manner, and therefore the Ernst Equation must be used. However, when the bottom layer possesses a much lower hydraulic conductivity than the top layer, it can be considered effectively impervious, reducing the system to a single-layer profile underlain by an impermeable stratum. In such cases, the Hooghoudt Equation may still be applied with negligible error.

In practical design, the Ernst Equation is primarily employed for two-layered soil profiles where the top layer is less permeable than the bottom layer ($K_t < K_b$). This condition allows Ernst's method to account for the contrasting conductivities between layers, yielding more realistic predictions of subsurface drainage behavior.

When the drains are located within the bottom soil layer, several simplifying assumptions can be made to reduce the general form of the Ernst Equation:

The vertical resistance in the bottom layer can be neglected compared with that in the top layer, since the hydraulic conductivity of the bottom layer (K_b) is typically much greater than that of the top layer (K_t).

The transmissivity of the top layer may also be neglected because $K_t < K_b$ and, in general, $D_t < D_b$. Consequently, in Equation 2.31, the summation term $\sum(KD)_h$ may be replaced by $K_b D_b$.

The radial flow is confined to the portion of the soil below the drain level (D_r), and therefore the geometry factor a is taken as unity ($a = 1$).

Applying these simplifications, Equation 5.31 reduces to:

$$h = q \left(\frac{D_v}{K_v} + \frac{L^2}{8K_b D_b} + \frac{L}{\pi K_b} \ln \frac{D_r}{u} \right) \quad (2.32)$$

When the drains are positioned within the top soil layer, further simplification is possible. In this case:

- There is no vertical flow in the bottom layer; therefore, the vertical component of head loss is simply represented by $D_v = h$. This formulation reflects the limiting case where drainage flow occurs entirely within the upper, less permeable layer, while the lower layer provides minimal hydraulic interaction.
- When the horizontal flow component is considered, the transmissivity of the top layer cannot be neglected. In this case, the total transmissivity term in Equation 2.29 is expressed as: $\sum(KD)_h = K_b D_b + K_t D_t$, where $D_t = D_r + \frac{1}{2}h$
- Furthermore, the radial flow is confined to the region of the top soil layer below the drain level, and the geometry factor (a) depends on the ratio of the hydraulic conductivities of the top and bottom layers

Accordingly, Equation 2.31 can be simplified to the following form:

$$h = q \left(\frac{D_v}{K_t} + \frac{L^2}{8(K_b D_b + K_t D_t)} + \frac{L}{\pi K_t} \ln \frac{a D_r}{u} \right) \quad (2.33)$$

Example 2.5

An experimental field has a soil profile consisting of two distinct layers. Subsurface pipe drains with a diameter of 0.12 m are to be installed in the upper layer, at a depth of 1.1 m above the interface between the two layers (Fig. 2.8). The relevant parameters for the design are as follows:

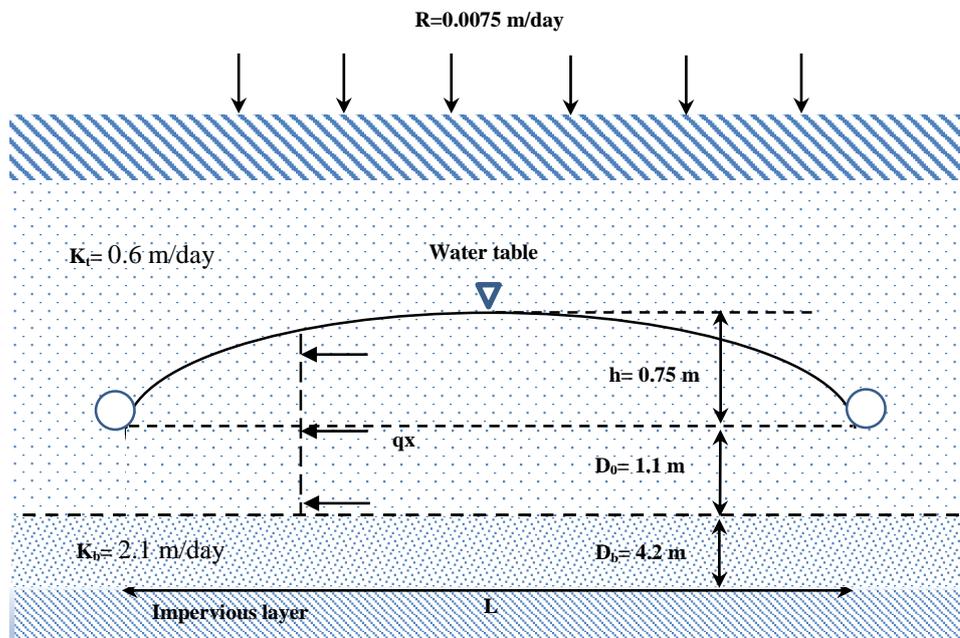


Fig. 2.8 Drain spacing in a two-layered soil profile with the drain in top layer

$$q = \frac{0.0075m}{d}, h = 0.75m, K_t = \frac{0.6m}{d}, K_b = \frac{2.1m}{d}, D_0 = 1.1m, D_b = 4.2m$$

Solution

$$D_v = h \text{ (vertical loss across the upper layer in the limiting top – layer case)} \\ = 0.75\text{m}$$

Drain is ($D_0 = 1.1\text{m}$) above the layer interface \Rightarrow thickness of top layer below the drain: $D_r = D_0 = 1.1\text{m}$

$$D_t = D_r + \frac{h}{2} = 1.1 + \left(\frac{0.75}{2}\right) = 1.475\text{m}$$

$$u = \pi d = \pi(0.12) = 0.377\text{m}$$

Transmissivity for horizontal flow:

$$\sum (KD)_h = K_b D_b + K_t D_t = (2.1)(4.2) + (0.6)(1.475) = 9.705\text{m}^2\text{d}^{-1}$$

Geometry factor a: $\frac{K_b}{K_t} = 3.5$, $\frac{D_b}{D_t} = \frac{4.2}{1.475} = 2.85$, bilinear interpolation in Table 2.2

gives $a = 3.83$

For radial flow in the top layer: $K_r = K_t = 0.6 \text{ m d}^{-1}$

Solve the quadratic for spacing L

Write $AL^2 + BL + C = 0$ with

$$A = \frac{q}{8 \sum (KD)_h} = 9.66 \times 10^{-5}$$

$$B = \frac{q}{\pi K_t} \ln \frac{a D_r}{u} = 9.6 \times 10^{-3}$$

$$C = \frac{q D_v}{K_t} - h = -0.740625$$

$$\text{Discriminant: } \Delta = B^2 - 4AC = 3.7834 \times 10^{-4}$$

$$\text{Positive root: } L = \frac{-B + \sqrt{\Delta}}{2A} = 50.99 \text{ m, take } 51 \text{ m}$$

$$\text{Check (decompose the head) } q \left(\frac{D_v}{K_v} + \frac{L^2}{8 \sum (KD)_h} + \frac{L}{\pi K_r} \ln \frac{a D_r}{u} \right)$$

$$h_v = \frac{q h}{k_t} = 0.00938 \text{ m}$$

$$h_h = \frac{q L^2}{8 \sum (KD)_h} = 0.251 \text{ m}$$

$$h_r = \frac{q L}{\pi K_r} \ln \frac{a D_r}{u} = 0.489$$

$$\text{Sum } h_v + h_h + h_r = 0.75 \text{ m} \checkmark$$

The non-rounded value of L was used in the calculations.

2.6 Unsteady-State Flow Equation

In subsurface drainage, the unsteady-state (transient) flow equation describes the time-dependent movement of water toward drains when the water table fluctuates after rainfall or irrigation. Unlike steady-state conditions, where inflow equals outflow, unsteady flow accounts for changes in water storage within the soil. The governing equation is derived from Darcy's law combined with the continuity

equation, resulting in a partial differential form that relates hydraulic head, time, and spatial position.

This approach allows for the determination of drain spacing under transient conditions, providing more realistic designs when equilibrium has not yet been reached, particularly in fine-textured soils or during the early stages of drainage. The Glover–Dumm equation is commonly applied to estimate water table decline and to determine the optimum drain spacing under unsteady flow conditions.

2.6.1 The Glover–Dumm Equation

In the case of unsteady (transient) flow, the groundwater movement is not constant with time; instead, it varies as water is either stored within, or released from, the soil. This temporal change in storage manifests as a rise or fall of the water table. To analyze this condition, the Dupuit–Forchheimer approach can be applied to derive a differential equation describing unsteady flow.

Consider a soil column bounded at the bottom by an impervious layer and at the top by the water table. When the water table fluctuates, the change in storage within the soil profile per unit surface area is expressed as (Figure 2.9):

$$\Delta W = \mu \Delta h \cdot dx \cdot dy \quad (2.34)$$

Where

ΔW : Change in water storage per unit surface area over the time considered (m)

μ : Drainable pore space (–)

Δh : Change in the level of the water table during the time considered (m)

For an infinitesimally small time interval dt , the corresponding change in storage can be written as:

$$dW = \mu \frac{\partial h}{\partial t} dx dy \quad (2.35)$$

Applying the principle of continuity, the total difference between the outgoing and incoming flow in the x - and y -directions must equal the rate of change in storage. Thus, the continuity equation for transient groundwater flow becomes:

$$-K \left[\frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial h}{\partial y} \right) \right] dx dy = \mu \frac{\partial h}{\partial t} dx dy \quad (2.36)$$

To simplify the continuity equation, we assume that the total depth of flow, h , is significantly greater than the incremental changes Δh . Hence, h may be regarded as a constant and represented by D , denoting the average thickness of the water-transmitting layer. Considering that flow occurs only in one horizontal direction,

Equation (2.36) can therefore be expressed in the following simplified differential form for unsteady flow:

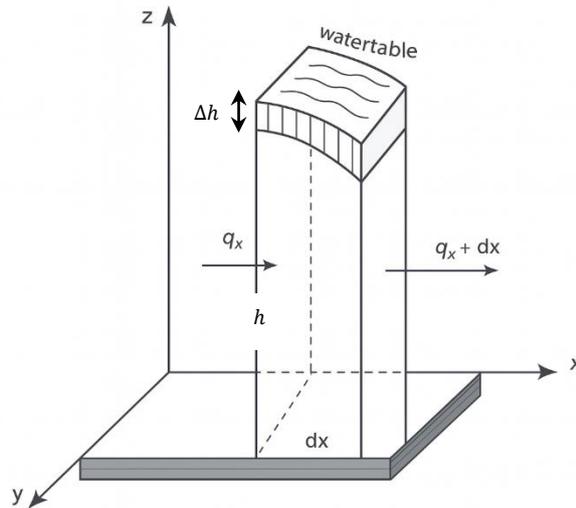


Fig. 2.9 Element of soil column for derivation of the Glover–Dumm equation

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t} \quad (2.37)$$

Dumm (1954) applied this differential equation to describe the recession of the water table following an instantaneous rise to a height h_0 above the drain level (Figure 2.10). His analytical solution derived from a formulation originally developed by Glover expresses the decline of an initially horizontal water table as a function of time, distance, drain spacing, and soil properties. The resulting equation is given as:

$$h(x, t) = \frac{4h_0}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} e^{-\alpha n^2 t} \sin\left(\frac{n\pi x}{L}\right) \quad (2.38)$$

Where

$$\alpha = \frac{Kd\pi^2}{\mu L^2}$$

$h(x, t)$: Height of the water table at distance x and time t (m)

h_0 : Initial height of the water table at $t = 0$ (m)

α : Reaction factor (d^{-1})

K : Hydraulic conductivity (m/d)

d : Equivalent depth of the soil layer below the drain level (m)

L : Drain spacing (m)

t : Time elapsed after the instantaneous rise of the water table (d)

The height of the water table midway between the drains is determined by substituting $x = 1/2 L$ into Equation (2.38). Thus,

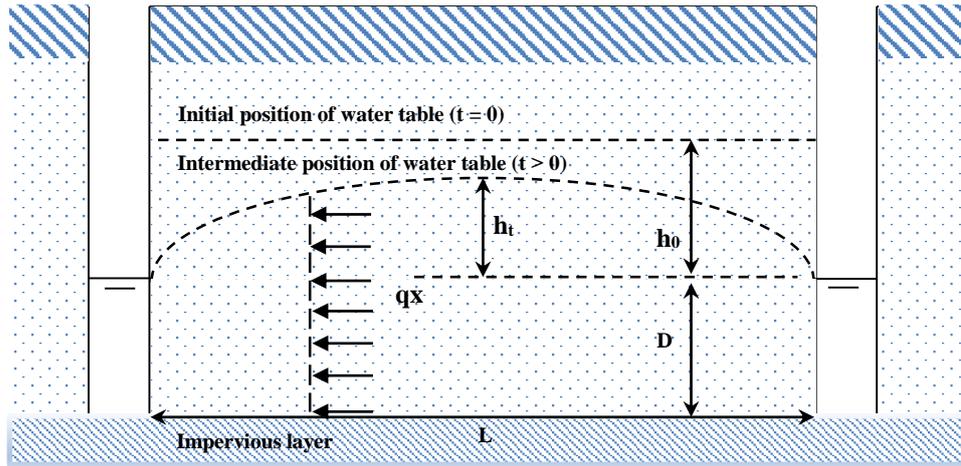


Fig. 2.10 Boundary conditions for the Glover–Dumm equation illustrating the fall of an initially horizontal water table

$$h_t = h(x = 1/2 L) = \frac{4h_0}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} e^{-\alpha n^2 t} \quad (2.39)$$

Where

h_t : Height of the water table midway between drains at $t > 0$ (m)

When $\alpha t > 0.2$, the second and subsequent terms of Equation (2.39) become negligible. Therefore, the equation can be simplified to:

$$h_t = \frac{4}{\pi} h_0 e^{-\alpha t} = 1.27 h_0 e^{-\alpha t} \quad (2.40)$$

If the initial water table is not horizontal but follows a fourth-degree parabolic profile, Dumm (1960) modified the expression to:

$$h_t = 1.16 h_0 e^{-\alpha t} \quad (2.41)$$

By substituting Equation ($\alpha = \frac{Kd\pi^2}{\mu L^2}$) into Equation (2.41), we obtain an expression for the drain spacing, L , as follows:

$$L = \pi \left(\frac{Kdt}{\mu} \right)^{1/2} \left(\ln 1.16 \frac{h_0}{h_t} \right)^{-1/2} \quad (2.42)$$

This expression is known as the Glover–Dumm Equation.

The drain discharge at any time t , expressed per unit surface area, can be derived from Darcy's law as:

$$q_t = - \frac{2Kd}{L} \left[\frac{dh}{dx} \right]_{x=0} \quad (2.43)$$

Where

q_t : Drain discharge per unit surface area at time $t > 0$ (m/d)

By differentiating Equation (2.38) with respect to x , and neglecting all terms where $n > 1$, then substituting $x = 0$ and combining the result with Equation (2.43), we obtain:

$$q_t = \frac{8Kd}{L^2} h_0 e^{-\alpha t} \quad (2.44)$$

Substituting Equation (2.40) into the above expression gives:

$$q_t = \frac{2\pi Kd}{L^2} h_t \quad (2.45)$$

The original Glover–Dumm Equation assumes horizontal flow only and therefore does not account for the radial resistance encountered by water as it converges toward drains that do not extend to the impervious layer. However, in analogy with the steady-state approach, this limitation can be addressed by incorporating Hooghoudt’s concept of equivalent depth (d) into Equation (2.42).

By doing so, the analysis accounts for the additional resistance produced by the converging flow toward the drains, thereby improving the applicability of the Glover–Dumm formulation to practical field conditions.

Example 2.6

In an irrigated area, a subsurface drainage system is required to control the water table under the following conditions (see schematic similar to Figure 2.11):

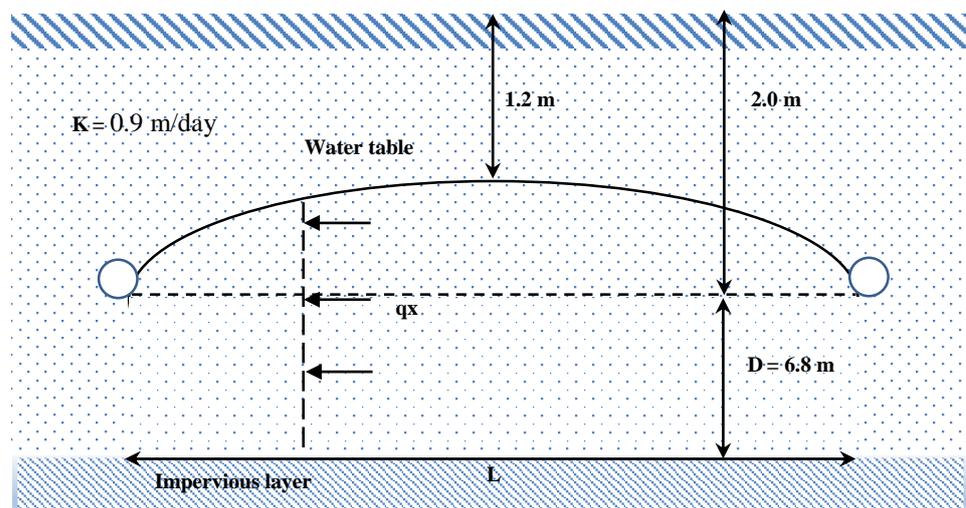


Fig. 2.11 Drain spacing under unsteady-state condition

- The maximum permissible water-table height is 1.2 m below the soil surface.
- Irrigation is applied every 8 days, and the field application loss percolating to the water table is 20 mm per irrigation.
- Drains are installed at a depth of 2.0 m below the soil surface.

- PVC laterals with a radius $r_0 = 0.10$ m are used.
- The impervious layer is at a depth of 8.8 m below the soil surface.
- Average hydraulic conductivity ($K = 0.9$ m d⁻¹).
- Drainable porosity ($\mu = 0.06$).

Determine a suitable drain spacing L using the Glover–Dumm equation. Use the accompanying equivalent-depth table to obtain the equivalent depth d for the given D , r_0 , and trial values of L .

Solution

Step 1: Geometry and recharge

Depth from drain level to the impervious layer: $D = 8.8 - 2.0 = 6.8$ m

Instantaneous recharge head rise from one irrigation: $\Delta h = \frac{R_i}{\mu} = \frac{0.02}{0.06} = 0.333$ m

Initial water-table height above drain level immediately after irrigation (limited by the permissible level):

$$h_0 = 2.0 - 1.2 = 0.8 \text{ m}$$

Required water-table height after $t = 8$ days (i.e., drawdown over the interval):

$$h_t = h_0 - \Delta h = 0.8 - 0.333 = 0.467 \text{ m}$$

Step 2: Glover–Dumm spacing formula

$$L = \pi \left(\frac{Kdt}{\mu} \right)^{1/2} \left(\ln 1.16 \frac{h_0}{h_t} \right)^{-1/2} = \pi \left(\frac{0.9 \cdot d \cdot 8}{0.06} \right)^{1/2} \left(\ln 1.16 \frac{0.8}{0.467} \right)^{-1/2}$$

$$L = 41.5\sqrt{d}$$

Step 3: Use the table to get d (trial and error)

Use the column for $r_0 = 0.10$ m. For each trial L , read d at $D = 6.8$ m by interpolating between the $D = 6.5$ and $D = 7.0$ rows.

Trial 1. Assume $L = 80$ m.

From the table at $L = 80$ m: Linear interpolation, $d \approx 4.17$ m

Compute spacing: $L = 41.5\sqrt{4.17} = 84.7$ m

Trial 2. Assume $L = 85$ m: Linear interpolation, $d \approx 4.26$ m

Compute spacing: $L = 41.5\sqrt{4.26} = 85.7$ m

Now the computed L (≈ 85.7 m) essentially matches the trial value $L = 85$ m \rightarrow convergence achieved.

3. Design of Subsurface Drainage Systems

3.1 Introduction

Efficient control of groundwater and excess soil moisture is essential for maintaining favorable soil conditions and ensuring the long-term productivity of agricultural lands and the stability of engineering structures. When natural drainage is insufficient, subsurface drainage systems are installed below the ground surface to intercept, collect, and remove excess groundwater from the root zone or foundation level. These systems help maintain the water table at a desirable depth, prevent waterlogging, improve soil aeration, and mitigate salinity buildup caused by inadequate leaching.

Subsurface drainage plays a vital role in enhancing crop yield and soil health by providing an optimal balance between air and water in the root zone. In civil engineering applications, it also protects roads, embankments, and foundations from the adverse effects of high groundwater pressures. The design of an effective drainage system requires a sound understanding of soil-water relationships, groundwater flow mechanisms, and the hydraulic performance of drainage materials.

This chapter provides a comprehensive overview of the design principles, criteria, and methods used for subsurface drainage systems. It covers the determination of drain depth and spacing, the hydraulic design of lateral and collector drains.

3.2 Design Considerations

The design of subsurface drainage systems primarily depends on determining the optimum depth and spacing of drains, which together control the position of the groundwater table and the rate at which excess water is removed from the soil. Proper selection of these parameters ensures that the root zone remains adequately aerated while minimizing installation and maintenance costs.

3.2.1 Drain Depth and Spacing

The depth of drains is influenced by several interrelated factors, including crop root depth, soil hydraulic properties, groundwater regime, and outlet conditions. In agricultural drainage, the depth is typically selected to maintain the water table below the effective root zone during the growing season. Shallow drains may not provide sufficient aeration or salinity control, while excessively deep drains increase construction costs and may lead to reduced system efficiency.

Typical drain depths range between **1.0 to 2.5 meters**, depending on soil type and crop requirements. In coarse-textured soils with high hydraulic conductivity, deeper drains can be effective, whereas in fine-textured or poorly permeable soils, shallower drains are often preferred to achieve better control of the water table. The availability and elevation of suitable outlets also play a critical role in determining the feasible drain depth.

Drain spacing refers to the horizontal distance between parallel lateral drains. It governs how quickly groundwater is removed from the soil and is primarily

determined using hydraulic design equations based on Darcy's law and steady-state or unsteady-state flow assumptions. Common analytical approaches include:

- Hooghoudt's equation for steady-state flow conditions, which considers soil hydraulic conductivity, drain depth, and desired water table drawdown.
- Ernst's equation and other modifications for layered soils or anisotropic conditions.
- Glover–Dumm equations for transient or unsteady-state conditions.

All these analytical methods were explained and illustrated with numerical examples in Chapter 2, where their theoretical derivations, assumptions, and practical applications were discussed in detail. In this chapter, these methods are applied as design tools for determining appropriate drain depth and spacing under field conditions.

In practice, the final selection of depth and spacing involves balancing technical efficiency, economic feasibility, and long-term sustainability. Field data on soil hydraulic conductivity, stratification, and water table behavior should be used whenever possible. Computer-aided design tools and drainage models can also assist in optimizing layout configurations under variable soil and climatic conditions.

The depth of a subsurface (subsoil) drain is primarily determined by groundwater conditions, the required lowering of the groundwater table, and the available hydraulic gradient. In general, the following guideline is recommended:

Interceptor drains designed to intercept and remove seepage from a specific soil layer or aquifer should be installed so that they clearly penetrate the target stratum. The drain should extend approximately 150 to 300 mm into the underlying impervious layer to ensure effective interception and prevent bypass flow.

When a subsurface drainage system is designed to lower the general groundwater level, the required depth of the drains depends on whether the layout consists of a single drain or a multi-drain system, as illustrated in Fig. 3.1.

The analysis of drainage performance in these cases relies on a thorough understanding of the soil's hydraulic properties and the application of appropriate theoretical solutions. In critical situations, the design should be conducted by a qualified engineer with specialized geotechnical expertise.

For less critical conditions, the drawdown curve for a single drain may be approximated using the representative characteristics presented in Table 3.1. In systems employing multiple drains, the recommended drain spacings provided in Table 3.2 can serve as a practical guide for preliminary design in non-critical applications.

Particular attention should be given to clay soils, which often exhibit extremely low permeability. In such cases, effective drawdown may not occur, and professional geotechnical consultation is strongly advised before proceeding with the design.

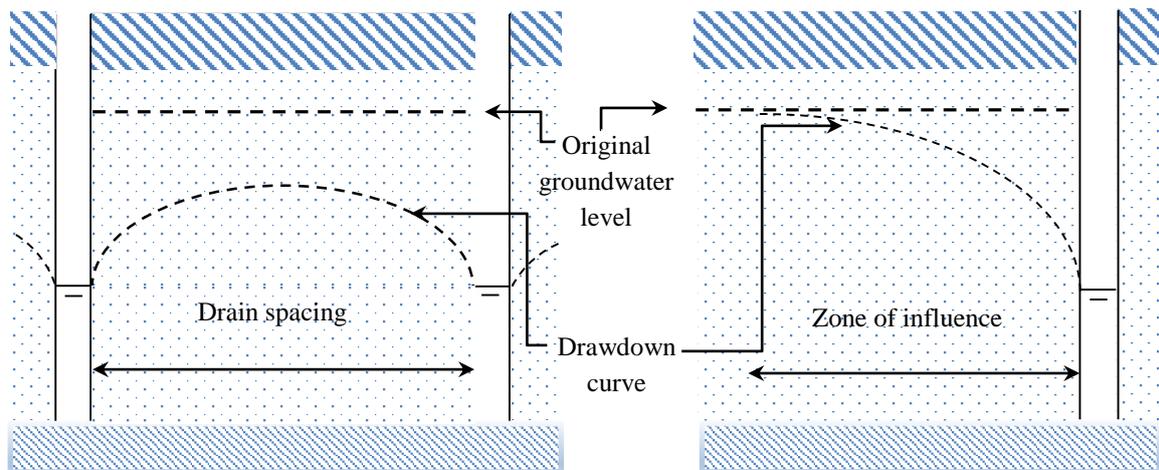


Fig. 3.1 Water table drawdown to single and multi-drain systems

Table 3.1 Typical drawdown characteristics for various soil types (Standards Australia/Standards New Zealand, 1998)

| Soil Type | Zone of Influence (m) | Typical Gradient of Drawdown Curve |
|---------------|-----------------------|------------------------------------|
| Coarse gravel | 150 | — |
| Medium gravel | 50 | 1:200 – 1:100 |
| Coarse sand | 40 | 1:100 – 1:33 |
| Medium sand | 15 – 30 | 1:50 – 1:20 |
| Fine sand | 8 – 15 | 1:20 – 1:5 |
| Silt/clay | Variable | 1:5 – 1:2.5 |

Table 3.2 Typical drain spacings for common soil types (Standards Australia/Standards New Zealand, 1998)

| Soil Type | Depth (m) | Spacing (m) |
|--|-----------|-------------|
| Sand | 1–2 | 50–90 |
| Sandy loam | 1–1.5 | 30–40 |
| Clay loam (<i>i.e., a clayey silt</i>) | 0.5–1 | 12–16 |

Where a subsoil drainpipe or geocomposite drain discharges into a pit or pump-out sump, provisions should be made for convenient inspection of flow. This allows effective monitoring of the subsoil drainage system's performance. In critical installations, the design should also include facilities for backflushing, enabling the removal of any blockages and ensuring the continued efficiency of the drain.

3.2.2 Pipe Gradient

The pipe gradient is one of the most critical factors in subsoil drain design, as it governs both the hydraulic conveyance capacity of the drain and the suction-induced inflow from the surrounding soil. A properly designed gradient ensures adequate flow

velocities to prevent sediment deposition and maintain the desired rate of water removal without causing erosion or structural instability.

In subsurface drainage, water movement into the pipe is driven by the difference in hydraulic head between the soil water and the drain interior. This head difference creates a suction effect, which promotes water inflow through perforations or slots in the pipe wall.

A gentle and uniform gradient sustains this suction along the entire drain length, ensuring balanced inflow and preventing air locks or localized saturation zones.

If the pipe gradient is too low, several operational and hydraulic problems can arise:

- Reduced hydraulic head difference, which decreases the suction effect and lowers the rate of inflow from the surrounding soil.
- Sediment accumulation inside the pipe due to insufficient flow velocity, resulting in reduced discharge capacity and potential clogging.
- Air entrapment and partial blockage, particularly in flat or irregular sections where flow becomes discontinuous.

Conversely, if the pipe gradient is excessively steep, the following conditions may develop:

- Excessive flow velocity, which can lead to internal scouring, erosion of bedding material, or displacement of filter envelopes surrounding the pipe.
- Non-uniform drawdown in the surrounding soil profile, causing uneven drainage and potentially unstable suction conditions along the system.

Proper gradient selection, therefore, must balance the need for adequate flow velocity with the requirement to maintain a stable hydraulic head and uniform suction distribution along the entire drain length.

The pipe gradient must satisfy two primary conditions:

1. The flow velocity should be sufficient to maintain self-cleansing conditions (typically ≥ 0.3 m/s).
2. The slope should be low enough to prevent erosion of pipe bedding or filter layers.

To avoid these issues, appropriate design gradients should be selected based on pipe diameter, soil type, and expected discharge. Table 3.3 provides recommended minimum gradients for typical subsoil drainage applications, illustrating practical ranges that maintain efficient flow while minimizing the risk of erosion and sediment deposition.

The Manning equation (Eq. 1.1, see chapter one) is commonly used to check hydraulic adequacy. For design verification, the slope S can be rearranged as:

$$S = \left(\frac{Qn}{AR^{2/3}} \right)^2 \quad (3.1)$$

This expression enables engineers to determine the minimum slope required to achieve the desired discharge while maintaining acceptable flow conditions.

Table 3.3 Recommended minimum gradients for subsoil drainage pipes

| Pipe Diameter (mm) | Minimum Gradient (m/m) | Typical Application |
|--------------------|------------------------|---|
| 65–80 | 0.0010–0.0020 | Short lateral drains in permeable soils |
| 100 | 0.0008–0.0015 | Lateral or collector drains (general use) |
| 150 | 0.0005–0.0010 | Collector drains or rising mains |
| 225–300 | 0.0003–0.0008 | Main drains and sumps |
| >300 | 0.0002–0.0005 | Major outfall or pumped discharge lines |

Note. Data adapted from *FAO (2007)*; *USDA (1973)*; *AS/NZS 3500.3.2 (1998)*; *CIRIA (1996)*; and *Smedema & Rycroft (1983)*.

3.2.3 Drain Envelopes

A drain envelope is a layer of material placed around a subsurface drain, typically a perforated or slotted pipe, to improve its hydraulic performance and structural stability. The envelope serves several important purposes:

- It prevents excessive migration of fine soil particles into the drain, thereby maintaining long-term function and preventing clogging.
- It increases the permeability of the soil immediately surrounding the drain, enhancing the rate at which water can enter the pipe.
- It enlarges the effective surface area available for drainage, improving overall system efficiency.
- It stabilizes the trench and provides mechanical support and bedding for the drainpipe.
- It helps reduce local hydraulic gradients that could otherwise cause soil instability or piping near the drain.

Although the term filter is often used to describe drain envelopes, it is not entirely accurate. The envelope's function is not to block all soil particles doing so would quickly lead to clogging but rather to balance filtration and permeability, allowing water to pass freely while minimizing soil intrusion.

a) Granular Filter Materials

Granular filters (Fig. 3.2), commonly composed of clean, well-graded sands, gravels, or crushed rock, are frequently used as drain envelopes. Suitable materials should meet the following general requirements:

- Be well-graded, containing a range of particle sizes to ensure good packing and adequate permeability.
- Contain less than 5% passing the 75 μm sieve when natural sand is used.
- For screened crushed rock, particle sizes should typically range from 3 mm to 20 mm.
- Be chemically stable and inert, not subject to breakdown or reaction with soil or groundwater.
- Be sufficiently coarse to prevent washing into the drain pipe or through any overlying geotextile.

b) Geotextile Filters

Geotextiles may be used as drain envelopes where appropriate (Fig. 3.3). These synthetic fabrics act as filters while maintaining permeability. The geotextile's permeability should generally be at least ten times greater than that of the surrounding soil to ensure unrestricted drainage.

However, in soils with high iron content or where oxidation and biological activity promote ferruginous deposits, geotextiles are prone to clogging. In such conditions, granular filters are preferred. The selection of suitable materials and design should be made under the guidance of a professional engineer with geotechnical expertise.

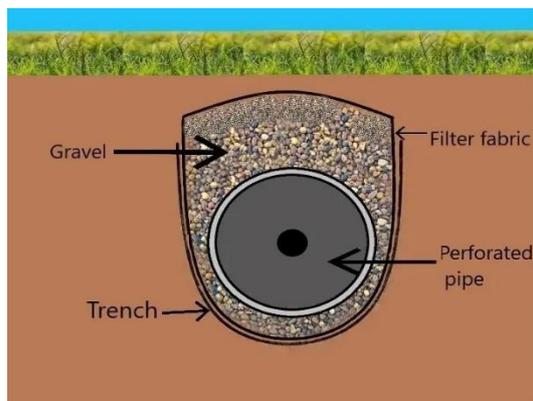


Fig. 3.2 Pipe drain with granular envelope



Fig. 3.3 Typical pipe drain with granular and geotextile envelope

3.2.4 Design Outflow

The design discharge of an agricultural drainage system is a key parameter that must be adapted to the prevailing climatic, hydrological, and soil conditions. It represents the maximum rate of drainage outflow that the system must safely convey without causing either waterlogging or salinization problems. The appropriate design discharge varies considerably among climatic zones, as summarized in Table 3.4.

Table 3.4 Indicative drainage discharge capacities for selected climatic areas

| Climate | Design Discharge, q (mm/day) |
|----------------------------------|--------------------------------|
| Humid temperate climates | 7–15 |
| Humid tropical climates | 10–15 |
| Irrigated lands in arid climates | 1–4 |

1. Humid Temperate Regions

In humid temperate regions, the design discharge is generally based on an event with a recurrence interval of 2 to 5 years. This ensures that excess rainfall is effectively removed, thereby preventing waterlogging and maintaining favorable aeration conditions for crops. Under these circumstances, crops are rarely subjected to prolonged saturated soil conditions.

2. Arid and Semi-Arid Regions

In arid and semi-arid climates, the primary objective of drainage is to prevent soil salinization rather than to remove excess rainfall. Since irrigation is the main source of water input, the drainage design must provide sufficient capacity for leaching accumulated salts. A discharge capacity of 1–4 mm/day is generally adequate for this purpose. However, where irrigation intensity is high or groundwater seepage contributes additional inflow, the design discharge should be increased accordingly, and pipe diameters adjusted to handle the extra load.

3. Humid Tropical and Monsoonal Regions

In humid tropical or monsoon climates, rainfall intensities are often so high that the infiltration capacity of the soil becomes the limiting factor. Surface runoff occurs readily, and subsurface drains alone cannot manage the rapid inflow of water. In such conditions, a combined surface and subsurface drainage system is essential. The surface system provides immediate relief during heavy storms, while the subsurface system lowers the groundwater table to an acceptable depth after rainfall (under non-steady-state conditions). During the dry season, this subsurface system also helps prevent salt accumulation within the root zone.

4. Dependence on Local Conditions

The exact value of the design discharge (q) is strongly dependent on local climatic and irrigation conditions, soil characteristics, and drainage objectives. Therefore, outflow intensities are often derived from local experience or field data. In areas lacking such data, the use of drainage simulation models is recommended to estimate appropriate discharge rates.

Where significant seepage occurs such as in areas where relief wells are used to intercept aquifer inflows the additional volume of seepage water must be included in the design discharge, ensuring that the drainage system has adequate capacity to convey this extra flow.

3.3 Design of Lateral Drains

The design of lateral drains is a critical step in the development of an agricultural subsurface drainage system. Lateral drains collect excess groundwater from the soil profile and convey it to collector or main drains. Their design requires determining the drainage coefficient, lateral discharge, and pipe size that will ensure adequate removal of water within the permissible period after rainfall or irrigation.

The drainage coefficient (q) represents the design rate of water removal from the surface or root zone, expressed as millimeters per day (mm/day). It defines the quantity of water that must be drained from the soil per day to maintain an optimal water table depth for crop growth. The value of q depends primarily on climatic conditions (Table 3.4), rainfall or irrigation intensity, soil infiltration characteristics, and the tolerance of crops to waterlogging.

For irrigated lands, q can be estimated using a water-balance approach (NRCS, 2001):

$$q = \frac{(P+C)}{100} \frac{i}{24F} \quad (3.2)$$

Where

P : Deep percolation or leaching fraction (% of applied irrigation water)

C : Canal and conveyance losses (%)

i : Average irrigation depth (mm)

F : Irrigation frequency (days)

The equation provides q in mm/hr and can be converted to mm/day by multiplying by 24.

After determining the design drainage coefficient, the discharge of each lateral drain can be calculated based on the area it drains. Each lateral typically serves a strip of land extending halfway between itself and the adjacent drains.

For parallel drains spaced at S meters apart and with a length L meters, the contributing area for one lateral is:

$$A_L = S \cdot L \quad (3.3)$$

$$A_{L,ha} = \frac{A_L}{10000} \quad (3.4)$$

Where

S : Spacing between laterals (m)

L : Length of the lateral (m)

A_L : Contributing area (m²)

$A_{L,ha}$: Contributing area (ha)

The required discharge capacity at the outlet of a lateral (Q_L) is calculated using the selected drainage coefficient and the contributing area. In SI units, the discharge (m³/s) is given by:

$$Q_L = 2.78 \times 10^{-7} q A_L \quad (3.5)$$

or, for the end lateral receiving the extra half-spacing:

$$Q_L = 2.78 \times 10^{-7} q S \left(L + \frac{S}{2} \right) \quad (3.6)$$

Where

q : Drainage coefficient (mm/hr)

Equation (3.6) accounts for the additional half-spacing area drained by the last (outlet) lateral, which typically conveys slightly higher discharge than the other laterals.

Once the discharge is determined, the required pipe diameter is computed so that the lateral can safely convey the design flow without becoming pressurized. Drain pipes are usually designed to flow under gravity (non-pressure) conditions to prevent soil particle movement and structural instability of the envelope material.

The discharge capacity of a pipe flowing partially full can be determined using Manning's equation:

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2} \quad (3.7)$$

Where

Q : Discharge (m³/s),

n : Manning's roughness coefficient,

A : Flow area (m²),

R : Hydraulic radius (m), and

S_0 : Pipe slope (m/m).

Manning's roughness coefficient (typically 0.011–0.015 for smooth PVC or corrugated plastic drains)

For design, the slope (S_0) should be sufficient to maintain self-cleansing velocities (usually ≥ 0.3 m/s) and to avoid sediment deposition within the lateral. The computed flow depth should be less than 70–80% of the pipe diameter to ensure free-flowing, non-pressurized operation.

Although Manning's equation is derived for steady, uniform flow where the discharge and energy slope remain constant along the pipe, it can still be validly applied in the design of perforated field drains with gradually increasing discharge. In a perforated drain, water enters continuously through the openings, causing the flow rate to increase toward the outlet. This means the assumption of constant discharge is technically violated, but the variation in discharge between adjacent sections of the drain is usually very small. Over short distances, each segment of the pipe can be considered as carrying an almost uniform flow, with the hydraulic gradient and velocity changing only slightly. Therefore, the drain can be treated as a series of short reaches, each behaving as a locally uniform flow section. In this approach, Manning's

equation is applied either reach-by-reach or using the average discharge corresponding to the mean flow along the pipe. Because the inflow distribution is gradual and the flow remains subcritical and stable, this simplification introduces only minor errors and provides results that are sufficiently accurate for practical agricultural drainage design.

When the pipe is partially full, the upper portion above the water surface forms a complementary (empty) circular segment. The water surface lies above the pipe center; the flow cross-section forms a circular segment as shown in Fig. 3.4 (a).

The flow area A_1 and wetted perimeter P_1 are given by:

$$A_1 = \pi r^2 \left(1 - \frac{\theta}{360}\right) + r \cdot y \cdot \sin \frac{\theta}{2} \quad (3.8)$$

$$P_1 = 2\pi r \left(1 - \frac{\theta}{360}\right) \quad (3.9)$$

Hence, the hydraulic radius for this case is:

$$R_1 = \frac{A_1}{P_1} \quad (3.10)$$

Where

r : Pipe radius

y : Vertical distance from the water surface to the pipe center

When the water level is below the pipe center, as shown in Fig. 3.4 (b). In this case, the area A_2 and the wetted perimeter P_2 are calculated as:

$$A_2 = \pi r^2 - A_1 = \pi r^2 \frac{\theta}{360} - r \cdot y \cdot \sin \frac{\theta}{2} \quad (3.11)$$

$$P_2 = 2\pi r - P_1 = 2\pi r \frac{\theta}{360} \quad (3.12)$$

These expressions are essential for determining hydraulic characteristics in partially full circular conduits, such as storm sewers, sanitary drains, and culverts, where flow depth varies with discharge.

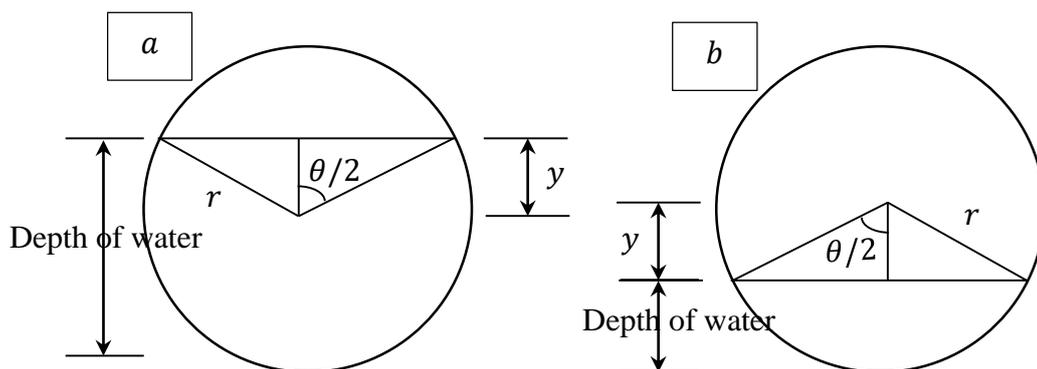


Fig. 3.4 Geometry of partially full circular conduits: (a) flow above center, (b) flow below center.

When designing subsurface drains or circular conduits operating under partially full flow, it is essential to determine whether the water surface lies above or below the

pipe centerline. This decision governs the selection of the appropriate set of geometric equations used to compute the flow area, wetted perimeter, and hydraulic radius.

The position of the water surface relative to the centerline depends on the filled area of the flow cross-section compared with the semi-circular area of the pipe. The total cross-sectional area of the pipe, $A_f = \pi r^2$, is divided into two complementary segments: the filled area (occupied by water) and the empty area (above the water surface). When the filled area is greater than one-half of the pipe's total area, the water surface lies above the center of the pipe, and the flow geometry corresponds to the case shown in Figure 3.4(a). Conversely, when the filled area is less than one-half of the total area, the water surface lies below the pipe center, as illustrated in Figure 3.4(b).

For analytical purposes, this distinction determines whether the designer should apply the equations for flow above the center (Equations 3.8–3.10) or flow below the center (Equations 3.11–3.12).

This logical procedure ensures that the correct expressions for the segmental area and wetted perimeter are applied in hydraulic computations. The approach also facilitates accurate estimation of the hydraulic radius and velocity distribution for both flow conditions, which is critical in drainage design, stormwater systems, and sewer hydraulics.

In the hydraulic design of subsurface agricultural drainage systems, Manning's equation is commonly used to estimate the flow characteristics within perforated drain pipes. Although this equation was originally developed for open channel flow, it provides a convenient and practical means of describing the flow behavior inside drains when certain simplifying assumptions are adopted. Its use serves two main purposes: first, to simplify the analysis of the complex flow conditions within the pipe by applying a well-known empirical relationship; and second, to provide a clear picture of the variation in flow depth and velocity along the drain as water enters gradually through the perforations. In the case of agricultural drainage design, the variation in flow between adjacent sections of a drain is generally very small because the inflow through the perforations is distributed gradually along the length of the pipe. Over relatively short distances, the hydraulic gradient and the velocity of flow remain nearly constant. Therefore, each small section of the pipe can be considered to carry a locally uniform discharge. Based on this concept, the entire drain can be viewed as a series of short spans, each behaving as a uniform flow segment. By analyzing successive short sections along the drain, the flow depth and velocity distribution can be estimated, while accounting for the gradual increase in discharge due to lateral inflow. This method provides a sufficiently accurate and practical approach for engineering design, as the actual flow, although slightly nonuniform, can be well approximated by the assumption of locally uniform flow.

Example 3.1

For the design of a lateral drain in an irrigated field, the following data are provided. The spacing between drains is $S = 40$ m, and each lateral has a length of $L = 200$ m.

The drainpipes are made of corrugated plastic or PVC material, with Manning's roughness coefficient taken as $n = 0.013$. The design should ensure that the pipe operates under partially full flow conditions, with the flow depth kept below 70% of the pipe diameter ($d < 0.7D$). The ground or pipe slope is to be selected as part of the design process. To determine the drainage coefficient using the NRCS water-balance approach, the irrigation parameters are as follows: deep percolation losses $P = 25\%$, canal and conveyance losses $C = 20\%$, average irrigation depth $i = 200$ mm, and irrigation frequency $F = 7$ days.

Solution

1. Drainage coefficient q

$$q = \frac{(P+C)}{100} \frac{i}{24F} = \frac{(25+20)}{100} \frac{200}{24 \times 7} = \frac{0.536 \text{ mm}}{\text{hr}} = 12.86 \text{ mm/day}$$

2. Contributing area of one lateral

$$A_L = S \cdot L = 40 \times 200 = 8000 \text{ m}^2 = 0.8 \text{ ha}$$

3. Design discharge of the lateral

Typical lateral:

$$Q_L = 2.78 \times 10^{-7} q A_L = 2.78 \times 10^{-7} \times 0.536 \times 8000 = 1.192 \times 10^{-3} \text{ m}^3/\text{s}$$

End (outlet) lateral (extra half-spacing):

$$Q_L = 2.78 \times 10^{-7} q S \left(L + \frac{S}{2} \right) = 1.31 \times 10^{-3} \text{ m}^3/\text{s}$$

We'll design for the end lateral (the controlling, slightly larger flow):

4. Select a trial pipe and slope; check with Manning (partially full)

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}, \text{ for a full circular pipe: } A = \frac{\pi D^2}{4}, R = \frac{D}{4}, \text{ and assume } S_0 = 0.006.$$

$$1.31 \times 10^{-3} \frac{\text{m}^3}{\text{s}} = \frac{1}{0.013} \frac{\pi D^2}{4} \left(\frac{D}{4} \right)^{2/3} \times 0.006^{1/2} \Rightarrow D = 0.066 \text{ m}$$

Take design diameter = 0.4 m,

Calculate filled area = $3.42 \times 10^{-3} \text{ m}^2$, and total area = 0.126 m^2 .

So, filled area $<$ total area / 2, \therefore the water surface lies below the pipe center

$$A_2 = \pi r^2 \frac{\theta}{360} - r \cdot y \cdot \sin \frac{\theta}{2}$$

$$34.2 = \pi r^2 \frac{\theta/2}{180} - r \cdot y \cdot \sin \frac{\theta}{2}, \text{ area (cm}^2\text{), dividing both sides of the equation by } r$$

$$1.71 = \pi(20) \frac{\theta/2}{180} - y \cdot \sin \frac{\theta}{2} \quad (1)$$

$$\cos \frac{\theta}{2} = \frac{y}{r} = \frac{y}{20} \quad (2)$$

To determine the relationship between the depth of flow and the cross-sectional flow area, geometric relationships of the circular segment are illustrated in the following table.

| Assume y | Calculate $\left(\frac{\theta}{2}\right)$, Eq. (2) | Right hand of Eq. (1) |
|----------------|---|-----------------------|
| $y = r/2 = 10$ | 60 | 12.28 |
| $y = 15$ | 41.4 | 4.55 |

| | | |
|------------|------|------|
| $y = 16$ | 36.9 | 3.28 |
| $y = 17.5$ | 28.9 | 1.62 |

Therefore, 1.62 is the closest value to 1.71, and the corresponding depth of water in the pipe is $20 - 17.5 = 2.5$ cm.

The cross-sectional area (A), wetted perimeter (P), and hydraulic radius (R) of a partially full circular conduit can be determined using the geometric and trigonometric relationships. It would therefore appear reasonable to employ these computed values of A and R in the Manning equation, together with the pipe slope and the Manning roughness coefficient (n) corresponding to full-flow conditions, to estimate either the discharge for a specified flow depth or the normal depth for a given discharge under partially full flow conditions.

However, experimental evidence accumulated as early as the mid-twentieth century demonstrated that measured flow rates in partially full pipes often deviate significantly from those predicted by direct application of the Manning equation using the full-flow roughness coefficient. To address this discrepancy, T. R. Camp (1946) developed a correction procedure that introduces a variation of the Manning roughness coefficient with flow depth, expressed as the ratio $n/n(\text{full})$ as a function of y/D , where y is the flow depth and D is the pipe diameter.

Camp's work resulted in a widely used graphical relationship illustrating the variation of the dimensionless parameters $Q/Q(\text{full})$, $V/V(\text{full})$, and $n/n(\text{full})$ as functions of the depth ratio y/D . This relationship is presented in Fig. 3.5, which shows how these parameters vary over the full range of partial-flow conditions.

Before the widespread availability of spreadsheet software, which now facilitates the computation of A , P , and R through direct trigonometric and geometric relationships, engineers frequently relied on Camp's graph for analyzing partially full pipe flow. Under this approach, the full-flow velocity (V_{full}) and discharge (Q_{full}) are first determined from the Manning equation for the pipe under full-flow conditions. The corresponding velocity (V) and discharge (Q) for any partial depth y can then be obtained by reading the appropriate ratios directly from Fig. 3.5.

This figure provides a practical means for estimating flow characteristics in partially full circular conduits without requiring detailed trigonometric computation. For modern design practice, these relationships can be readily implemented in spreadsheet or computational tools for greater precision and efficiency.

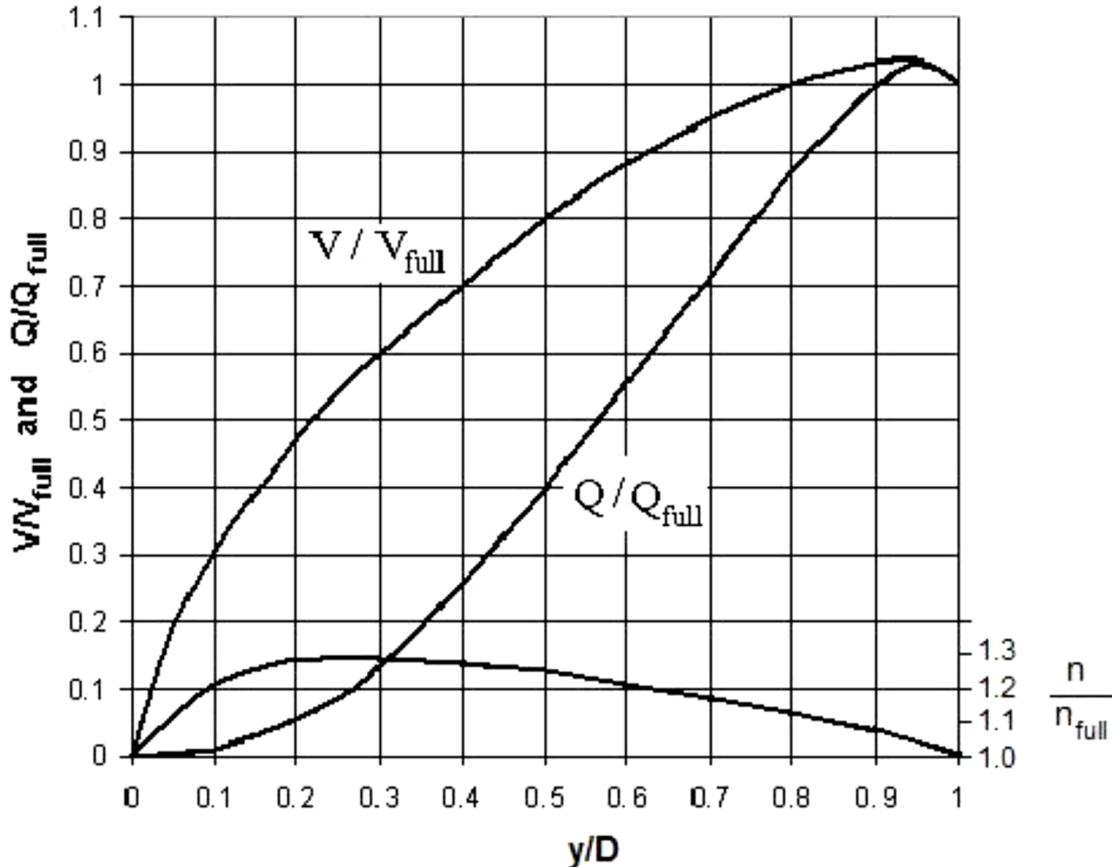


Fig. 3.5 Relationship between dimensionless discharge (Q/Q_{full}), velocity (V/V_{full}), and Manning roughness ratio (n/n_{full}) as functions of relative depth of flow (y/D), based on Camp (1946). Adapted from Steel and McGhee

Example 3.2

A lateral drain pipe has an internal diameter of 525 mm. When the pipe flows completely full, the discharge and mean velocity have been determined as:

$$Q_{full} = 0.258 \frac{m^3}{s}, V_{full} = 1.155 \text{ m/s}$$

Estimate the average velocity and discharge in this lateral drain when it is flowing:

- a. at a depth of $y = 210$ mm
- b. at a depth of $y = 373$ mm

Solution

1. Determine the depth ratio:

$$\frac{y}{D} = \frac{210}{525} = 0.4, \frac{373}{525} = 0.7$$

From hydraulic relationships for partially full circular conduits (Fig. 3.5), obtain the corresponding ratios:

For case (1)

$$\frac{y}{D} = 0.4, \frac{V}{V_{full}} = 0.7, \frac{Q}{Q_{full}} = 0.27$$

$$V = 0.7 \times 1.155 = 0.808 \frac{m}{s}, \quad Q = 0.27 \times 0.258 = 0.07m^3/s$$

For case (2)

$$\frac{y}{D} = 0.7, \quad \frac{V}{V_{full}} = 0.95, \quad \frac{Q}{Q_{full}} = 0.7$$

$$V = 0.95 \times 1.155 = 1.097 \frac{m}{s}, \quad Q = 0.7 \times 0.258 = \frac{0.181m^3}{s}$$

Although the “Flow in Partially Full Pipes” chart can be used to determine the average velocity and discharge in a circular conduit under partially full flow conditions, as illustrated in Example 3.3, it is often more practical to perform these computations using a spreadsheet program such as Microsoft Excel.

To facilitate such calculations, a set of empirical relationships has been developed to express the ratio of the flow parameter n/n_{full} as a function of the relative depth of flow (y/D), within the range:

| | |
|-----------------------|--|
| $0 \leq y/D < 0.03$ | $n/n_{full} = 1 + (y/D)/(0.3)$ |
| $0.03 \leq y/D < 0.1$ | $n/n_{full} = 1.1 + (y/D - 0.03)/(12/7)$ |
| $0.1 \leq y/D < 0.2$ | $n/n_{full} = 1.22 + (y/D - 0.1)/(0.6)$ |
| $0.2 \leq y/D < 0.3$ | $n/n_{full} = 1.29$ |
| $0.3 \leq y/D < 0.5$ | $n/n_{full} = 1.29 - (y/D - 0.3)(0.2)$ |
| $0.5 \leq y/D \leq 1$ | $n/n_{full} = 1.25 - (y/D - 0.5)(0.5)$ |

Example 3.3

Water flows through a metal drain pipe having an internal diameter of 300 mm (12 in). The Manning’s roughness coefficient for full-pipe flow in this corrugated metal pipe is $n_{full} = 0.022$, Determine the equivalent Manning’s roughness coefficient corresponding to a flow depth of 100 mm (4 in).

Solution

The given parameters are: $y = 100$ mm, $D = 300$ mm, $y/D = 0.333$

Since y/D lies between 0.3 and 0.5, the corresponding empirical relationship for the ratio n/n_{full} is given by:

$$n/n_{full} = 1.29 - (y/D - 0.3)(0.2) = 1.29 - (0.333 - 0.3)(0.2) = 1.28$$

$$n = 1.28 \times 0.022 = 0.028$$

Example 3.4

Determine the flow rate and average velocity for water flowing at a depth of 100 mm in a 300 mm diameter metal pipe, as described in Example 3.3. The slope of the pipe is 0.0085.

Solution

$$h = 100 \text{ mm}, \quad y = (D/2) - h = 150 - 100 = 50 \text{ mm}$$

$$\frac{\theta}{2} = \cos^{-1} \frac{50}{150} \Rightarrow \frac{\theta}{2} = 70.5^\circ$$

$$A_2 = \pi r^2 \frac{\theta}{360} - r \cdot y \cdot \sin \frac{\theta}{2}$$

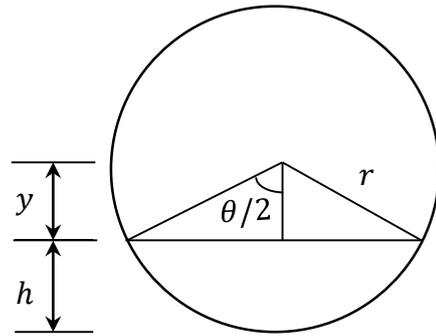
$$A_2 = \pi(150)^2 \frac{141}{360} - (150)(50) \sin \frac{141}{2}$$

$$A_2 = 20615 \text{ mm}^2, 0.0206 \text{ m}^2$$

$$P_2 = 2\pi r \frac{\theta}{360} = 2\pi(150) \frac{141}{360} = 369 \text{ mm}, 0.369 \text{ m}$$

$$R_2 = \frac{A_2}{P_2} = \frac{20615}{369} = 55.8 \text{ mm}, 0.0558 \text{ m}$$

From Example 6.3, $n = 0.028$



$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} = \frac{1}{0.028} (0.0206)(0.0558)^{2/3} (0.0085)^{1/2} = 9.905 \times 10^{-3} \text{ m}^3/\text{s}$$

$$V = \frac{Q}{A} = \frac{9.905 \times 10^{-3}}{0.0206} = 0.48 \text{ m/s}$$

Example 3.5

The lateral drain is constructed using PVC pipe with a Manning's roughness coefficient of $n = 0.013$. The available longitudinal grade along the drain is $S_0 = 0.002$ (equivalent to a 0.20% slope). Parallel laterals are spaced at $S = 40$ m, each having a length of $L = 300$ m. The design considers the end lateral that receives an additional half-spacing of drainage area. For the peak or rainy condition, the design drainage coefficient is taken as (10.3 mm/day). The design objective is to select a practical pipe diameter and/or operating condition such that the mean velocity at the outlet satisfies the self-cleansing criterion of $V \geq 0.30$ m/s under the chosen drainage coefficient q .

Solution

$$q = 10.3 \frac{\text{mm}}{\text{day}} = 0.429 \text{ mm/hr}$$

1. Outlet discharge of the end lateral

$$Q_L = 2.78 \times 10^{-7} q S \left(L + \frac{S}{2} \right) = 2.78 \times 10^{-7} (0.429)(40)(320) = 0.00153 \text{ m}^3/\text{s}$$

2. Calculate diameter and capacity check with Manning (full-flow)

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} \Rightarrow 0.00153 = \frac{1}{0.013} \frac{\pi D^2}{4} \left(\frac{D}{4} \right)^{2/3} 0.002^{1/2} \Rightarrow D = 0.086 \text{ m}$$

Take $D = 0.10$ m

$$\text{Total area} = 7.85 \times 10^{-3} \text{ m}^2, \text{ filled area} = 5.809 \times 10^{-3} \text{ m}^2$$

Filled area $>$ (1/2) total area, the water surface lies above the center of the pipe.

$$A_1 = \pi r^2 \left(1 - \frac{\theta}{360} \right) + r \cdot y \cdot \sin \frac{\theta}{2}$$

$$58.09 = \pi(5)^2 \left(1 - \frac{\theta/2}{180} \right) + (5)y \sin \frac{\theta}{2}$$

$$11.62 = \pi(5) \left(1 - \frac{\theta/2}{180} \right) + y \sin \frac{\theta}{2} \quad (1)$$

$$\cos \frac{\theta}{2} = \frac{y}{r} = \frac{y}{5} \quad (2)$$

| Assume y | Calculate $\left(\frac{\theta}{2}\right)$, Eq. (2) | Right hand of Eq. (1) |
|-----------------|---|-----------------------|
| $y = r/2 = 2.5$ | 60 | 15.26 |
| $y = 2$ | 66.4 | 11.74 |

Therefore, 11.74 is the closest value to 11.62, and the corresponding depth of water in the pipe is $2 + 5 = 7.0$ cm.

Calculate, $V_{full} = 0.294$ m/s

From hydraulic relationships for partially full circular conduits (Fig. 3.5), obtain the corresponding ratios:

$$\frac{y}{D} = 0.7, \quad \frac{V}{V_{full}} = 0.95,$$

$$V = 0.95 \times 0.294 = 0.28 \frac{m}{s},$$

Result: $V \approx 0.28$ m/s < 0.30 m/s \rightarrow self-cleansing not satisfied.

Keep $D = 100$ mm, increase slope, $S_0 = 0.003$,

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} \Rightarrow 0.00153 = \frac{1}{0.013} \frac{\pi D^2}{4} \left(\frac{D}{4}\right)^{2/3} 0.003^{1/2} \Rightarrow D = 0.079 \text{ m}$$

Take $D = 0.10$ m

Total area = 7.85×10^{-3} m², filled area = 4.9×10^{-3} m²

Filled area > (1/2) total area, the water surface lies above the center of the pipe.

$$49 = \pi(5)^2 \left(1 - \frac{\theta/2}{180}\right) + (5)y \sin \frac{\theta}{2}$$

$$9.8 = \pi(5) \left(1 - \frac{\theta/2}{180}\right) + y \sin \frac{\theta}{2} \quad (1)$$

$$\cos \frac{\theta}{2} = \frac{y}{r} = \frac{y}{5} \quad (2)$$

| Assume y | Calculate $\left(\frac{\theta}{2}\right)$, Eq. (2) | Right hand of Eq. (1) |
|-----------|---|-----------------------|
| $y = 2.5$ | 60.0 | 12.63 |
| $y = 2$ | 66.4 | 11.74 |
| $y = 1.5$ | 72.5 | 10.8 |
| $y = 1.0$ | 78.5 | 9.8 |

The corresponding depth of water in the pipe is $1 + 5 = 6.0$ cm.

$$\frac{y}{D} = 0.6, \quad \frac{V}{V_{full}} = 0.89$$

Calculate, $V_{full} = 0.36$ m/s

$$V = 0.89 \times 0.36 = 0.32 \frac{m}{s}$$

Result: $V \approx 0.32$ m/s > 0.30 m/s \rightarrow self-cleansing satisfied.

Or, it can be calculated using the Manning equation.

$$A_1 = 49 \text{ cm}^2$$

$$P_1 = 2\pi r \left(1 - \frac{\theta}{360}\right) = 2\pi(5) \left(1 - \frac{157}{360}\right) = 17.7 \text{ cm}$$

Hence, the hydraulic radius for this case is:

$$R_1 = \frac{A_1}{P_1} = \frac{49}{17.7} = 2.768 \text{ cm} = 0.02768 \text{ m}$$

$$\text{From Fig. 3.5, } \frac{y}{D} = 0.6, \frac{n}{n_{full}} = 1.2 \Rightarrow n = 1.2 \times 0.013 = 0.0156$$

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} = \frac{1}{0.0156} (0.02768)^{2/3} 0.003^{1/2} = 0.32 \text{ m/s}$$

3.4 Design of Collector Drains

The collector drain is a crucial component of a subsurface drainage system designed to control the water table and remove excess groundwater from agricultural fields. It serves as an intermediary between smaller lateral drains and the main drain or outlet, ensuring the efficient conveyance of drained water toward a disposal point such as a canal, pump station, or natural water body.

Collector drains receive water from lateral subdrains (also called field drains or tile drains) that are installed within the crop root zone. These laterals collect groundwater through perforated pipes or porous materials and discharge it into the subsurface collector, which then conveys the accumulated water toward the main collector drain.

The main functions are:

1. Conveyance: Transfer water from several lateral subdrains to the main collector.
2. Integration: Act as an intermediate conduit linking local field drainage to regional or main drainage infrastructure.
3. Hydraulic Balance: Maintain a uniform and controlled water table depth, preventing both waterlogging and excessive drying.
4. Protection of Fields: Ensure that salts and stagnant water are flushed away to sustain soil productivity.

The Fig. 3.6 illustrates the layout and flow of a subsurface drainage system leading to a main collector and eventually to a sea or lake outlet. The system begins with lateral subdrains, which are the smallest pipes or channels installed beneath the soil surface, usually arranged in parallel lines within the field. Their main function is to collect groundwater through infiltration and direct it toward the subsurface collector, as indicated by the arrows in the figure. The subsurface collectors receive water from several lateral subdrains and convey it toward the main collector. They act as intermediate drains that gather and unify the drainage flow and are typically placed at a greater depth with a larger diameter than the laterals. The main collector drain is the primary conduit of the system, responsible for gathering water from multiple subsurface collectors and transporting it to the disposal outlet, such as a pump station or an open drain. In the figure, the main collector runs diagonally toward the pump station located near the irrigation canal. The pump station plays a critical role by lifting the collected drainage water and discharging it into the main open drain, which ultimately conveys it to the sea or lake. In low-lying or coastal areas, pumping is essential since gravity flow alone may not be sufficient to remove the drainage water effectively. Finally, the main open drain and outlet receive the pumped water and

deliver it to the final disposal point, ensuring that all excess water is efficiently removed from the system to maintain optimal soil conditions.

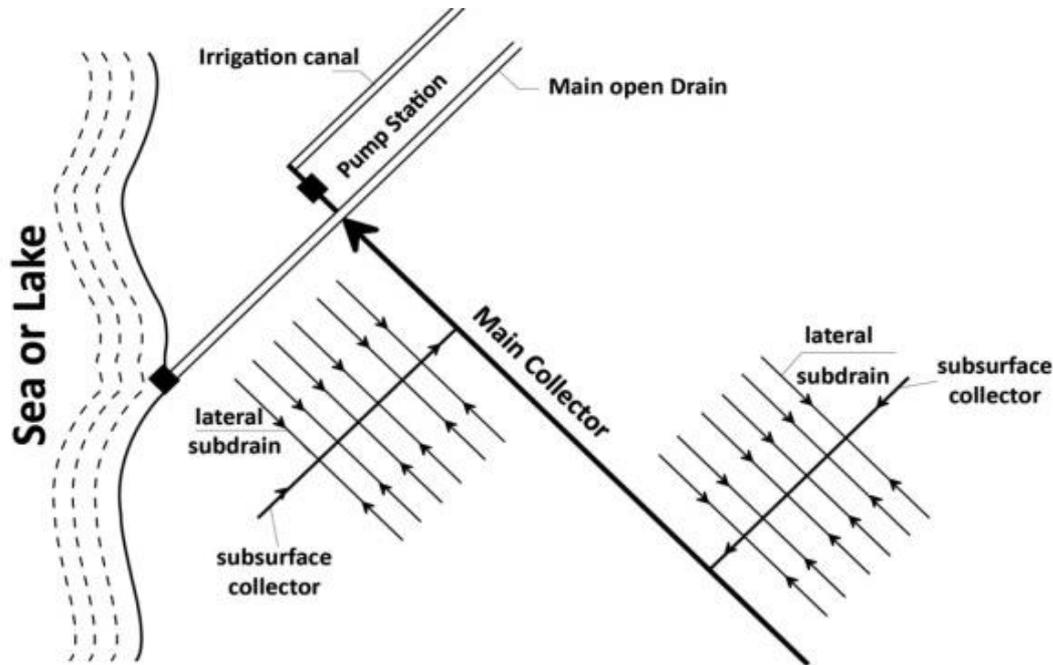


Fig. 3.6 Schematic layout of a subsurface drainage system with collectors and main drain leading to the disposal outlet

Field topography and the location or elevation of the outlet are among the primary factors influencing the planning and design of a drainage system layout. Topography, in particular, has a significant impact on the range of feasible layout alternatives. Although several layout options may exist for a given field (as illustrated in Fig. 3.7), the selection of the most suitable layout should be guided by the specific drainage objectives. These objectives may include removing excess water from isolated low-lying areas, improving overall field drainage, or intercepting seepage along a hillside. Designers should adopt a comprehensive and forward-looking approach when planning drainage systems, taking into account both current and potential future needs. Even when the system is implemented in phases, planning should be integrated rather than fragmented. Future expansions or modifications will be more efficient and cost-effective if the main drains are initially designed with adequate capacity and placed in optimal locations.

When determining the appropriate layout pattern for a specific field or topographic condition, field drains should be aligned as closely as possible with the natural contours of the land. This alignment allows the laterals to effectively intercept water as it moves downslope. In contrast, collector drains are typically positioned along steeper gradients to enhance the efficiency of water conveyance and to facilitate the proper arrangement of the lateral drains (as illustrated in Fig. 3.8).

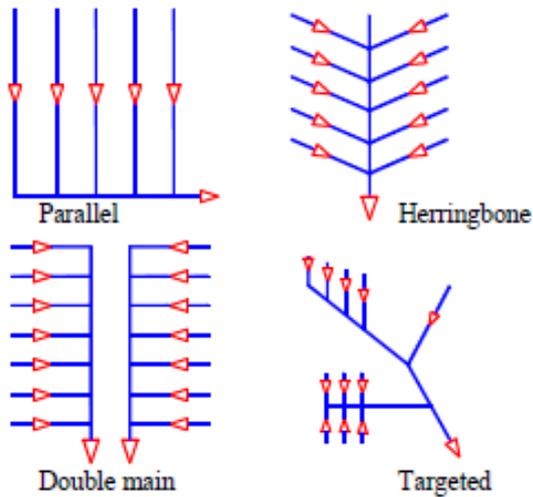


Fig. 3.7 Alternative layout patterns for subsurface drainage systems

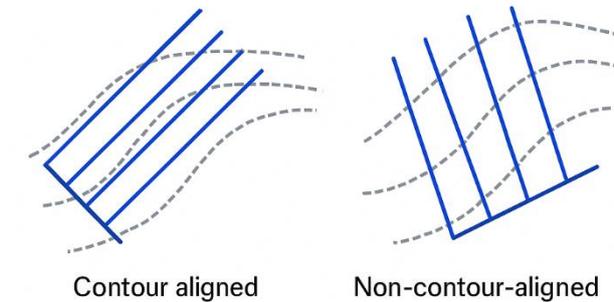


Fig. 3.8 Alignment of field laterals relative to land contours

The collector receives inflow continuously from attached laterals along its length. Therefore, its discharge increases progressively downstream, from zero at the upper end to a maximum at the outlet. The collector receives inflow only at lateral junctions; therefore, its discharge increases stepwise downstream from zero at the upstream end to a maximum at the outlet. The design procedure must account for this gradual increase. If laterals are spaced at a uniform interval S_L along the collector.

Let

$Q_{c,max}$: Maximum discharge (flow rate) that the collector drain must convey at its downstream end or outlet (m^3/s)

Q_c : Discharge in collector at any distance x from its upstream end (m^3/s)

Q_L : Discharge contributed by a single lateral (m^3/s)

n : Total number of laterals entering the collector

$L_c = nS_L$: length of collector (m)

$x_i = iS_L$: Position of i^{th} junction

1. Exact stepwise formulation (uniform laterals)

For $k = 1, 2, \dots, n$ and $(k-1)S_L < x \leq kS_L$

$$Q_c(x) = kQ_L \quad (3.13)$$

$$Q_{c,max} = n \cdot Q_L \quad (3.14)$$

2. General stepwise formulation (non-uniform laterals)

If laterals have unequal discharges $Q_{L,i}$ and positions x_i ,

$$Q_c(x) = \sum_{i=1}^n Q_{L,i} H(x - x_i) \quad (3.15)$$

Where

$H(\cdot)$: Heaviside step function (0 for negative argument, 1 for non-negative)

If all laterals drain equal areas A_L , then:

$$Q_{c,max} = 2.78 \times 10^{-7} q n A_L \quad (3.16)$$

or equivalently, substituting $A_L = S_L L_L$:

$$Q_{c,max} = 2.78 \times 10^{-7} q n S_L L_L \quad (3.17)$$

Where

S_L : Spacing between laterals (m),

L_L : Length of lateral (m), and

q : Drainage coefficient (mm/hr).

The collector pipe is designed to flow partly full under gravity. Manning's formula is used to estimate capacity.

For a full circular pipe, $A = \pi D^2/4$ and $R = D/4$. However, subsurface drains are not operated under pressure, so the flow depth should not exceed 70–80 % of the pipe diameter.

The slope of the collector must satisfy two main hydraulic conditions:

1. Self-cleansing velocity to prevent sediment deposition:
 $V \geq 0.3$ m/s for plastic or concrete drains.
2. Non-erosive velocity to prevent structural damage:
 $V \leq 2.0$ m/s

These limits ensure stable operation throughout seasonal fluctuations in discharge.

Collector drains are installed deeper than laterals to maintain gravity flow from the lateral outlets.

Because inflow from laterals enters the collector discretely (at junctions), the discharge along the collector increases stepwise rather than continuously. Two consistent approaches can be used in design:

1. Reach-by-Reach (Exact) Design

Divide the collector into n reaches corresponding to the spacing between lateral junctions:

$$(k-1)S_L, kS_L$$

Within each reach, the discharge remains constant.

2. Single-Size Collector (Simplified)

For practical construction and maintenance, a single pipe size is often adopted for the entire collector.

Then, perform two essential verifications:

- Downstream-most reach: Check capacity under $Q_{c,max}$ using Manning's equation to ensure non-pressurized, free-flow conditions.
- Upstream-most reach: Check that the flow velocity satisfies the self-cleansing criterion, typically $V \geq 0.30$ m/s.

Example 3.6

A PVC collector drain (smooth) with Manning roughness $n_{full} = 0.013$. The longitudinal grade S_0 is to be selected. Parallel field laterals have spacing across the field $S = 40$ m and length $L = 200$ m. The spacing of lateral junctions along the collector is $S_L = 50$ m. A total of $n = 10$ laterals enter the collector, giving a collector length $L_c = nS_L = 500$ m. The drainage coefficient for the peak/rainy condition is $q = 10.3$ mm day⁻¹. Target hydraulics are free, partially full flow under non-pressurized conditions ($y/D \leq 0.7 - 0.8$), with self-cleansing velocity $V \geq 0.30$ m s⁻¹ and non-erosive velocity $V \leq 2.0$ m s⁻¹.

Determine:

1. The discharge from one lateral Q_L and the maximum collector discharge at the outlet $Q_{c,max} = nQ_L$;
2. a practical single pipe size D and slope S_0 that satisfy the self-cleansing and non-erosive criteria;
3. at the downstream reach (flow $Q_{c,max}$), the partial-flow velocity;
4. The normal depth y ;
5. at the upstream reach (flow Q_L), verify $V \geq 0.30$ m s⁻¹.

Solution

1. Discharge from one lateral

Contributing area per lateral

$$A_L = SL_L = 40 \times 200 = 8000 \text{ m}^2$$

Lateral design discharge

$$Q_L = 2.78 \times 10^{-7} q A_L = 2.78 \times 10^{-7} \times 0.429 \times 8000 \approx 9.54 \times 10^{-4} \text{ m}^3/\text{s}$$

2. Collector design discharges

$$Q_{c,max} = n \cdot Q_L = 10 \times 9.54 \times 10^{-4} = 9.54 \times 10^{-3} \text{ m}^3/\text{s}$$

Upstream-most reach (just after first lateral):

$$Q = Q_L = 9.54 \times 10^{-4} \text{ m}^3/\text{s}$$

We adopt the single-size collector approach (Eq. 3.17) but check both the downstream-most reach at $Q_{c,max}$ and the upstream-most reach at Q_L for velocity criteria.

3. Choose a practical trial diameter and slope, compute full-flow capacity

We will tune the slope to satisfy the velocity at the upstream reach (worst for self-cleansing). A slope of 0.5% is a practical agricultural grade, so try:

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} \Rightarrow 9.54 \times 10^{-3} = \frac{1}{0.013} \frac{\pi D^2}{4} \left(\frac{D}{4}\right)^{2/3} 0.005^{1/2} \Rightarrow D = 0.14 \text{ m}$$

Take $D = 0.16$ m

Check downstream

$$\text{Total area} = 0.02 \text{ m}^2, \text{ filled area} = \pi 0.14^2 / 4 = 0.015 \text{ m}^2$$

Filled area > (1/2) total area, the water surface lies above the center of the pipe.

$$A_1 = \pi r^2 \left(1 - \frac{\theta}{360}\right) + r \cdot y \cdot \sin \frac{\theta}{2}$$

$$150 = \pi 8^2 \left(1 - \frac{\theta}{360}\right) + 8 \cdot y \cdot \sin \frac{\theta}{2}$$

$$18.75 = \pi 8 \left(1 - \frac{\theta/2}{180}\right) + y \cdot \sin \frac{\theta}{2} \quad (1)$$

$$\cos \frac{\theta}{2} = \frac{y}{r} = \frac{y}{8} \quad (2)$$

| Assume y | Calculate $\left(\frac{\theta}{2}\right)$, Eq. (2) | Right hand of Eq. (1) |
|----------|---|-----------------------|
| y = 4 | 60.0 | 44.46 |
| y = 2 | 75.5 | 16.52 |
| y = 2.25 | 73.66 | 17.0 |
| y = 3.0 | 67.8 | 18.44 |

Therefore, 18.44 is the closest value to 18.75, and the corresponding depth of water in the pipe is 3 + 8 = 11.0 cm.

$$\text{Calculate, } V_{full} = 0.64 \text{ m/s}$$

From hydraulic relationships for partially full circular conduits (Fig. 3.5), obtain the corresponding ratios:

$$\frac{y}{D} = \frac{11}{16} = 0.69, \quad \frac{V}{V_{full}} = 0.95,$$

$$V = 0.94 \times 0.64 = 0.60 \frac{\text{m}}{\text{s}}$$

Result: $V \approx 0.60 \text{ m/s} > 0.30 \text{ m/s} \rightarrow$ self-cleansing satisfied.

Check upstream

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} \Rightarrow 9.54 \times 10^{-4} = \frac{1}{0.013} \frac{\pi D^2}{4} \left(\frac{D}{4}\right)^{2/3} 0.005^{1/2} \Rightarrow D = 0.06 \text{ m}$$

$$\text{Total area} = 0.02 \text{ m}^2, \text{ filled area} = \pi 0.06^2 / 4 = 2.83 \times 10^{-3} \text{ m}^2$$

Filled area < (1/2) total area, the water surface lies below the center of the pipe.

$$A_2 = \pi r^2 \frac{\theta}{360} - r \cdot y \cdot \sin \frac{\theta}{2}$$

$$28.27 = \pi 8^2 \frac{\theta/2}{180} - 8 \cdot y \cdot \sin \frac{\theta}{2}, \text{ area (cm}^2\text{), dividing both sides of the equation by } r$$

$$3.53 = \pi (8) \frac{\theta/2}{180} - y \cdot \sin \frac{\theta}{2} \quad (1)$$

$$\cos \frac{\theta}{2} = \frac{y}{r} = \frac{y}{8} \quad (2)$$

| Assume y | Calculate $\left(\frac{\theta}{2}\right)$, Eq. (2) | Right hand of Eq. (1) |
|----------|---|-----------------------|
| y = 6 | 41.4 | 1.81 |
| y = 5 | 51.3 | 3.26 |

Therefore, 3.26 is the closest value to 3.53, and the corresponding depth of water in the pipe is $8 - 5 = 3.0$ cm.

$$\frac{y}{D} = \frac{3}{16} = 0.19, \quad \frac{V}{V_{full}} = 0.48,$$

$$V = 0.48 \times 0.64 = 0.31 \frac{m}{s},$$

Result: $V \approx 0.31 \text{ m/s} > 0.30 \text{ m/s} \rightarrow$ self-cleansing satisfied.

4. Vertical Drainage (Drainage Wells)

4.1 Introduction

Vertical drainage, also known as drainage wells, vertical drains, or deep wells is a specialized groundwater-control technique used to lower the water table and improve soil drainage by providing a vertical pathway for water to flow from shallow, poorly drained layers into deeper, more permeable strata. Unlike surface and subsurface horizontal drains, which rely on lateral movement of water, vertical drainage systems take advantage of differences in hydraulic conductivity between geological layers, allowing excess groundwater to bypass low-permeability horizons.

Vertical drainage has become increasingly important in regions with fine-textured soils, deep water tables, or complex stratification where conventional surface drains, tile drains, or interceptor drains are ineffective or prohibitively expensive. The method is widely used in irrigated agriculture, urban development, land reclamation, and geotechnical dewatering for construction projects.

The primary objective of a vertical drainage system is to accelerate the removal of excess groundwater in order to:

- Prevent waterlogging and soil salinity.
- Improve aeration in the root zone for better crop growth.
- Stabilize foundations and embankments by reducing pore-water pressures.
- Enhance the efficiency of surface and subsurface irrigation systems.

A typical drainage well penetrates through an upper confining layer of low permeability (e.g., clay or silt) and discharges water into an underlying aquifer composed of sand or gravel with much higher hydraulic conductivity. As a result, drainage wells act as controlled vertical conduits, redistributing groundwater within the soil profile rather than removing it from the system.

For vertical drainage to be effective, several hydrogeologic conditions must be satisfied:

1. Presence of an upper poorly permeable layer where water accumulates (perched or shallow water table).
2. Existence of a deep permeable aquifer with sufficient transmissivity to receive and convey the drained water.
3. A hydraulic gradient between the upper and lower layers that allows downward flow.
4. Adequate well construction, including screens, gravel packing, casing, and development to ensure efficient operation.

Without these conditions, vertical drainage becomes ineffective or may even lead to problems such as clogging, reduced well yield, or groundwater mounding.

4.2 Principles of Vertical Drainage

Vertical drainage operates on the concept of establishing a controlled pathway that enables excess groundwater from perched or shallow saturated zones to move downward into deeper, more transmissive formations. By installing a properly designed drainage well, the system creates a hydraulic connection that bypasses the natural restrictions imposed by low-permeability layers, allowing groundwater to migrate vertically rather than relying solely on slower horizontal movement.

At the core of vertical drainage is the induced vertical hydraulic gradient. When a well is constructed to intersect both the upper saturated layer and an underlying aquifer with a lower potentiometric level, a preferential flow path is created. Water from the higher-head zone naturally drains downward through the wellbore and into the deeper aquifer, where it is dispersed rapidly due to its higher permeability and greater transmissive capacity. Thus, the drainage well acts as a conduit that accelerates the redistribution of groundwater and effectively lowers the water table in the overlying soil.

In practical application, vertical drainage systems rely on careful well design and construction to maintain efficient operation. Key design considerations include proper sizing and placement of screens to match the hydraulic properties of the intercepted strata, adequate gravel packing for enhanced filtration and stability, and secure casing to ensure structural integrity. Well development procedures, such as surging or pumping, are essential to remove fines, improve hydraulic connectivity, and achieve optimal well performance.

Once in operation, vertical drainage wells contribute to improved soil aeration, reduced waterlogging, and enhanced root zone conditions benefits of particular importance in irrigated agriculture and areas with stratified soils. Additionally, vertical drainage can serve as an effective complement to horizontal subsurface drains by providing rapid relief during peak recharge periods or in locations where lateral groundwater movement is constrained.

4.2.1 Concept of Water Table Control by Pumping

Water table control by pumping is a fundamental mechanism underlying the operation of vertical drainage wells. The principle is based on reducing the groundwater head in an upper saturated zone by continuously or intermittently extracting water through a well that extends into a deeper, permeable aquifer. As water is pumped from the well, the hydraulic head within the deep aquifer decreases locally, creating a cone of depression around the well. This drop in head induces downward flow from the overlying saturated layers, effectively lowering the water table in the area of influence.

The pumping process alters the natural groundwater flow system by enhancing the vertical hydraulic gradient between the upper water table and the lower aquifer. As pumping continues, groundwater moves through the well screen from both the upper and lower formations, depending on the well design and the relative hydraulic heads. In systems where the well is screened across both strata, water from the upper zone

drains directly into the deeper aquifer through the wellbore, thereby accelerating the dissipation of excess moisture in the root zone.

The extent of water table lowering depends on several factors, including the pumping rate, the transmissivity of the deeper aquifer, the hydraulic characteristics of the overlying layers, and the spacing of additional wells if operated as part of a well field. A sustained pumping rate large enough to maintain a differential head will continue to promote vertical drainage, preventing the reestablishment of high water tables even during periods of recharge.

Water table control by pumping offers several advantages in soils with restricted horizontal permeability or in areas where conventional subsurface drains cannot achieve sufficient drawdown. By actively managing groundwater levels, pumping helps restore soil aeration, improve trafficability, and create favorable conditions for plant growth. In many agricultural settings, pumping wells are operated strategically during irrigation seasons or periods of high rainfall to maintain the desired water table depth.

4.2.2 Advantages and Limitations Compared to Horizontal Drainage

While horizontal subsurface drainage remains the most widely used method for controlling shallow water tables in agricultural lands, vertical drainage presents a viable alternative in situations where traditional systems are less effective or impractical. Understanding how vertical drainage compares with horizontal drainage is essential for selecting the appropriate strategy for site-specific groundwater management.

Vertical drainage offers several notable advantages, particularly in soils with pronounced stratification or restricted horizontal permeability. By utilizing wells that penetrate deeper, highly transmissive aquifers, vertical drainage can achieve drawdowns that are difficult or impossible to obtain with buried drains. The limited land area required for well installation also makes vertical systems attractive in intensively cultivated fields or built-up areas where constructing a network of horizontal drains would cause significant disruption. In addition, the capacity for pumping allows vertical systems to respond rapidly to sudden increases in recharge, providing greater operational flexibility during wet periods or irrigation seasons.

However, vertical drainage also exhibits important limitations when compared with horizontal drainage. Its effectiveness depends heavily on the presence of a suitable deep aquifer capable of receiving the drained water, a condition not universally available. Construction and operation costs are generally higher due to drilling requirements and the need for continuous or intermittent pumping. Vertical wells also demand more rigorous maintenance to prevent clogging and ensure sustained performance. Finally, the zone of influence of a single well is relatively limited, often necessitating multiple wells to achieve uniform water table control across large areas.

In summary, vertical drainage complements rather than replaces horizontal drainage. Its use is most justified where deeper aquifers provide favorable hydraulic conditions or where horizontal drains cannot achieve the desired performance. A clear

understanding of both systems' capabilities allows engineers to design integrated drainage solutions tailored to local hydrogeologic and land-use conditions.

4.3 Types of Wells for Drainage

Wells used for vertical drainage vary in design, depth, and function depending on the groundwater conditions they are intended to control. They range from shallow wells that relieve perched or near-surface water tables to deep drainage wells that penetrate permeable aquifers capable of transmitting large volumes of water. Understanding the distinctions between these well types is essential for selecting the appropriate system for effective groundwater management.

4.3.1 Shallow Wells

Shallow wells represent the simplest form of vertical drainage and are typically constructed to intercept and lower perched or locally elevated water tables that lie relatively close to the ground surface. These wells extend only a short distance below the upper saturated zone and do not penetrate deeper regional aquifers. As a result, they primarily serve to drain excess water that accumulates above a restrictive layer, such as a compacted clay horizon or a shallow hardpan.

Construction of shallow wells is generally straightforward and economical. They may be excavated by hand, augered, or drilled using light equipment, and their depths usually range from a few meters up to about 10–15 meters, depending on the thickness of the perched zone. The wells are commonly finished with perforated casing or a short screen section placed opposite the water-bearing layer, while gravel packing is added to enhance intake capacity and minimize sediment entry.

These wells are particularly suited for:

- Relieving waterlogging in stratified soils where horizontal movement is constrained.
- Dewatering construction sites or small agricultural plots with shallow perched water.
- Supplementing horizontal drains by providing localized relief in problem areas.

However, their effectiveness is inherently limited by the modest depth of influence and the absence of connection to a deeper, high-transmissivity aquifer. For larger-scale or long-term groundwater control, deeper drainage wells may be required.

4.3.2 Deep Wells

Deep wells are designed to penetrate permeable aquifers located well below the ground surface and serve as the primary means of achieving substantial and sustained lowering of groundwater levels in vertical drainage systems. Unlike shallow wells, which drain only perched or near-surface water tables, deep wells establish a direct

hydraulic connection between upper saturated zones and a deeper, more transmissive formation capable of accepting and conveying large quantities of water.

These wells are typically drilled using rotary or cable-tool methods to depths that may range from several tens to hundreds of meters, depending on local hydrogeology. They are constructed with screened intervals placed within the target aquifer, supported by properly graded gravel packing to ensure efficient inflow and minimize clogging. Strong, corrosion-resistant casings are required to maintain structural integrity under varying hydraulic pressures and long-term pumping conditions.

Deep wells usually operate with mechanical pumping to induce a sufficient downward hydraulic gradient, creating a cone of depression that extends upward into the shallow saturated zones. This drawdown lowers the water table over a much larger radius of influence compared with shallow wells, making deep wells particularly effective in areas where horizontal drainage is limited by low lateral permeability or deep-rooted crops require significant drainage depth.

While deep wells offer high drainage capacity and broad-area control, they also demand careful design, continuous monitoring, and regular maintenance due to their complexity and higher operational costs.

4.4 Flow to Wells

Flow to wells refers to the movement of groundwater toward a pumped or drained well as a result of the hydraulic gradient created by water withdrawal. When a well begins to operate, groundwater flows radially inward, forming a characteristic drawdown pattern around the well. The behavior of this flow depends on the type of aquifer, confined or unconfined, as well as whether conditions are steady or unsteady. Understanding the principles governing flow to wells provides the basis for analyzing well performance, estimating discharge, and designing effective drainage and dewatering systems.

4.4.1 Flow in Confined Aquifers

Confined aquifers are groundwater-bearing layers bounded above and below by relatively impermeable strata (aquitards or aquicludes). Water within these aquifers is stored under pressure, so the hydraulic head is represented by a piezometric surface rather than a free water table. When a pumped well fully penetrates a confined aquifer, the withdrawal of water lowers the piezometric head and induces radial horizontal flow toward the well.

Before pumping, the piezometric surface is generally horizontal, reflecting uniform pressure conditions throughout the aquifer. Once pumping begins at a constant rate, a cone of depression forms in the piezometric surface, creating a radial hydraulic gradient. Groundwater then flows horizontally through the confined layer toward the well screen, as illustrated in Fig. 4.1. The figure shows the initial and pumped piezometric surfaces, the fully penetrating well, and the constant-thickness confined aquifer situated between low-permeability layers.

Because the saturated thickness of a confined aquifer remains constant during pumping, the analysis of flow is simplified compared with unconfined systems. Flow behavior is controlled primarily by two properties: the hydraulic conductivity (K) and the aquifer thickness (b). Together, these define the transmissivity ($T = Kb$), which governs the capacity of the aquifer to transmit water to the well.

The conceptual principles described in this section provide the basis for analyzing confined well hydraulics. The full mathematical derivation of the steady-state radial flow equation for confined aquifers (Thiem equation) is presented later in Section 4.4.3.

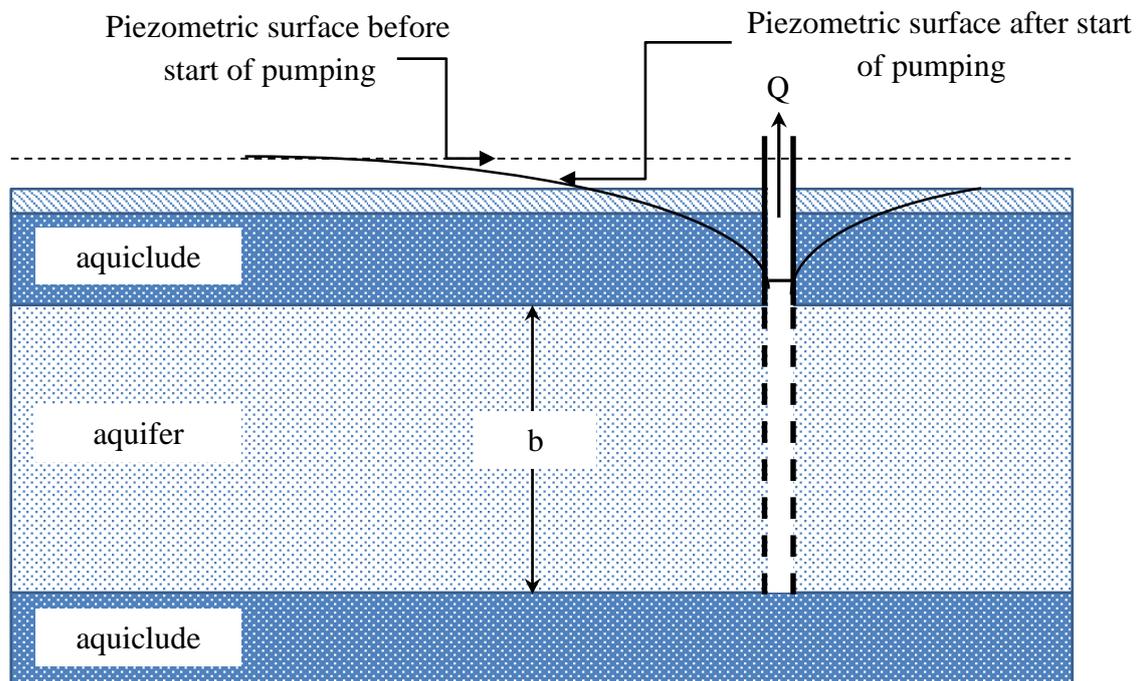


Fig. 4.1 Cross-section of a pumped confined aquifer

Fig. 4.1 illustrates the hydraulic behavior of a fully penetrating well pumping from a confined aquifer. The aquifer is bounded above and below by aquicludes. An aquiclude is a geological layer composed of rock or sediment that acts as a barrier to groundwater flow. It typically consists of materials with very low porosity and very low permeability, such as clay or shale. Although an aquiclude may store water due to its fine-grained nature, it has negligible ability to transmit water, making its transmitting capacity extremely low. Prior to pumping, the piezometric surface is horizontal, reflecting uniform pressure conditions. When pumping begins at a constant discharge Q , the piezometric surface declines near the well, forming a cone of depression whose curvature represents the radial flow of water toward the well screen. Because the confining layers prevent vertical drainage, flow within the aquifer remains predominantly horizontal, and the saturated thickness b remains unchanged during pumping. The figure clearly shows the drawdown pattern, the steady lowering

of hydraulic head around the well, and the structural relationship between the aquifer and its confining boundaries.

4.4.2 Flow in Unconfined Aquifers

In unconfined aquifers, the upper boundary is the water table, which is free to rise or fall in response to pumping. Unlike confined systems, the saturated thickness of an unconfined aquifer changes during pumping because water is withdrawn directly from the saturated zone. As pumping begins, the water table declines, forming a characteristic cone of depression that represents the vertical and radial movement of groundwater toward the well.

Because the water table itself is a sloping free surface, flow in unconfined aquifers involves both horizontal and vertical components. The hydraulic conductivity of the upper, newly drained zone may differ from that of the deeper saturated zone, and this variability often makes unconfined flow more complex than confined flow. The drawdown causes the saturated thickness to decrease near the well, resulting in a nonlinear relationship between discharge and hydraulic head.

Fig. 4.2 illustrates these behaviors by showing the initial water table prior to pumping and the depressed water table after pumping begins. The flow toward the well occurs through the saturated portion of the aquifer, while vertical drainage from the unsaturated zone contributes gradually to the pumped discharge. This dual flow regime distinguishes unconfined aquifers from confined ones and forms the basis for the analytical treatment of unconfined well hydraulics presented later in Section 4.4.3.

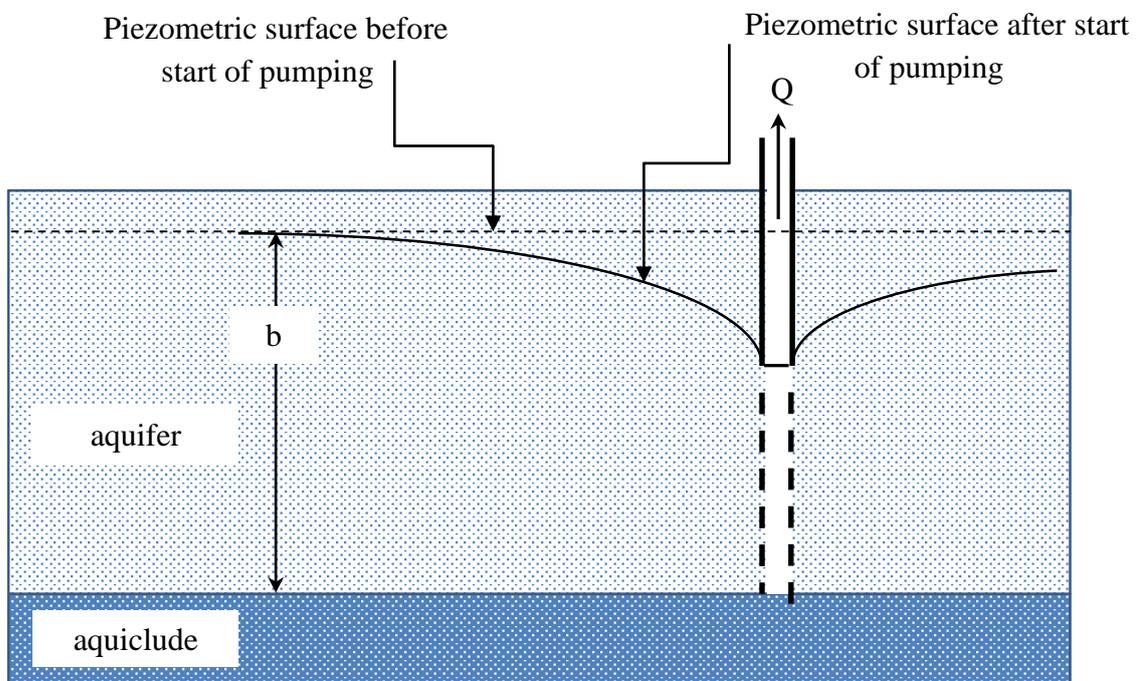


Fig. 4.2 Cross-section of a pumped unconfined aquifer

4.4.3 Steady-State Flow Conditions

Steady-state flow in a confined aquifer is attained when a pumped well has operated long enough for the drawdown to stabilize, such that the hydraulic head at any point no longer changes with time. Under these conditions, the rate of groundwater flow toward the well equals the pumping rate, and the cone of depression assumes a fixed shape. Because a confined aquifer maintains a constant saturated thickness during pumping, the analysis of radial flow becomes relatively straightforward.

A classic analytical solution for steady-state flow to a fully penetrating well in a confined aquifer is provided by the Thiem equation. This solution is based on the following assumptions:

1. The aquifer is confined above and below by impermeable layers.
2. The aquifer extends infinitely in the horizontal direction.
3. The aquifer is homogeneous, isotropic, and of uniform thickness.
4. The piezometric surface is horizontal prior to pumping.
5. Pumping occurs at a constant discharge rate Q .
6. The well penetrates the full thickness of the aquifer so that flow into the well is entirely horizontal (Fig. 4.3).

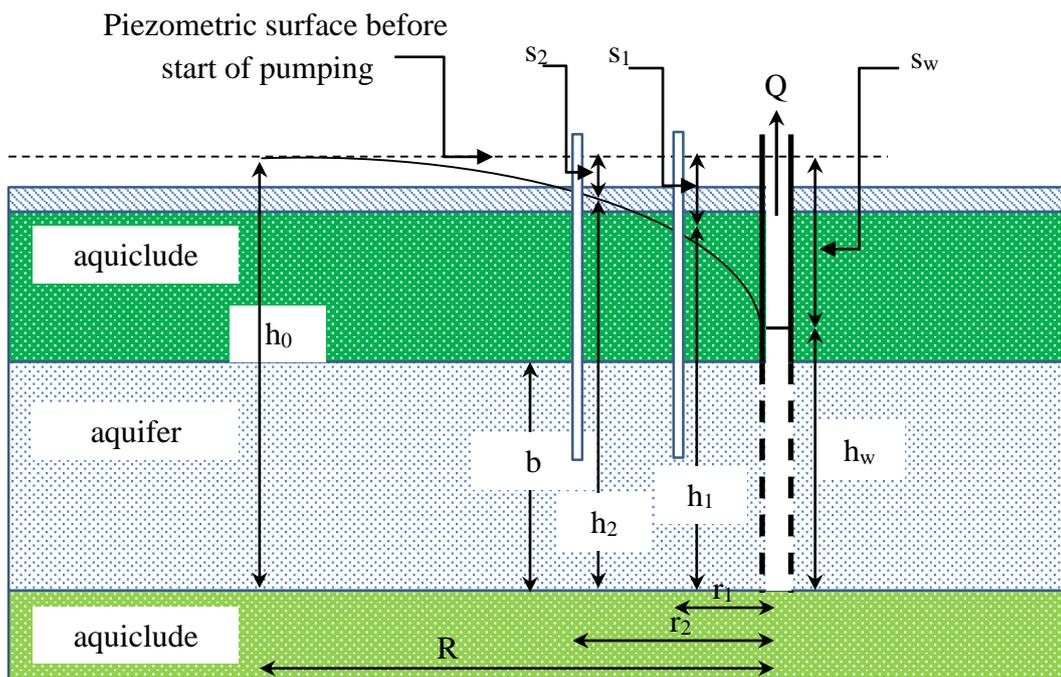


Fig. 4.3 Steady-state drawdown and horizontal flow toward a fully penetrating well in a confined aquifer

The well penetrates the full thickness of the aquifer so that flow into the well is entirely horizontal (Fig. 4.3). Under these conditions, the hydraulic behavior around the well can be described analytically by applying Darcy's law to radial flow in a confined system. This leads to the classical Thiem equation, derived as follows.

Under radial steady-state flow, Darcy's law can be expressed as:

$$q = K \frac{\partial h}{\partial r} \quad (4.1)$$

Where

q : Specific discharge in the radial direction,

K : Hydraulic conductivity,

h : Hydraulic head,

r : Radial distance from the well.

For a cylindrical control surface of radius r and thickness b :

$$Q = 2\pi r b q \quad (4.2)$$

Substituting Eq. (4.1) into Eq. (4.2) gives:

$$Q = 2\pi r b K \frac{\partial h}{\partial r} \quad (4.3)$$

Rearranging:

$$\frac{Q}{2\pi K b} \frac{1}{r} dr = dh \quad (4.4)$$

Integrating between two piezometers located at distances r_1 and r_2 , with corresponding heads h_1 and h_2 :

$$\frac{Q}{2\pi K b} \ln\left(\frac{r_2}{r_1}\right) = h_2 - h_1 \quad (4.5)$$

Because drawdown $s = h_0 - h$, the equation may be expressed in field-measured form as:

$$\frac{Q}{2\pi K b} \ln\left(\frac{r_2}{r_1}\right) = s_1 - s_2 \quad (4.6)$$

Equation (4.6) is the Thiem equation, widely used for steady-state pumping-test analysis and for estimating aquifer transmissivity.

Rearranging the Thiem equation yields:

$$T = Kb = \frac{Q}{2\pi(s_1 - s_2)} \ln\left(\frac{r_2}{r_1}\right) \quad (4.7)$$

To minimize errors caused by well losses at the pumped well, the use of two or more piezometers is recommended.

For practical applications, the Thiem equation is often applied using two boundary conditions:

1. At the well radius r_w , the hydraulic head is h_w .
2. At the radius of influence R_0 , the head equals the original piezometric level h_0

Using these conditions, the steady-state drawdown in the pumped well becomes:

$$s_w = h_0 - h_w = \frac{Q}{2\pi T} \ln\left(\frac{R_0}{r_w}\right) \quad (4.8)$$

This form is valuable for estimating well efficiency, determining required pumping rates, and predicting long-term drawdown for confined aquifer drainage or dewatering systems.

The radius of influence R_0 is the outer limit of the cone of depression, representing the distance from the pumping well beyond which the drawdown becomes negligible and the piezometric head remains essentially unchanged from its original value h_0 . In other words, R_0 defines the boundary of the area affected by pumping and marks the extent of the aquifer region contributing water to the well. Although R_0 cannot be directly observed in the field, it is typically estimated from steady-state analytical solutions, empirical relationships, or data obtained from pumping tests.

When a well pumps from an unconfined aquifer, the water table itself declines and forms a sloping free surface, creating a characteristic cone of depression (Fig. 4.4). Because the saturated thickness decreases near the well, the hydraulic behavior differs from that of confined aquifers and requires special analytical treatment. A widely used approach is that developed by Dupuit and later refined by Forchheimer, which simplifies the flow field under steady-state conditions.

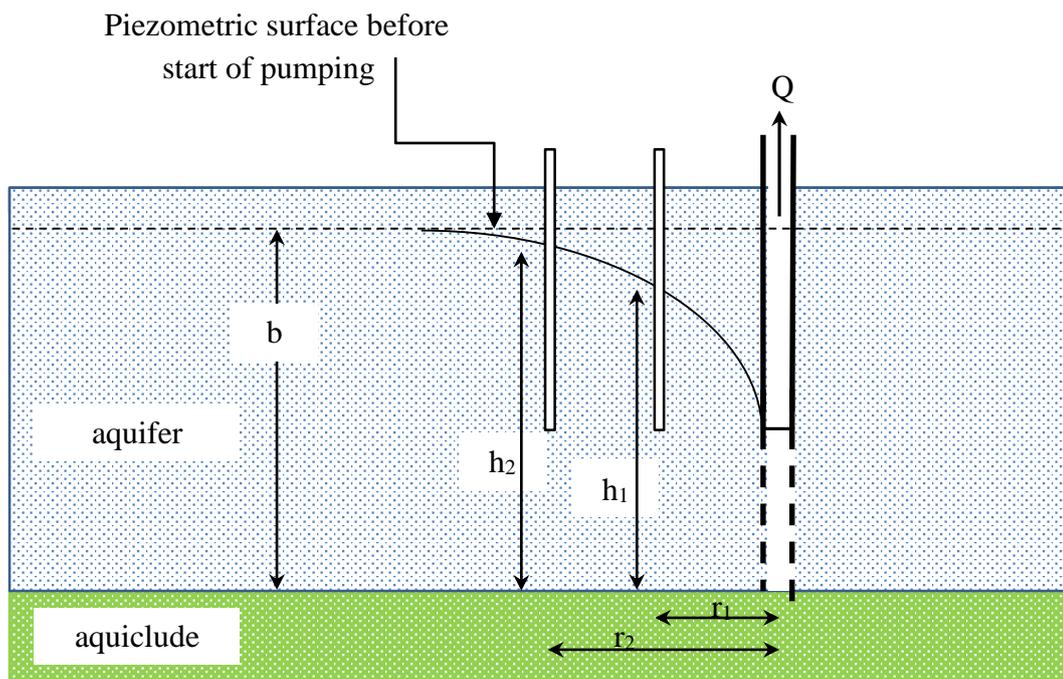


Fig. 4.4 Steady-state drawdown and radial flow toward a fully penetrating well in an unconfined aquifer

Assumptions

The Dupuit–Forchheimer analysis is based on the following assumptions:

1. The aquifer is unconfined.
2. It extends laterally to great distances (infinite aerial extent).

3. The aquifer is homogeneous, isotropic, and of uniform thickness.
4. The initial water table is horizontal before pumping begins.
5. The aquifer is pumped at a constant rate Q .
6. The well fully penetrates the aquifer so that water is received from the entire saturated thickness (Fig. 4.4).
7. Flow lines are horizontal and parallel to the underlying impermeable layer.
8. The hydraulic gradient is taken as equal to the slope of the water table, which must be small for the approximation to hold.

Under steady radial flow, Darcy's law for an unconfined aquifer is expressed as:

$$q = K \frac{\partial h}{\partial r} \quad (4.9)$$

The continuity equation for a cylindrical surface of radius r gives:

$$Q = 2\pi r h q \quad (4.10)$$

Eliminating q and substituting Eq. (4.9):

$$Q = 2\pi r h K \frac{\partial h}{\partial r} \quad (4.11)$$

Rearranging:

$$\frac{Q}{2\pi K} \frac{1}{r} dr = h dh \quad (4.12)$$

Integrating between two observation radii r_1 and r_2 , with corresponding water table elevations h_1 and h_2 , yields the Dupuit formula:

$$\frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right) = h_2^2 - h_1^2 \quad (4.13)$$

This equation describes the steady-state water-table configuration around a pumping well in an unconfined aquifer.

Limitations of the Dupuit–Forchheimer Method

While useful and widely applied, the Dupuit approximation has important limitations:

- Near the well, the water table curvature becomes steep, violating the assumption of small slope; thus Eq. (4.13) does not accurately describe the immediate vicinity of the well.
- Steady-state conditions in unconfined aquifers develop slowly, often requiring a long period of pumping before a stable cone of depression is established.
- Vertical components of flow, which become significant near the well, are neglected in this method.

Despite these limitations, the Dupuit–Thiem equation provides a practical and reasonably accurate tool for estimating aquifer properties and analyzing flow toward wells under steady-state conditions.

Example 4.1

A fully penetrating well is pumped at a constant discharge rate of ($Q = 1200 \text{ m}^3/\text{day}$), from a confined aquifer of uniform thickness (20 m). After the well has operated long enough, steady-state conditions are reached and the drawdown becomes time-

invariant. Two observation piezometers installed in the aquifer record the following data:

Distance to first piezometer: $r_1 = 50\text{m}$, measured head $h_1 = 36.856\text{ m}$

Distance to second piezometer: $r_2 = 150\text{ m}$, measured head $h_2 = 38.356\text{ m}$

The original (static) piezometric level before pumping was 40.0 m . The effective radius of the pumped well is $r_w = 0.20\text{ m}$, and the radius of influence of pumping is estimated to be $R_0 = 500\text{ m}$.

Required:

1. Determine the transmissivity T and hydraulic conductivity K of the aquifer using the Thiem equation for steady-state flow.
2. Compute the steady-state drawdown in the pumped well, s_w , and determine the corresponding water level in the well.

Solution

1. Determination of Transmissivity and Hydraulic Conductivity

$$s = h_0 - h, s_1 = 40 - 36.856 = 3.144\text{ m}, s_2 = 40 - 38.356 = 1.644\text{ m}$$

$$T = Kb = \frac{Q}{2\pi(s_1 - s_2)} \ln\left(\frac{r_2}{r_1}\right)$$

$$T = \frac{1200}{2\pi(1.5)} \ln\left(\frac{150}{50}\right) \approx 140\text{ m}^2/\text{day}$$

$$\text{Hydraulic conductivity: } K = \frac{T}{b} = \frac{140}{20} = 7\text{ m/day}$$

2. Drawdown in the Pumped Well

Applying the Thiem equation with boundary conditions at $r = r_w$ and $r = R_0$:

$$s_w = \frac{Q}{2\pi T} \ln\left(\frac{R_0}{r_w}\right) = \frac{1200}{2\pi(140)} \ln\left(\frac{500}{0.2}\right) \approx 10.7\text{ m}$$

The steady-state water level in the well is therefore:

$$h_w = h_0 - s_w = 40 - 10.7 = 29.3\text{ m}$$

Example 4.2

A fully penetrating well of effective radius $r_w = 0.20\text{ m}$, is installed in a confined aquifer. The aquifer has a known transmissivity of $T = 250\text{ m}^2/\text{day}$, based on previous pumping tests and laboratory data. The initial (static) piezometric level in the aquifer is 40.0 m above datum. The well is pumped at a constant discharge of $Q = 2000\text{ m}^3/\text{day}$, until steady-state conditions are reached. At steady state, the measured water level in the pumped well is $h_w = 30.3\text{ m}$ above datum.

Required:

1. Determine the radius of influence, R_0 , of the pumping well.
2. Compute the steady-state drawdown at a piezometer located 100 m from the well.

Solution

1. Radius of Influence R_0

$$s_w = h_0 - h_w = 40 - 30.3 = 9.7 \text{ m}$$

For a confined aquifer under steady-state flow, the Thiem equation for the pumped well is:

$$s_w = \frac{Q}{2\pi T} \ln\left(\frac{R_0}{r_w}\right)$$

Rearrange to solve for the radius of influence R_0 :

$$\ln\left(\frac{R_0}{r_w}\right) = \frac{2\pi T}{Q} s_w$$

$$R_0 = r_w \exp\left(\frac{2\pi T}{Q} s_w\right) = 0.2 \exp\left(\frac{2\pi(250)}{2000} 9.7\right) = 407 \text{ m}$$

2. Drawdown at $r = 100 \text{ m}$

The steady-state drawdown at any radial distance r in a confined aquifer is given by:

$$s(r) = \frac{Q}{2\pi T} \ln\left(\frac{R_0}{r}\right) = \frac{2000}{2\pi(250)} \ln\left(\frac{407}{100}\right) \approx 1.8 \text{ m}$$

The corresponding piezometric head at $r = 100 \text{ m}$ is:

$$h(100) = h_0 - s(100) = 40.0 - 1.8 = 38.2 \text{ m}$$

Example 4.3

A fully penetrating well pumps from an unconfined sandy aquifer under steady-state conditions. The aquifer is underlain by an impermeable layer, and flow satisfies the Dupuit–Forchheimer assumptions. The initial (static) water table is horizontal (15.0 m, above the impermeable base). The well is pumped at a constant discharge of $Q = 900 \text{ m}^3/\text{day}$. After sufficient time, steady-state conditions are reached. Two observation wells, screened over the full saturated thickness, record the following water-table elevations above the impermeable base:

Distance to first piezometer: $r_1 = 40\text{m}$, measured head $h_1 = 13.2 \text{ m}$

Distance to second piezometer: $r_2 = 120 \text{ m}$, measured head $h_2 = 14.4 \text{ m}$

Assume that the aquifer is homogeneous, isotropic, and of large areal extent, and that flow complies with the Dupuit–Forchheimer assumptions.

Required:

Determine the hydraulic conductivity K of the aquifer using the Dupuit equation.

Solution

For steady radial flow to a well in an unconfined aquifer, the Dupuit formula between two observation radii r_1 and r_2 with corresponding water-table elevations h_1 and h_2 (measured above the impermeable base) is:

$$\frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right) = h_2^2 - h_1^2$$

Solving for K :

$$K = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)} = \frac{900 \ln\left(\frac{120}{40}\right)}{\pi(207.36 - 174.24)} = 9.5 \text{ m/day}$$

4.4.4 Unsteady Flow Conditions

When a well penetrating a confined aquifer is pumped at a constant discharge rate, the influence of pumping does not reach the entire aquifer instantaneously. Instead, the decline of hydraulic head propagates outward gradually with time. As pumping continues, an expanding cone of depression is formed, and the zone affected by the pumping well known as the radius of influence increases continuously.

In confined aquifers, water released to the well under unsteady conditions originates primarily from the elastic storage of the aquifer matrix and the compressibility of the water itself. Because the available volume of stored water grows with the expanding cone of depression, the hydraulic head continues to decline as long as pumping proceeds, although the rate of decline decreases with time due to the increasing area contributing water.

Thus, whenever the pumping duration is insufficient to reach hydraulic equilibrium, or when the aquifer is too extensive for stabilization to occur within the time scale of pumping, the flow regime is described as unsteady, transient, or non-equilibrium radial flow.

Fig. 4.5 illustrates the geometry of instantaneous drawdown around a well fully penetrating a confined aquifer of thickness b . Consider radial flow through an annular cylindrical element of soil located at a distance r from the center of the well, having thickness dr .

Flow Across the Annular Ring

Let:

- Q_1 : flow entering the outer face of the annular ring
- Q_2 : flow leaving the inner face
- h : piezometric head at radius r
- b : confined aquifer thickness
- S : storage coefficient of the aquifer

As pumping continues, the cone of depression deepens, and the water stored in the annular volume decreases. The difference between inflow and outflow therefore represents the rate of water released from storage within that cylindrical shell.

From the principle of continuity,

$$Q_1 - Q_2 = \frac{\partial V}{\partial t} \quad (4.14)$$

which states that the difference between inflow and outflow equals the rate of change of the volume of water stored within the annular element.

At the inner surface r , the radial hydraulic gradient is

$$i_r = \frac{\partial h}{\partial r} \quad (4.15)$$

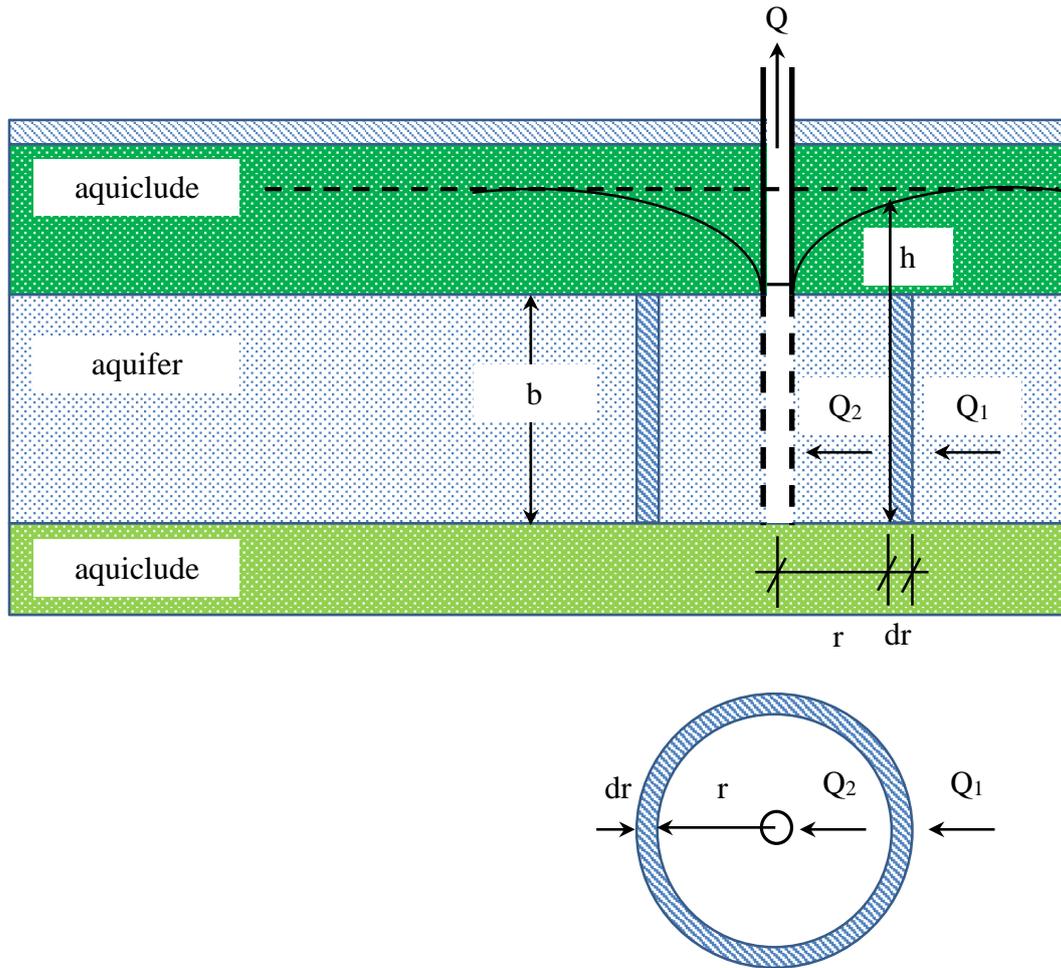


Fig. 4.5 Flow geometry of unsteady radial flow toward a pumping well in a confined aquifer

At the outer surface $r + dr$, the gradient can be written by a Taylor expansion as

$$i_{r+dr} = \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \quad (4.16)$$

Using Darcy's law, for radial flow through a cylindrical surface of radius r and thickness b , the area is $2\pi rb$. Thus:

Inflow at $r + dr$:

$$Q_1 = K \left(\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \right) 2\pi(r + dr)b \quad (4.17)$$

Outflow at r :

$$Q_2 = K \left(\frac{\partial h}{\partial r} \right) 2\pi r b \quad (4.18)$$

The storage coefficient S of a confined aquifer is defined as the volume of water released from storage per unit surface area of aquifer per unit decline in head. For the annular element:

$$\begin{aligned} dV &= S(2\pi r dr) dh \\ \frac{\partial V}{\partial t} &= S(2\pi r dr) \frac{\partial h}{\partial t} \end{aligned} \quad (4.19)$$

Substitute Eqs. (4.17) and (4.18) into the continuity Eq. (4.14), and equate to Eq. (4.19):

$$K \left(\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \right) [2\pi(r + dr)b] - K \left(\frac{\partial h}{\partial r} \right) [2\pi r b] = S(2\pi r dr) \frac{\partial h}{\partial t}$$

Divide both sides by $2\pi b dr$:

$$K \left(\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \right) \left[\frac{r+dr}{dr} \right] - K \left(\frac{\partial h}{\partial r} \right) \left[\frac{r}{dr} \right] = \frac{Sr}{b} \frac{\partial h}{\partial t}$$

Expand the term on the left and neglect higher-order small quantities involving dr^2 .

After simplification, we obtain:

$$K \left(r \frac{\partial^2 h}{\partial r^2} + \frac{\partial h}{\partial r} \right) = \frac{Sr}{b} \frac{\partial h}{\partial t}$$

Divide through by Kr :

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{Kb} \frac{\partial h}{\partial t}$$

Recognizing that transmissivity T is defined as $T = Kb$

we finally obtain the governing differential equation for unsteady radial flow in a confined aquifer:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \quad (4.20)$$

One of the most widely applied transient solutions is the Theis (1935) non-equilibrium equation, derived by analogy between groundwater flow in radial coordinates and heat conduction. By conceptualizing the pumping well as a mathematical line sink of constant discharge and imposing appropriate boundary conditions, Theis obtained the following expression for drawdown:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} du \quad (4.21)$$

This integral represents the well function, $W(u)$, and the equation is commonly written in the form:

$$s = \frac{Q}{4\pi T} W(u) \quad (4.22)$$

The dimensionless parameter u is defined as:

$$u = \frac{r^2 S}{4Tt} \quad (4.23)$$

Where

s : Drawdown at distance r and time t

Q : Constant pumping rate

T : Aquifer transmissivity

S : Aquifer storativity (A dimensionless parameter that describes how much water an aquifer releases or stores per unit surface area per unit change in hydraulic head).

r : Distance from pumping well to observation point

t : Time since pumping began

A. Assumptions of the Theis Solution

The validity of the Theis method depends on several simplifying assumptions:

1. The potentiometric surface is initially horizontal (no regional gradient).
2. The aquifer is confined and extends laterally to an effectively infinite distance.
3. The aquifer is homogeneous, isotropic, and of uniform saturated thickness.
4. Pumping occurs at a constant rate.
5. The pumped well fully penetrates the aquifer.
6. Water released from storage responds instantaneously to changes in hydraulic head.
7. Wellbore storage effects are negligible due to small well diameter.

B. Required Data for Applying the Theis Method

To determine transmissivity T and storativity S , the following information is needed:

1. A time series of drawdown measurements at an observation well.
2. The radial distance between the pumping well and the observation well.
3. The pumping rate maintained during the test.

These data are typically obtained during a constant-rate pumping test.

C. Graphical Procedure Using the Theis Type Curve

Theis introduced a type-curve matching technique, which remains a standard method of transient aquifer analysis. The approach relies on comparing field data to the theoretical form of $W(u)$ versus $1/u$, plotted on logarithmic scales.

Step 1: Plot Field Drawdown Data

On log–log graph paper:

- Plot measured drawdown s on the vertical axis.
- Plot time t on the horizontal axis.

This results in a smooth, rising curve representing the aquifer's transient response to pumping (similar to typical field data plots shown in Fig. 4.6).

Step 2: Plot the Theis Type Curve

On separate log–log paper (printed or tabulated (Table 4.1)), plot the theoretical relationship:

$$W(u) \text{ vs. } \frac{1}{u}$$

This curve is dimensionless and exhibits the characteristic shape shown in classical Theis type-curve charts (e.g., Fig. 4.7).

Step 3: Superimpose the Curves

Keeping axes parallel and scales aligned, overlay the transparent type curve on the field data plot. Shift the type curve horizontally and vertically until a satisfactory match is achieved across the majority of the data points (Fig. 4.8 illustrates a typical matching result).

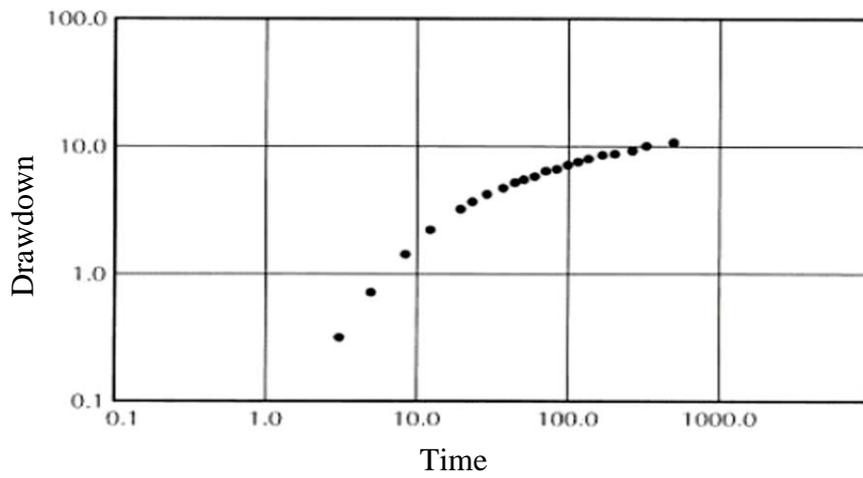


Fig. 4.6 Drawdown–time field data plotted on log–log paper

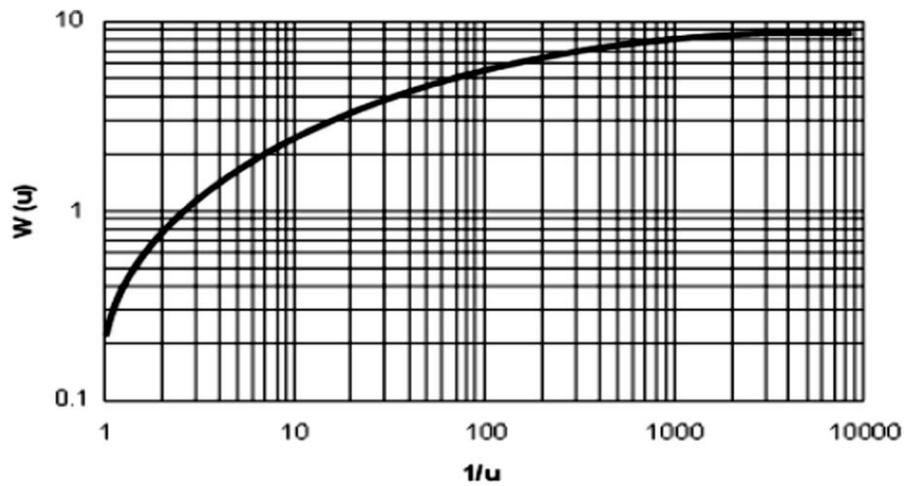


Fig. 4.7 Theis non-equilibrium type curve for a fully confined aquifer

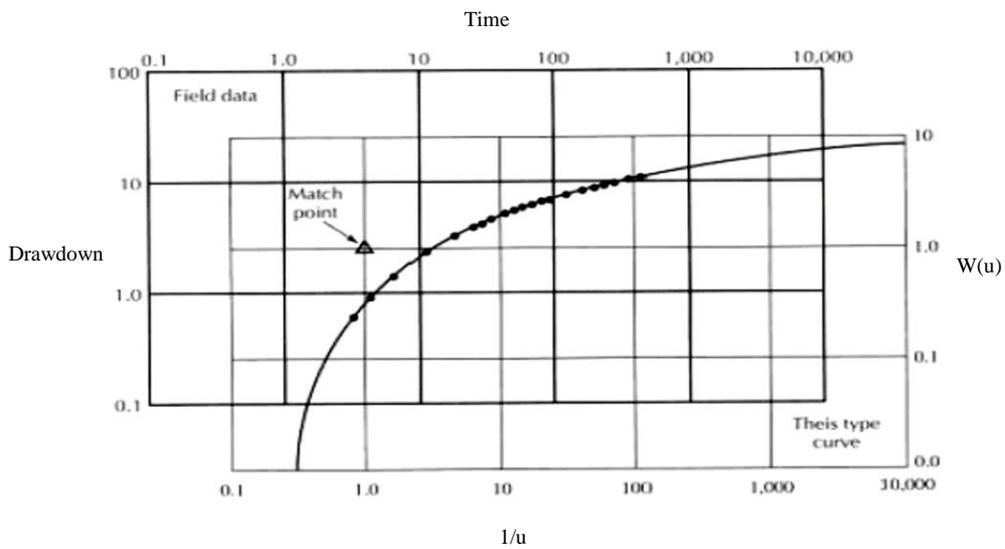


Fig. 4.8 Matching field data to the Theis type curve

Table 4.1 Function W(u) for selected values of u.

| u | W(u) | u | W(u) | u | W(u) | u | W(u) |
|---------------------|-------|--------------------|-------|--------------------|------|--------------------|-------|
| 1×10^{-10} | 22.45 | 7×10^{-8} | 15.90 | 4×10^{-5} | 9.55 | 1×10^{-2} | 4.04 |
| 2 | 21.76 | 8 | 15.76 | 5 | 9.33 | 2 | 3.35 |
| 3 | 21.35 | 9 | 15.65 | 6 | 9.14 | 3 | 2.96 |
| 4 | 21.06 | 1×10^{-7} | 15.54 | 7 | 8.99 | 4 | 2.68 |
| 5 | 20.84 | 2 | 14.85 | 8 | 8.86 | 5 | 2.47 |
| 6 | 20.66 | 3 | 14.48 | 9 | 8.74 | 6 | 2.30 |
| 7 | 20.50 | 4 | 14.15 | 1×10^{-4} | 8.63 | 7 | 2.15 |
| 8 | 20.37 | 5 | 13.93 | 2 | 7.94 | 8 | 2.03 |
| 9 | 20.25 | 6 | 13.75 | 3 | 7.53 | 9 | 1.92 |
| 1×10^{-9} | 20.15 | 7 | 13.60 | 4 | 7.25 | 1×10^{-1} | 1.823 |
| 2 | 19.45 | 8 | 13.46 | 5 | 7.02 | 2 | 1.223 |
| 3 | 19.05 | 9 | 13.34 | 6 | 6.84 | 3 | 0.906 |
| 4 | 18.76 | 1×10^{-6} | 13.24 | 7 | 6.69 | 4 | 0.702 |
| 5 | 18.54 | 2 | 12.55 | 8 | 6.55 | 5 | 0.560 |
| 6 | 18.35 | 3 | 12.14 | 9 | 6.44 | 6 | 0.454 |
| 7 | 18.20 | 4 | 11.85 | 1×10^{-3} | 6.33 | 7 | 0.374 |
| 8 | 18.07 | 5 | 11.63 | 2 | 5.64 | 8 | 0.311 |
| 9 | 17.95 | 6 | 11.45 | 3 | 5.23 | 9 | 0.260 |
| 1×10^{-8} | 17.84 | 7 | 11.29 | 4 | 4.95 | 1×10^0 | 0.219 |
| 2 | 17.18 | 8 | 11.16 | 5 | 4.73 | 2 | 0.061 |
| 3 | 16.74 | 9 | 11.04 | 6 | 4.54 | 3 | 0.013 |
| 4 | 16.46 | 1×10^{-5} | 10.94 | 7 | 4.39 | 4 | 0.004 |
| 5 | 16.23 | 2 | 10.24 | 8 | 4.26 | 5 | 0.001 |
| 6 | 16.05 | 3 | 9.84 | 9 | 4.14 | | |

Source: Adapted from L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials with Special Reference to Discharging Well Methods, U.S. Geological Survey Water-Supply Paper 887, 1942.

Step 4: Select a Match Point

Choose a point where the type curve and the field data coincide.

Record:

- W(u) and 1/u from the type curve,
- the corresponding s and t from the field data.

Selecting a match point near W(u) = 1 and 1/u = 10 simplifies calculations, though any suitable point is acceptable.

Step 5: Solve for Transmissivity and Storativity

Using the relationships:

$$s = \frac{Q}{4\pi T} W(u), \quad u = \frac{r^2 S}{4Tt}$$

Transmissivity T is calculated from the first equation, and storativity S is subsequently determined using the matched value of u.

D. Reliability of Early-Time and Late-Time Data

- Early-time points may be distorted by wellbore storage and pumping-well effects.
- Late-time points can be influenced by boundary effects or heterogeneity.

Therefore, the best-fit region is typically the mid-section of the curve where transient flow approximates theoretical conditions.

E. Practical Significance

Unsteady-flow analysis, particularly via the Theis method, plays a vital role in drainage engineering by enabling:

- evaluation of aquifer hydraulic properties,
- design of subsurface drainage systems,
- prediction of water-table drawdown during dewatering or reclamation,
- assessment of well performance under non-steady conditions.

The method's conceptual simplicity and general versatility make it an indispensable tool in groundwater hydraulics and drainage system design.

Cooper–Jacob approach (Time – Drawdown)

The transient drainage problem is mathematically analogous to a confined aquifer responding to pumping. The Theis solution introduces the exponential integral function $W(u)$, where:

$$W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \dots \quad (4.24)$$

For small values of u (long times), the higher-order terms become negligible and the expression simplifies to the Cooper–Jacob approximation:

$$s = \frac{Q}{4\pi T} \left(-0.5772 - \ln\left(\frac{r^2 S}{4Tt}\right) \right) \quad (4.25)$$

Changing to base-10 logarithms and rearranging gives:

$$s = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25T}{r^2 S} t\right) \quad (4.26)$$

This can be expressed as a linear equation of the form:

$$s = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25T}{r^2 S}\right) + \frac{2.3Q}{4\pi T} \log t \quad (4.27)$$

which may be written symbolically as:

$$Y = C + aX$$

Where

$$Y = s$$

$$X = \log t$$

$$a = \frac{2.3Q}{4\pi T}$$

$$C = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25T}{r^2 S}\right)$$

It follows that a plot of (s) against (log t) should be a straight line. Extending this line back to where it intersects the time axis (i.e. where $s = 0$ at $t = t_0$) gives

$$\frac{2.25T}{r^2S} t_0 = 1 \quad (4.28)$$

The gradient of the straight line (i.e. the increase in drawdown per log cycle, Δs) is

$$\Delta s = \frac{2.3Q}{4\pi T} \quad (4.29)$$

First, the transmissivity T is obtained from the slope using equation (4.29):

$$T = \frac{2.3Q}{4\pi \Delta s} \quad (4.30)$$

Once T is known, the storage coefficient S can be calculated from equation (4.28) by substituting T and the intercept time t_0 :

$$S = \frac{2.25Tt_0}{r^2} \quad (4.31)$$

Where

Q : Constant discharge of the pumping well,

r : Distance from the pumping (or drainage) well to the observation piezometer,

Δs : Increase in drawdown over one log cycle of time, and

t_0 : Time at which the straight line extrapolated through the data intersects the t-axis at $s = 0$.

Practical procedure (Cooper–Jacob straight-line method)

1. Plot the data

Draw a graph of drawdown s (vertical axis) versus $\log t$ (horizontal axis) for the observation well.

2. Identify the late-time straight segment

Select the portion of the curve that becomes approximately linear at late time (small u , typically $u \leq 0.05$). Early-time data affected by wellbore storage or partial penetration should be ignored.

3. Draw the best-fit straight line

Through the linear portion, draw a straight line that represents the Cooper–Jacob approximation.

4. Determine the slope Δs

Measure the increase in drawdown over one log cycle of time (e.g. between t and $10t$). This is Δs , which is used in equation (4.29) to compute T.

5. Find the intercept time t_0

Extend the straight line backwards until it intersects the t-axis (where $s = 0$). Read the corresponding time t_0 . Insert t_0 and T into equation (4.28) to obtain S.

This procedure provides a simple and robust means of estimating transmissivity and storage from pumping-test (or analogous drainage-test) data, provided that the assumptions of the Cooper–Jacob method are reasonably satisfied: u is small (long pumping times, relatively short distances), the aquifer is homogeneous and isotropic, and flow to the well (or drain) is radial under confined or quasi-confined conditions.

Cooper–Jacob approach (Distance – Drawdown)

If simultaneous observations are made of drawdown in three or more observation wells, the observation-well distance is plotted along the logarithmic x-axis, while drawdown is plotted along the linear y-axis. Under conditions where the Cooper–Jacob approximation is valid (i.e., small values of u), the drawdown versus distance plot becomes a straight line, from which aquifer parameters can be determined. For the Distance–Drawdown method, the transmissivity and storativity are calculated as follows.

Transmissivity

The slope of the straight line, represented by the change in drawdown per log cycle of distance, Δs , gives:

$$T = \frac{2.3Q}{4\pi\Delta s} \quad (4.32)$$

This is analogous to the time–drawdown method, except that distance r replaces time t .

Storativity

The straight line is extended back toward the pumping well until it intersects the distance axis at a point $r = r_0$, where:

$$s = 0 \Rightarrow \frac{2.25Tt}{r_0^2 S} = 1 \quad (4.33)$$

Solving for storativity S gives:

$$S = \frac{2.25Tt}{r_0^2} \quad (4.34)$$

Where

Q : Constant discharge from the pumping well,

T : Transmissivity,

S : Storativity (dimensionless),

r_0 : Distance where the extrapolated line meets the $s = 0$ axis,

t : Time since pumping began.

Procedure for the Distance–Drawdown Method

1. Plot field data

For a fixed pumping time t , plot drawdown s (linear scale) against distance r (logarithmic scale) for all observation wells.

2. Identify the linear segment

Select the portion of the curve that is approximately linear. Early-time observations or wells too close to the pumped well may deviate due to well losses or partial penetration.

3. Determine the slope Δs

Measure the change in drawdown corresponding to one log cycle of distance. Use equation (4.32) to calculate transmissivity T .

4. Extrapolate to $s = 0$

Extend the linear segment backward until it intersects the r -axis at $r = r_0$. Insert r_0 , T , and the pumping time t into equation (4.34) to determine storativity S .

Example 4.4

A fully penetrating well is pumped at a constant discharge from a confined aquifer. The aquifer is laterally extensive, homogeneous, and isotropic, and is bounded above and below by impermeable layers. The initial piezometric surface is horizontal.

- Aquifer thickness: $b = 20$ m
- Hydraulic conductivity: $K = 5 \times 10^{-4}$ m/s
- Storage coefficient: $S = 1 \times 10^{-4}$
- Constant pumping rate: $Q = 0.015$ m³/s
- Initial piezometric head (before pumping): $h_0 = 25$ m (referenced to an arbitrary datum)

Two observation wells are located at radial distances:

- Well A: $r_A = 30$ m
- Well B: $r_B = 150$ m

Assuming unsteady (transient) radial flow and neglecting well losses, determine the drawdown at each observation well after:

- $t = 1$ h
- $t = 8$ h

Solution

Use the Theis solution:

$$s(r, t) = \frac{Q}{4\pi T} W(u)$$

and the well function for small u may be approximated as

$$W(u) = -0.5772 - \ln(u), \quad u \leq 0.05$$

Aquifer transmissivity

$$T = Kb = (5 \times 10^{-4}) \times 20 = 0.01 \text{ m}^2/\text{day}$$

Dimensionless time parameter u

$$u = \frac{r^2 S}{4Tt}, \text{ well A, } r = 30 \text{ m, for } t = 1 \text{ h, } u_{A,1} = \frac{30^2 \times (1 \times 10^{-4})}{4(10^{-2})(3600)} = 6.25 \times 10^{-4}$$

$$\text{For } t = 8 \text{ h, } u_{A,8} = \frac{30^2 \times (1 \times 10^{-4})}{4(10^{-2})(28800)} = 7.81 \times 10^{-5}$$

$$\text{well B, } r = 150 \text{ m, for } t = 1 \text{ h, } u_{B,1} = \frac{150^2 \times (1 \times 10^{-4})}{4(10^{-2})(3600)} = 1.56 \times 10^{-2}$$

$$\text{for } t = 8 \text{ h, } u_{B,8} = \frac{150^2 \times (1 \times 10^{-4})}{4(10^{-2})(28800)} = 1.95 \times 10^{-3}$$

All values of u are small; the approximation

$W(u) \approx -0.5772 - \ln(u)$ is acceptable

Drawdown $s(r, t)$

$$s(30, 1) = \frac{0.015}{4\pi(0.01)} (-0.5772 - \ln(6.25 \times 10^{-4})) = 0.81 \text{ m}$$

$$s(30,8) = \frac{0.015}{4\pi(0.01)} (-0.5772 - \ln(7.81 \times 10^{-5})) = 1.06 \text{ m}$$

$$s(150,1) = \frac{0.015}{4\pi(0.01)} (-0.5772 - \ln(1.56 \times 10^{-2})) = 0.43 \text{ m}$$

$$s(150,8) = \frac{0.015}{4\pi(0.01)} (-0.5772 - \ln(1.95 \times 10^{-3})) = 0.68 \text{ m}$$

Example 4.5

A drainage well is installed in a confined aquifer to lower the piezometric level in a low-lying field. The well is pumped (or drained) at a constant discharge of $Q = 1200 \text{ m}^3/\text{day}$. An observation piezometer is located at a distance 50 m, from the drainage well. The drawdown s in the observation piezometer is recorded at different times t after the start of pumping. The measured data are given in the following table.

| Time since pumping started, (t) (min) | Drawdown, (s) (m) |
|---------------------------------------|-------------------|
| 5 | 0.05 |
| 10 | 0.22 |
| 20 | 0.38 |
| 50 | 0.60 |
| 100 | 0.77 |
| 200 | 0.93 |
| 500 | 1.15 |

Assuming that the conditions for the Cooper–Jacob approximation are satisfied (confined or quasi-confined aquifer, radial flow, homogeneous and isotropic medium, and sufficiently long pumping times), **determine**:

1. The aquifer transmissivity T (m^2/day)
2. The storage coefficient S (dimensionless).

Solution

The analysis follows the Cooper–Jacob straight-line method described previously.

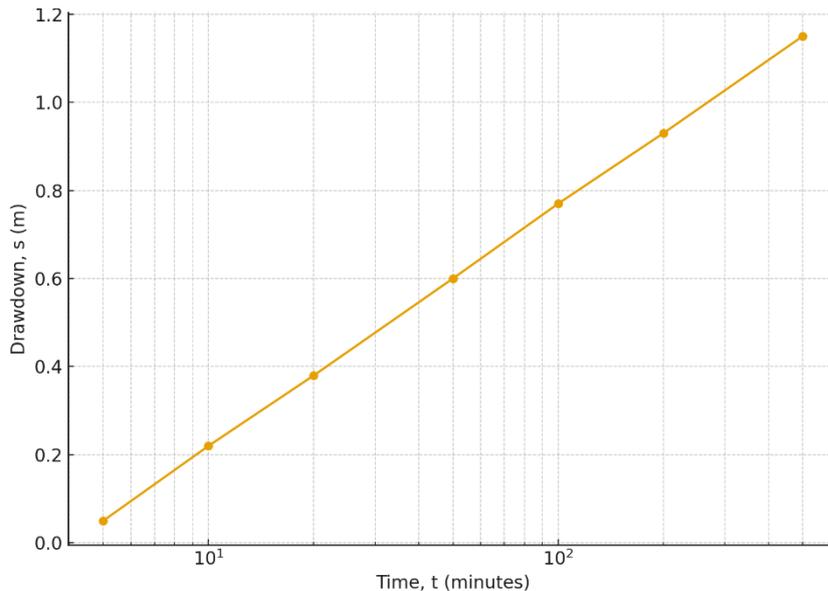
1. Plot s versus $\log t$
 - Vertical axis: drawdown s (m),
 - Horizontal axis: $\log_{10} t$ (time t in minutes).

At late time, the points fall approximately on a straight line. The early-time data affected by wellbore storage and other disturbances can be ignored if necessary. In this example, the points from $t = 5$ to $t = 500$ min already form an essentially straight line.

2. Determine the slope Δs

The Cooper–Jacob approximation leads to a straight-line relationship:

$$s = \frac{2.3Q}{4\pi T} \log \left(\frac{2.25T}{r^2 S} \right) + \frac{2.3Q}{4\pi T} \log t$$



To obtain Δs from the plot, select two points one logarithmic cycle apart in time. For example, from the plotted line:

At $t = 5$ min, $s \approx 0.05$ m

At $t = 50$ min, $s \approx 0.60$ m

Note that: $\log_{10} 50 - \log_{10} 5 = \log_{10} \left(\frac{50}{5}\right) = \log_{10} 10 = 1$

Thus, the increase in drawdown over one log cycle of time is

$$\Delta s = 0.6 - 0.05 = 0.55 \text{ m}$$

3. Calculate transmissivity T

$$T = \frac{2.3Q}{4\pi\Delta s} = \frac{2.3(1200)}{4\pi(0.55)} \approx 4 \times 10^2 \text{ m}^2/\text{day}$$

4. Determine the intercept time t_0

From the drawn straight line, extend the line backwards until it cuts the t -axis (where $s = 0$). Suppose this graphical extrapolation gives $t_0 = 4.0$ min

(You can show this in the figure by marking the extrapolated intersection at $s = 0$.)

Alternatively, you can obtain t_0 analytically from one of the points on the line. Using the point ($t = 50$ min, $s = 0.60$ m):

$$s = \Delta s \log_{10} \left(\frac{t}{t_0}\right)$$

$$0.6 = 0.55 \log_{10} \left(\frac{50}{t_0}\right) \Rightarrow t_0 = 4.05 \text{ min} = 2.81 \times 10^{-3} \text{ day}$$

5. Calculate the storage coefficient S

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25(4 \times 10^2)(2.81 \times 10^{-3})}{50^2} = 10^{-3}$$

Example 4.6

A pumping well fully penetrates a confined aquifer and is pumped at a constant discharge $1200 \text{ m}^3/\text{day}$, at time 1.0 day, after the start of pumping, drawdown is measured in several observation wells at different distances from the pumped well. The measured data are given in the following table.

| Observation well | Distance (r) (m) | Drawdown (s) (m) |
|------------------|------------------|------------------|
| 1 | 50 | 0.80 |
| 2 | 80 | 0.69 |
| 3 | 120 | 0.59 |
| 4 | 200 | 0.47 |
| 5 | 320 | 0.36 |
| 6 | 500 | 0.25 |
| 7 | 800 | 0.14 |

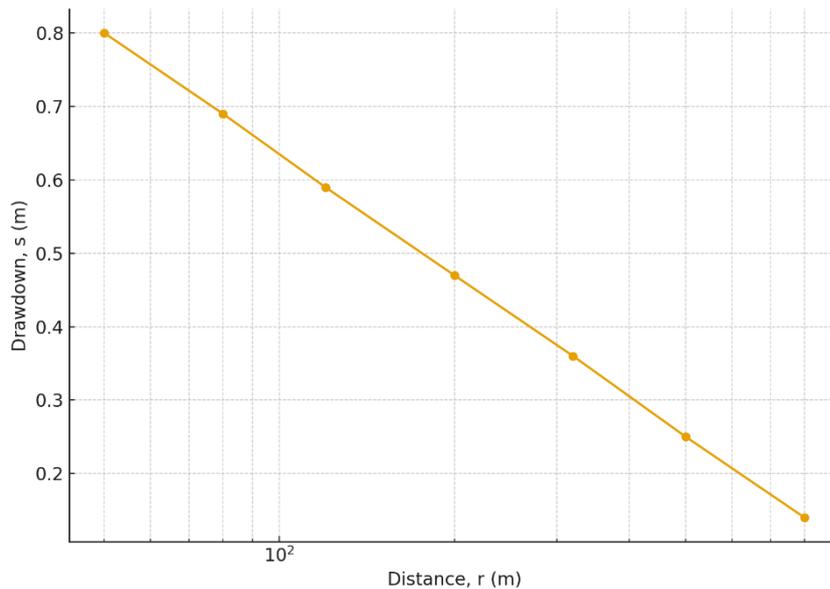
Assuming that the Cooper–Jacob approximation is valid (confined or quasi-confined aquifer, small u , homogeneous and isotropic conditions, radial flow), **determine:**

1. The aquifer transmissivity T (m^2/day)
2. The storage coefficient S (dimensionless).

Solution

1. Plot s versus $\log r$

- Vertical axis (linear scale) drawdown s (m),
- Horizontal axis (log scale) distance r (m).



2. Determine the slope Δs

From the straight line, measure the change in drawdown for one log cycle of distance.

Choose two wells whose distances differ by a factor of 10, for example:

Well 2: $r = 80$ m, $s = 0.69$ m

Well 7: $r = 800$ m, $s = 0.14$ m

the slope is simply

$$\Delta s = 0.69 - 0.14 = 0.55 \text{ m}$$

3. Transmissivity T

Using the Cooper–Jacob Distance–Drawdown transmissivity formula

$$T = \frac{2.3Q}{4\pi\Delta s} = \frac{2.3(1200)}{4\pi(0.55)} \approx 4 \times 10^2 \text{ m}^2/\text{day}$$

4. Extrapolate to $s = 0$ and find r_0

Extend the straight line on the $s - \log r$ plot back toward the pumping well until it intersects the distance axis at zero drawdown ($s = 0$).

$$r_0 = 1400 \text{ m}$$

5. Storativity S

$$S = \frac{2.25Tt}{r_0^2} = \frac{2.25(4 \times 10^2)(1)}{1400^2} \approx 4.6 \times 10^{-4}$$