Graphs

1- Using graphs to solve inequalities

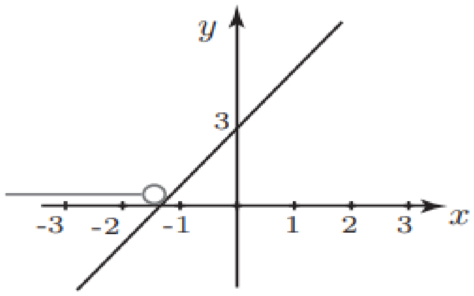
Inequalities can be solved very easily using graphs, and if you are in any way unsure about the algebra, it would be a good idea to do a graph to check. Let us see how this works .

Example 1:- Suppose we wish to solve $2x + 3 < 0$.

This inequality could be solved very easily doing algebra, but it makes a good graphical example.

First we sketch a graph of $y = 2x + 3$ as shown

x	$y = 2x + 3$	(x, y)
0	$y = 0 * 2 + 3 = 3$	(0,3)
-1	$y = (-1) * 2 + 3 = 1$	(-1,1)
$\frac{-3}{2}$	$y = \frac{-3}{2} * 2 + 3 = 0$	$(\frac{-3}{2}, 0)$



A graph of $y = 2x + 3$.

Example 2:- :- Suppose we wish to solve $3x - 2 < 4$.

Soultion:-

x		(x, y)

Exercises 4 :- Using graphs to solve inequalities

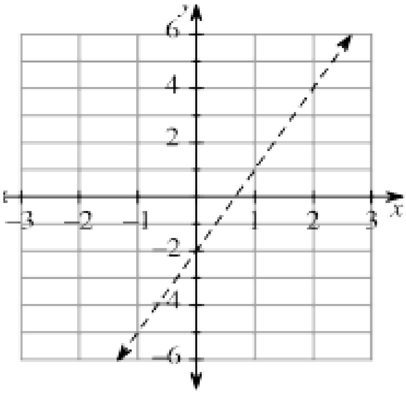
a) $2x+5 \leq 3$ b) $4x-1 > 1$ c) $x - 3 \leq 2$ d) $5x + 4 \geq 2$ g) $3\ \ x+ 6 > 1$

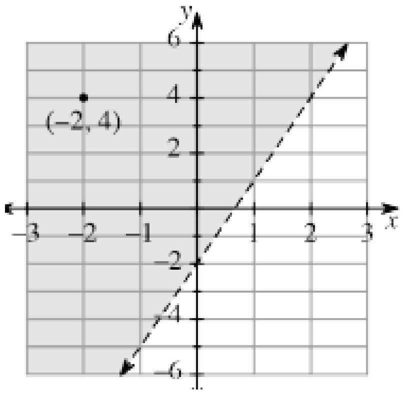
2- Graphing Linear Inequalities in Two Variables

Example 1 :- Graph the solutions to the inequality $y > 3x - 2$.

First, graph the line $y - 3x = -2$

x		(x, y)





3- Graphs of Functions

<u>Functions</u>

A function is a relation that uniquely associates members of one set with members of another set. More formally, a function from A to B is an object f such that every a $\in$ A is uniquely associated with an object f(a) in B.

A function is therefore a many-to-one (or sometimes one-to-one) relation. The set A of values at which a function is defined is called its domain, while the set f(A) $\subset$ B of values that the function can produce is called its range. Here, the set B is called the codomain of f.

In the context of univariate, real valued functions $f : A \subset R \to R$, the fact that domain elements are mapped to unique range elements can be expressed graphically by way of the vertical line test.

Example 1:- We can show this mathematically by writing

$$f(x) = x + 3$$

1- Types of functions

1- Constant function

Example 2

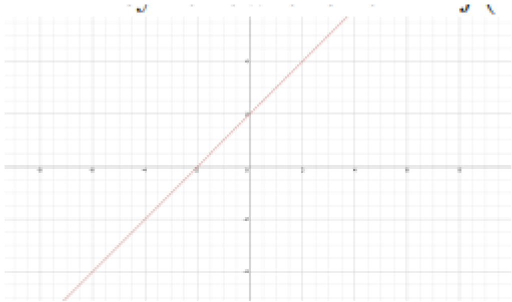
1) $f(x) = 3$, 2) $f(x) = 1/3$ 3) $f(x) = k$, where $k \in R$

2-Linear function

Example 3:-

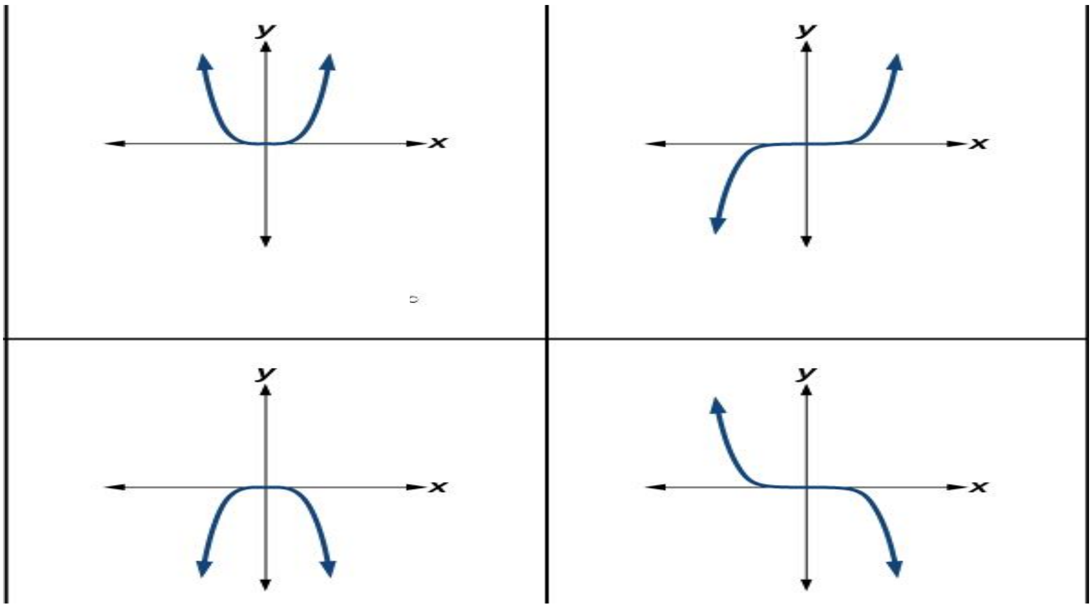
1) f(x) = a + bx 2) f(x) = x 3) f(x) = 2 + 5x

*** The graph of linear function is <u>line</u>**



3- Power Functions : Power functions are functions that have a leading term greater than one. The most common are $f(x) = x^2$ and $f(x) = x^3+3x+4$.

*** The graph of Power Functions is <u>curve</u>**

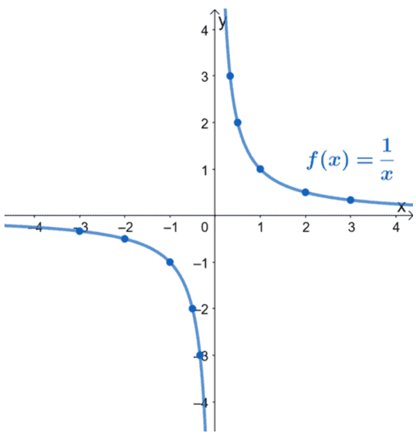


4- Rational function

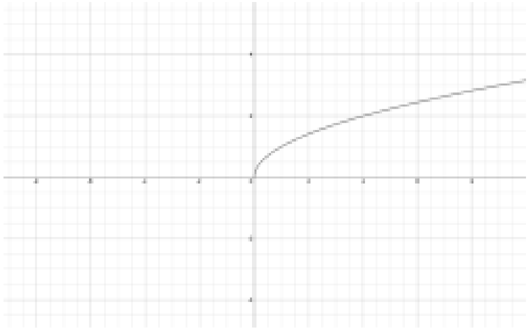
Example 4 :-

$$1 - \text{f(x)} = \frac{3x-4}{x}$$

$$2 - \text{f(x)} = \frac{1}{x}$$



5 - Root Functions : Root functions are function that involve roots, square or otherwise. The most common is , f(x) = $\sqrt{x}$



2 - Domain and Range

Definition :-The domain of a function is the set of values that we are allowed to plug into our function. This set is the x values in a function such as f(x).as D_f

Definition :- The range of a function is the set of values that the function assumes. This set is the values that the function shoots out after we plug an x value in. They are the y values. as R_f .

Example :- find Domain and Range for functions

1. f(x) = 5

Solutions

$$D_f = \{x : -\infty < x < \infty\} \text{ or } \quad D_f = (-\infty, \infty), \text{ and}$$

$$R_f = \{y : y = 5\} \text{ or } R_f = \{5\}$$

2. f(x) = x + 5

Solutions

$$D_f = \{x : -\infty < x < \infty\} \text{ or } D_f = (-\infty, \infty),$$

And $$\mathbf{y = x + 5 \Rightarrow x = y - 5}$$

$$\Rightarrow R_f = \{y : -\infty < y < \infty\} \text{ or } R_f = (-\infty, \infty),$$

3. f(x) = - x^2 + 4

Solutions

$$D_f = \{x : -\infty < x < \infty\} \text{ or } D_f = (-\infty, \infty),$$

$$\text{and } y = -x^2 + 4 \quad \rightarrow \quad x^2 = 4 - y \quad \rightarrow \quad x = \sqrt{4 - y}$$

$$\rightarrow 4 - y \geq 0 \quad \rightarrow \quad 4 \geq y \; Or \; \; y \leq 4$$

$R_f = \{y : -\infty < y \leq 4\}$ or $R_f = (-\infty, 4]$,

4. $y = \frac{2}{x^2-1}$

Solutions

$$x^2 - 1 = 0 \rightarrow x = \mp 1 \rightarrow D_f = \frac{R}{\{-1,1\}}$$

And $$y = \frac{2}{x^2-1} \rightarrow x^2 y - y = 2$$

$$x^2 y = 2 + y \rightarrow x^2 = \frac{2+y}{y}$$

$$x = \pm \frac{\sqrt{2+y}}{\sqrt{y}}$$

$$2 + y \geq 0 \ \ and\ y > 0 \rightarrow y \geq -2 \ \ and\ y > 0$$

$$R_f = [-2, \infty) \cap (0, \infty)$$

5. $f(x) = \sqrt{x+2}$

Solutions

$$x + 2 \geq 0 \rightarrow x \geq -2, \qquad D_f = [-2, \infty)$$

$$y = \sqrt{x+2} \rightarrow y^2 = x + 2 \rightarrow x = y^2 - 2$$

$$R_f = R \ or\ \ R_f = (-\infty, \infty)$$

6. $f(x) = \sqrt{9 - x^2}$

Solutions

$$9 - x^2 \geq 0 \rightarrow 9 \geq x^2 \rightarrow x^2 \leq 9,$$

$* \ \boldsymbol{x^2 \leq a^2 \rightarrow \quad |x| \leq a \rightarrow \quad -a \leq x \leq a}$

$$x^2 \leq 9 \rightarrow |x| \leq 3 \rightarrow \boldsymbol{-3 \leq x \leq 3} \qquad D_f = [-3,3]$$

$$y = \sqrt{9 - x^2} \rightarrow y^2 = 9 - x^2 \rightarrow x^2 = 9 - y^2$$

$$x \;\; = \sqrt{9-y^2} \rightarrow \;\; 9-y^2 \geq 0 \rightarrow \;\; y^2 \leq 9$$

$$\rightarrow \; -3 \leq y \leq 3 \; but \;\; y \; is \; root \; fcuntion \; so \; y \geq 0$$

$$then \quad 0 \leq y \leq 3$$

$$R_f = [0\,,3] \;\; or \;\; R_f = (-\infty\,,\infty)$$

Exercises 5 :- find Domain and Range for functions

1- $f(x) = \sqrt{x-4}$

2- $f(x) = \frac{1}{x-3}$

3- $f(x) = \frac{x-1}{(x+3)(x-3)}$

4- $f(x) = x + 4$

4- Graphs of Functions (Graphs of Curves)

To graph the curve of a function, we can following steps:-

1- Find the domain and range of the function

2- Check the symmetry of the function

3- Find (if any found) points of intersection with x-axis and y-axis.

4-Choose some another on the curve

5-Draw smooth line through the above points

Graphical Test for Symmetry

1-X-Axis Symmetry: If the point (x, y) the is on the graph, the point $(x, -y)$ is also point

2-Y-Axis Symmetry: If the point (x, y) the is on the graph, the point $(-x, y)$ is also point

3- Origin Symmetry: If the point (x, y) the is on the graph, the $(-x, -y)$ is also on the graph.

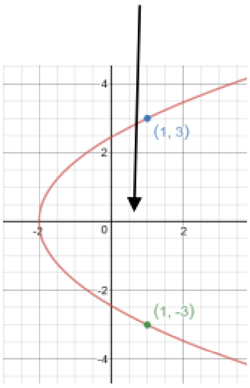
***Even** Functions have <u>Y-Axis</u> Symmetry!*

***Odd** Functions have <u>Origin</u> Symmetry!*

X-Axis Symmetry:

If the point (x, y) is on the graph, the point $(x, -y)$ is also on the graph.

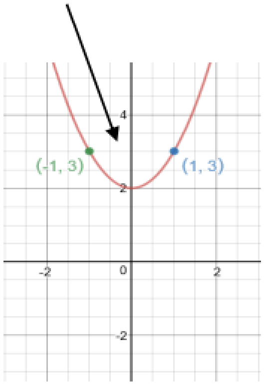
The X-Axis acts like a mirror.



Y-Axis Symmetry:

If the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph.

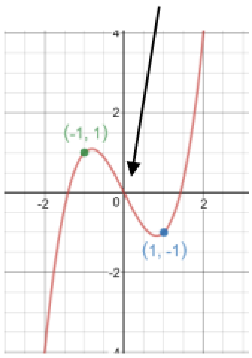
The Y-Axis acts like a mirror.



Origin Symmetry:

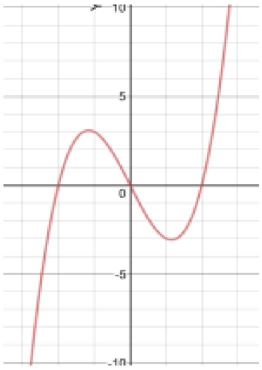
If the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph.

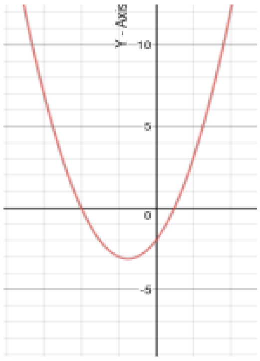
If you spin the picture upside down about the Origin, the graph looks the same!

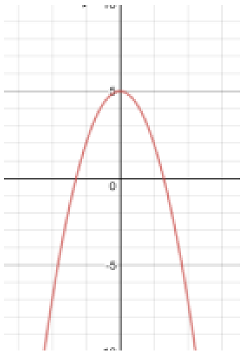


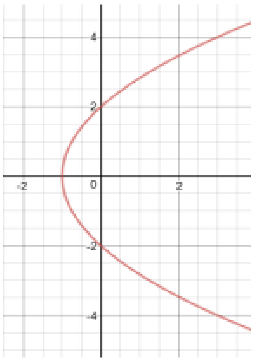
Equation Symmetry - Practice Problems

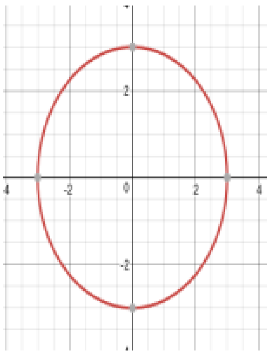
A. Graphically determine what type(s) of symmetry, if any, are present.

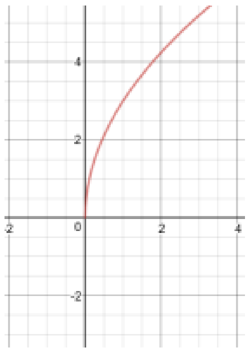
1. 

2. 

3. 

4. 

5. 

6. 

B. Algebraically check for symmetry with respect to the x-axis, y-axis, and the origin.

1. $y = x^2 + 4$
2. $y = -x^3 - x$
3. $y = 2x - 10$
4. $x = -y^2 + 4$
5. $x^2 + y^2 = 25$
6. $y = |x| + 2$

Example :- Find Domain and Range for functions and Graph the functions

$$y = f(x) = x^2 - 1$$

Solution: -

1) Find D_f, R_f of the function $D_f = (-\infty, \infty)$

To find R_f : we must convert the function from $y = f(x)$

$$y = x^2 - 1 \quad \rightarrow \; x^2 = y + 1 \; \rightarrow x = \pm\sqrt{y+1}$$

$$\rightarrow so \;\; y + 1 \geq 0 \quad \rightarrow \;\; y \geq -1 \qquad R_f = [-1, \infty)$$

2) Find x and y intercept

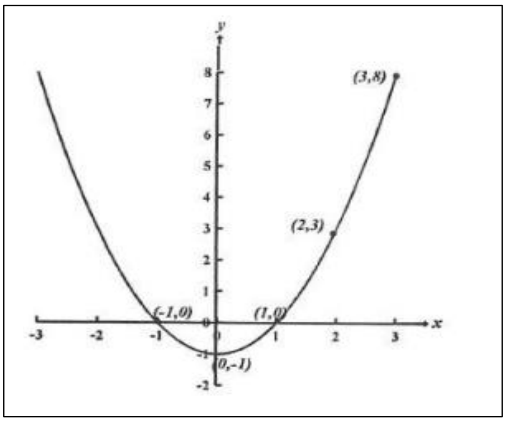
$a)\;\; x - intercept \;\; put \;\; y = 0 \;\; \rightarrow \;\; x^2 - 1 = 0 \;\; \rightarrow x = \pm 1 \;\; \rightarrow (-1,0), (1,0)$

$a)\ y - intercept\ \ put\ \ \ x = 0\ \ \rightarrow\ \ y = 0 - 1\ \ \rightarrow y = -1\ \ \rightarrow (0, -1)$

3) check the symmetry:

4) choose some another point on the curve

x	y
2	3
3	8



Example:- Find Domain and Range for functions and Graph the functions

1) f(x) = 5

<u>**Solutions**</u>

1) $D_f = \{x : -\infty < x < \infty\}$ or $D_f = (-\infty, \infty)$,

$R_f = \{5\}$

2) Find x and y intercept

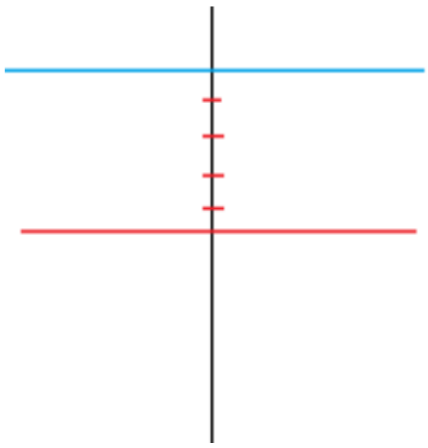
$a)\ \ x - intercept\ \ put\ \ \ y \neq 0\ \rightarrow\ there\ is\ no\ point\ .$

$b)\ \ y - intercept\ put\ x = 0\ \rightarrow\ \ \ y = 5\ \rightarrow\ \ \ (0\ ,5)$

3) check the symmetry:

4) choose some another point on the curve

x	y	(x,y)
-2	5	(-2,5)
-1	5	(-1,5)
0	5	(0,5)
1	5	(1,5)
2	5	(2,5)



2) $f(x) = x + 5$

Solutions

1) $D_f = \{x : -\infty < x < \infty\}$ or $D_f = (-\infty, \infty)$,

$R_f = \{y : -\infty < y < \infty\}$ or $R_f = (-\infty, \infty)$,

2) Find x and y intercept

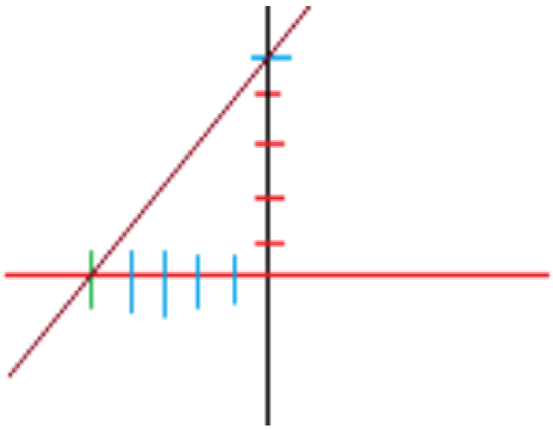
$a)\ x - intercept\ put\quad y = 0 \rightarrow x = -5 \quad (-5{,}0)$.

$b)\ \ y - intercept\ put\ \ x = 0 \rightarrow \quad y = 5 \rightarrow \quad (0\ {,}5)$

3) check the symmetry:

4) choose some another point on the curve

x	y	(x,y)
-2	3	(-2,3)
-1	4	(-1,4)
0	5	(0,5)
1	6	(1,6)
2	7	(2,7)



3) $f(x) = x$

Solutions

1) $D_f = \{x : -\infty < x < \infty\}$ or $D_f = (-\infty, \infty)$,

$R_f = \{y : -\infty < y < \infty\}$ or $R_f = (-\infty, \infty)$,

2) Find x and y intercept

$a)\ x - intercept\ put\quad y = 0 \rightarrow x = 0 \quad (0\ {,}0)$.

$b)\ \ y - intercept\ put\ \ x = 0 \rightarrow \quad y = 5 \rightarrow \quad (0\ {,}0)$

3) check the symmetry:

4) choose some another point on the curve

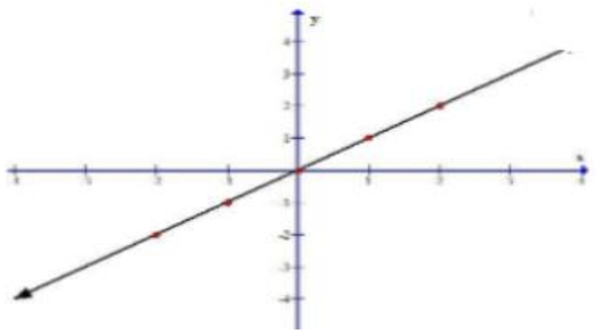
$f(-2) = -2$

$f(-1) = -1$

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$



4) $f(x) = x^2$

Solutions

1) $D_f = \{x : -\infty < x < \infty\}$ or $D_f = (-\infty, \infty)$,

$R_f = \{y : 0 \le y < \infty\}$ or $R_f = [0\ \infty)$,

2) Find x and y intercept

$a)\ x - intercept\ \ put\ \ \ y = 0\ \rightarrow\ x = 0\ \ (0, 0)\ .$

$b)\ \ y - intercept\ put\ x = 0\ \rightarrow\ \ \ y = 0\ \rightarrow\ \ \ (0\ ,0)$

3) check the symmetry:

4) choose some another point on the curve

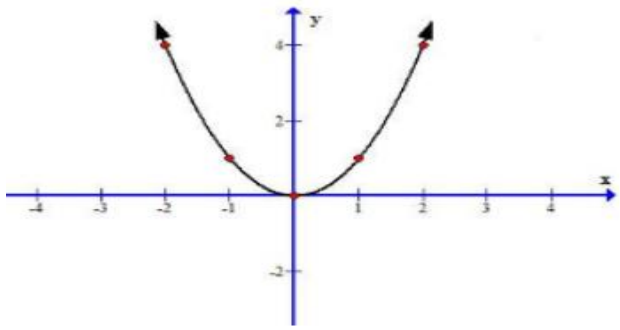
$f(-2) = (-2)^2 = 4$

$f(-1) = (-1)^2 = 1$

$f(0) = (0)^2 = 0$

$f(1) = (1)^2 = 1$

$f(2) = (2)^2 = 4$



5) $f(x) = x^3$

Solutions

1) $D_f = \{x : -\infty < x < \infty\}$ or $D_f = (-\infty, \infty)$,

$$y = x^3 \quad \rightarrow \quad x = \sqrt[3]{y}$$

$R_f = \{y : -\infty < y < \infty\}$

2) Find x and y intercept

$a)\ \ x - intercept\ \ put \quad y = 0 \ \rightarrow \quad x = 0 \ \rightarrow \ (0,0)$.

$b)\ \ y - intercept\ put \quad x = 0 \ \rightarrow \quad y = 0 \ \rightarrow \ (0\ ,0)$

3) check the symmetry:

4) choose some another point on the curve

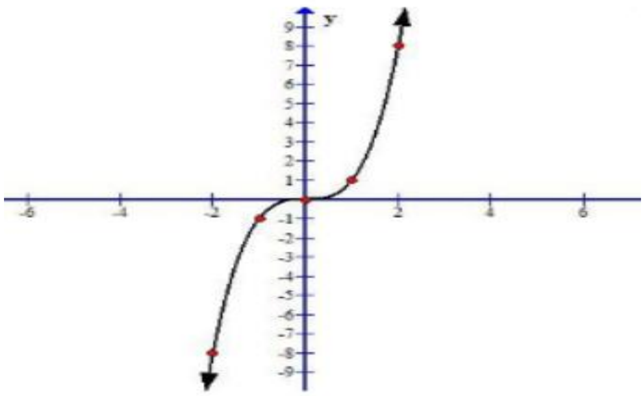
$f(-2) = (-2)^3 = -8$

$f(-1) = (-1)^3 = -1$

$f(0) = (0)^3 = 0$

$f(1) = (1)^3 = 1$

$f(2) = (2)^3 = 8$



6) $f(x) = \sqrt{x}$

Solutions

1) $D_f = \{x : 0 \le x < \infty\}$ or $D_f = [0, \infty)$,

$y = \sqrt{x} \quad \rightarrow \quad y^2 = x \rightarrow x = \ y^2$

$R_f = \{y :\ 0 \le y < \infty\}$ or $R_f = [0, \infty)$,

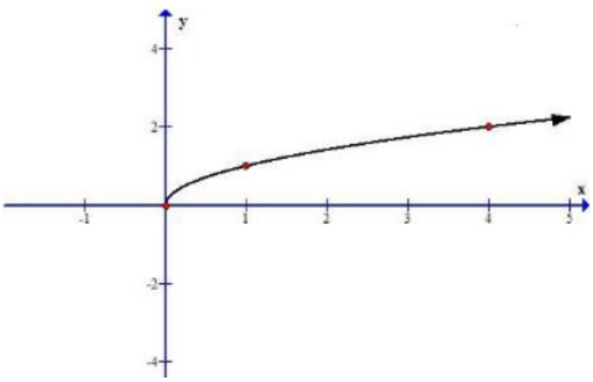
$f(-2) = \sqrt{-2} =$

$f(-1) = \sqrt{-1} =$

$f(0) = \sqrt{0} = 0$

$f(1) = \sqrt{1} = 1$

$f(4) = \sqrt{4} = 2$



7) $f(x) = |x|$

Solutions $D_f = \{x\ \infty \leq x < \infty\}$ or $D_f = (-\infty, \infty)$,

$R_f = \{y: 0 \leq y < \infty\}$ or $R_f = [0, \infty)$,

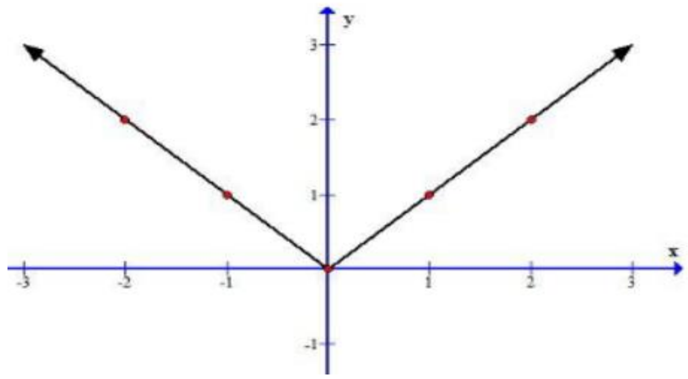
$f(-2) = |-2| = 2$

$f(-1) = |-1| = 1$

$f(0) = |0| = 0$

$f(1) = |1| = 1$

$f(2) = |2| = 2$



8) $f(x) = \frac{1}{x}$

Solutions $D_f = R/\{0\}$ **and** $R_f = R/\{0\}$,

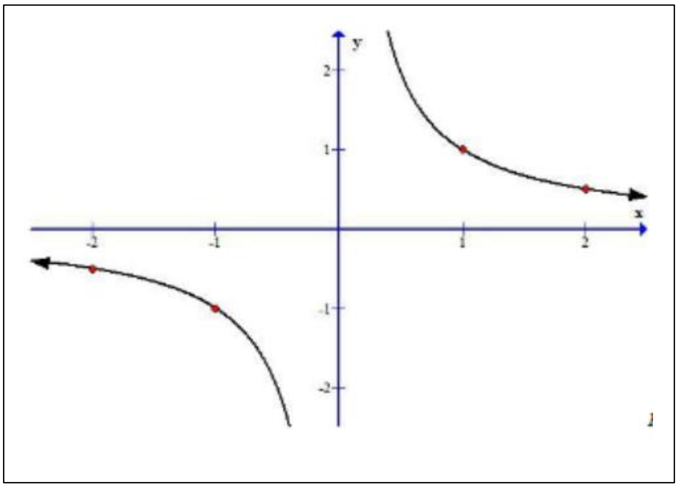
$f(-2) = \frac{1}{-2} = -\frac{1}{2}$

$f(-1) = \frac{1}{-1} = -1$

$f(0) = undefined$

$f(1) = \frac{1}{1} = 1$

$f(2) = \frac{1}{2} = \frac{1}{2}$



Exercises 6 :-Find Domain and Range for functions and Graph the functions

1- $f(x) = \sqrt{x-4}$

2- $f(x) = \frac{1}{x-3}$

3- $f(x) = \frac{x-1}{x+3}$

4- $f(x) = x + 4$

5- $f(x) = x^2 + 4x - 3$