The Fundamental Concepts of Vectors

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Outlines:

1.1 A Scalar and Vector Quantities

Quantities

Scalar

- Has both magnitude only
- Independent of any coordinates
- Such as Density, energy and temperature

Vector

- Has both magnitude and direction.
- Dependent of any coordinates
- Obeys the parallelogram rules.
- Such as Force , displacement

1.2 Vector Algebra

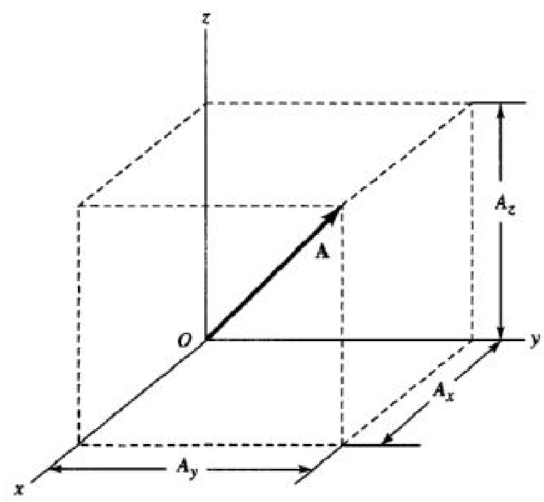


Figure 1.1: A vector A and its components in Cartesian coordinates.

$$A = Ax + Ay + Az$$

$$P1(x1, y1, z1) \ and \ P2(x2, y2, z2)$$

$Ax = x2 - x1$
$Ay = y2 - y1$
$Az = z2 - z1$

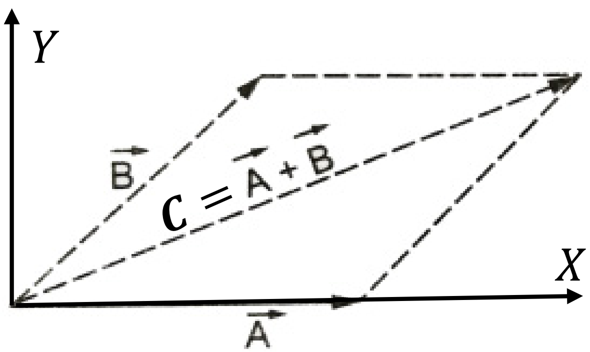
1.2.1 Equality of Vectors

$$\boxed{A = B}$$

$$\boxed{Ax = Bx\ , Ay = By, Az = Bz}$$

1.2.2 Vector Addition

$$A + B = (Ax;Ay;Az) + (Bx;By;Bz)$$
$$A + B = (Ax + Bx) + (Ay + By) + (Az + Bz)$$



1.2.3 Multiplication by a Scalar

If c is a scalar and A is a vector, then:

$$cA = c(Ax + Ay + Az) = cAx + cAy + cAz = A\,c$$

1.2.4 Vectors Subtraction

$$A - B = (Ax - Bx) + (Ay - By) + (Az - Bz)$$

1.2.6 The Commutative Law of Addition

This law holds for vectors; that is,

$$\mathbf{A + B = B + A}$$

Because $A_x + B_x = B_x + A_x$ and similarly for the y and z components.

1.2.7 The Associative Law

$$A + (B + C) = (A_x + (B_x + C_x), A_y + (B_y + C_y) , A_z + (B_z + C_z))$$
$$= ((A_x + B_x) + C_x , (A_x + B_x) + C_x , (A_x + B_x) + C_x)$$
$$= (A + B) + C$$

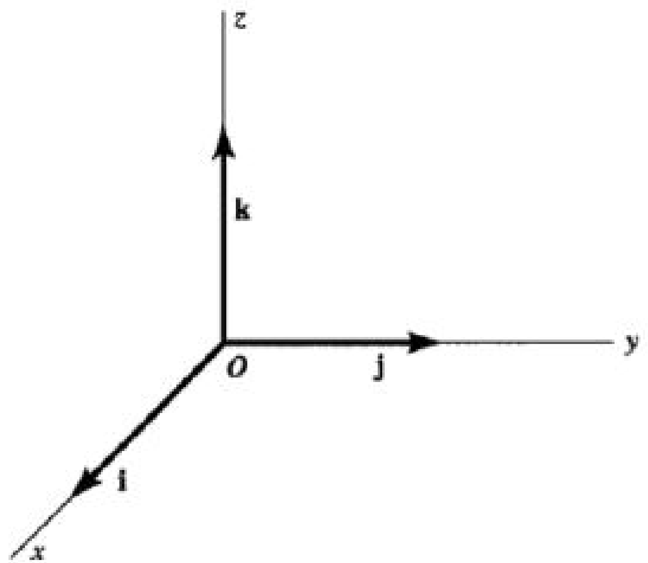
1.2.8 The Distributive Law

$$c(A + B) = c\ (Ax + Bx; Ay + By; Az + Bz)$$
$$= c\ (Ax + Bx); c(Ay + By); c(Az + Bz)$$
$$= c\,A + c\,B$$

1.2.9 Magnitude of a Vector

$$A = |A| = (A^2x + A^2y + A^2z\ )^{(1/2)}$$

1.2.10 Unit Coordinate Vectors



$e_x = (1, 0, 0)$ $e_y = (0, 1, 0)$ $e_z = (0, 0, 1)$

And $i=e_x$, $j=e_y$ and $k=e_z$

So , the any vector can be written as following:

$A = e_x A_x + e_y A_y + e_z A_z$

Example

A helicopter flies 100 m vertically upward, then 500 m horizontally east, then 1000 m horizontally north. How far is it from a second helicopter that started from the same point and flew 200 m upward, 100 m west, and 500 m north?

Solution:

Choosing up, east, and north as basis directions, the final position of the first helicopter is expressed vertically as A = (100,500,1000) and the second as B =(200,-100, 500), in meters. Hence, the distance between the final positions is given by the expression

|A – B| = ((100 -200); (500 + 100); (1000-500)) = 787.4m

1.3 Scalar Product

Given two vectors A and B, the scalar product or "dot" product, A.B, is the scalar defined by the equation

$$\boxed{A.B = AxBx + AyBy + AzBz}$$

From the above dentition,

1-Scalar multiplication is commutative(A.B=B.A)

2-It is also distributive (A.(B+C)=A.B+A.C)

1.3 Scalar Product

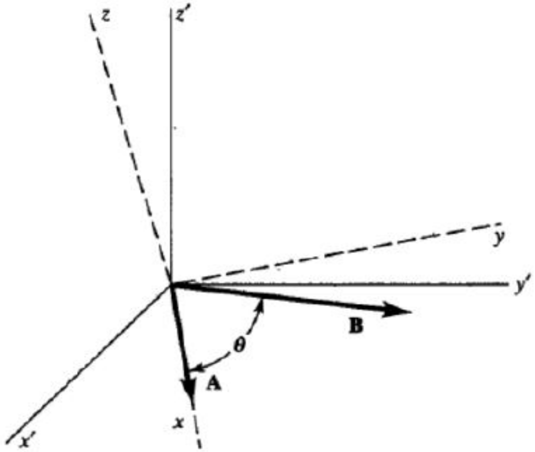


Figure 1.5: Evaluating a dot product between two vectors.

$$A.B = |A||B|\cos\theta \tag{1.15}$$

$$cos\theta = \frac{A.B}{AB} \Rightarrow \theta = cos^{-1}\frac{A.B}{AB}$$

Note: If A. B is equal to zero and neither A nor B is null, then cos is zero and A is perpendicular to B.)

$A^2 = |A^2| = A.A$

1.3 Scalar Product

From the definitions of the unit coordinate vectors i,j, and k, it is clear that the following relations hold

$i.i = j.j = k.k = 1$

$i.j = i.k = j.k = 0$

In addition, we can write any vector associated with its unit vectors by this form:

$$A = iA_x + jA_y + kA_z$$

Example 1.3.2 Law of Cosines: Consider the triangle whose sides are A, B, and C, as shown in Figure 1.6.Then C = A + B. Take the dot product of C with itself,

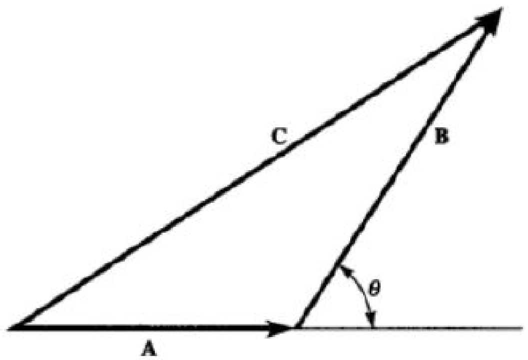


Figure 1.7: The law of cosines

C.C = (A + B)(A + B)
= A.A + 2A.B + B.B

By Replacing A. B with AB cos 0 to obtain which is the familiar law of cosines.

$C^2 = A^2 + 2AB \cos \Phi + B^2$

1.3 Scalar Product

Example 1.3.1: Suppose that an object under the action of a constant force undergoes a linear displacement. By definition, the work AW done by the force is given by the product of the component of the force F in the direction of multiplied by the magnitude of the displacement; that is,

where is the angle between F and 4s As. But the expression on the right is just the dot product of F and As, that is,

1.4 The Vector Product

Given two vectors A and B, the vector product or cross product, A x B, is defined as the vector whose components are given by the equation

$$A \times B = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.23}$$

$$A \times B = (A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x) \tag{1.24}$$

and

$$(A \times B) = -(B \times A) \tag{1.25}$$

$$A \times (B + C) = A \times B + A \times C \tag{1.26}$$

$$n(A \times B) = (nA) \times B = A \times (nB) \tag{1.27}$$

1.4 The Vector Product

According to the definitions of the unit coordinate vectors (Section 1.3), it follows that

$$\begin{aligned} i \times i = j \times j = k \times k &= 0 \\ j \times k = i &= -k \times j \\ i \times j = k &= -j \times i \\ k \times i = j &= -i \times k \end{aligned} \qquad (1.28)$$

These latter three relations define a right-handed triad. For example, $i \times j = (0-0, 0-0, 1-0) = (0,0,1) = k$

In general, the cross product expressed in ijk form is

$$A \times B = (A_y B_z - A_z B_y, A_x B_z - A_z B_x, A_x B_y - A_y B_x) \qquad (1.29)$$

Each term in parentheses is equal to a determinant,

$$A \times B = i \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + j \begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix} + k \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \qquad (1.30)$$

1.4 The Vector Product

and finally

$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.31}$$

$$A \times B = ABsin\theta \tag{1.32}$$

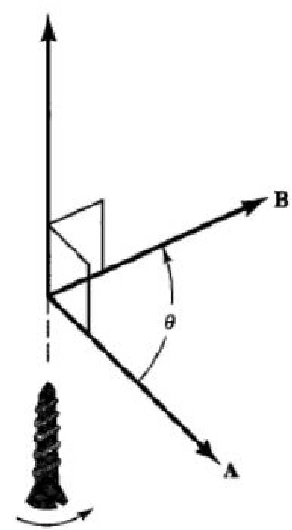


Figure 1.8: The cross product of two vectors.