Information Theory

Aims

The aims of this course are to introduce the principles and applications of information theory. The course will study how information is measured in terms of probability and entropy, and the relationships among conditional and joint entropies; how these are used to calculate the capacity of a communication channel and coding.

Objectives

At the end of the course students should be able to

• calculate the information content of a random variable from its probability distribution
• relate the joint, conditional, and marginal entropies of variables in terms of their coupled probabilities
• define channel capacities and properties using Shannon's Theorems
• construct efficient codes for data on imperfect communication channels
• generalise the discrete concepts to continuous signals on continuous channels
• understand Fourier Transforms and the main ideas of efficient algorithms for them

Recommended book

*** Cover, T.M. & Thomas, J.A. (1991). *Elements of information theory*. New York: Wiley.**

1. Chapter one

1.1. Overview:

What is Information Theory?

Key idea: **The movements and transformations of information, just like those of a fluid, are constrained by mathematical and physical laws.**

These laws have deep connections with:

• probability theory, statistics, and combinatorics
• thermodynamics (statistical physics)
• spectral analysis, Fourier (and other) transforms
• sampling theory, prediction, estimation theory
• electrical engineering (bandwidth; signal-to-noise ratio)
• complexity theory (minimal description length)
• signal processing, representation, compressibility

As such, information theory addresses and answers the two fundamental questions of communication theory:

1. What is the ultimate data compression?
(answer: the entropy of the data, H, is its compression limit.)
2. What is the ultimate transmission rate of communication?
(answer: the channel capacity, C, is its rate limit.)

All communication schemes lie in between these two limits on the compressibility of data and the capacity of a channel. Information theory can suggest means to achieve these theoretical limits. But the subject also extends far beyond communication theory.

Important questions... to which Information Theory offers answers:
• How should information be measured?
• How much additional information is gained by some reduction in uncertainty?
• How do the *a priori* probabilities of possible messages determine the informativeness of receiving them?
• What is the information content of a random variable?
• How does the noise level in a communication channel limit its capacity to transmit information?
• How does the bandwidth (in cycles/second) of a communication channel limit its capacity to transmit information?
• By what formalism should prior knowledge be combined with incoming data to draw formally justifiable inferences from both?
• How much information in contained in a strand of DNA?