

L6 physics of cardiovascular system

Pressure and Blood Flow

If the velocity of a fluid flowing in a tube increased by reducing the radius of the tube it will reach a critical velocity V_c when laminar flow changes into turbulent flow. **Critical velocity** is the velocity of a liquid flow upto which its flow is streamlined and after which its flow becomes turbulent. The critical velocity depends on **coefficient of viscosity (η)**, **density of the liquid (ρ)**, and **radius of the tube (r)**. Using the methods of dimensions, one can show that :

$$V_c = \frac{K\eta}{\rho r}$$

ρ : density r :radius η : viscosity K : Reynolds's number ≈ 1000 for many fluids (blood)

The critical velocity will be lower if there are restrictions or obstructions in the tube .

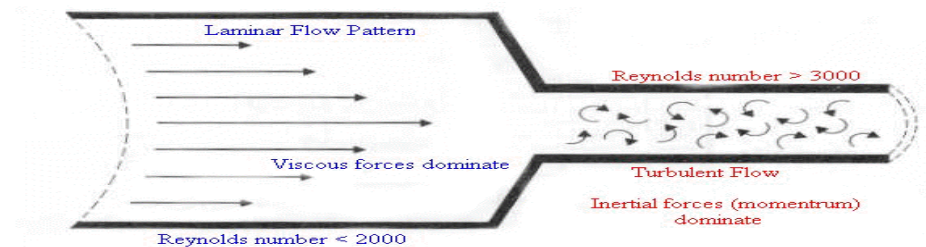
Reynolds Number (K)

The Reynolds number is a pure number that determines the nature of flow through a pipe. The critical velocity ' V_c ' is given by

$$V_c = \frac{K\eta}{\rho r}$$

then $K = (V_c \rho r) / \eta$

If the value of **Reynolds Number K** lies between **0 to 2000**, the flow of liquid is streamline or laminar (the layers of liquid glide or slip over one another like sheets or laminar). If the value of **K** is **greater than 3000**, the flow is turbulent. **For values in between**, the flow is not steady and changes from one to another.



Example: Find the kinetic energy (KE) of 2gm of blood leaving aorta of radius 1.2cm.

$$K.E = (Mv^2)/2$$

$$V_c = \frac{K\eta}{\rho r}$$

$$V_c = \frac{K\eta}{\rho r} = (1000 \times 3.5 \times 10^{-3}) / (1.04 \times 10^3 \times 1.2 \times 10^{-2})$$

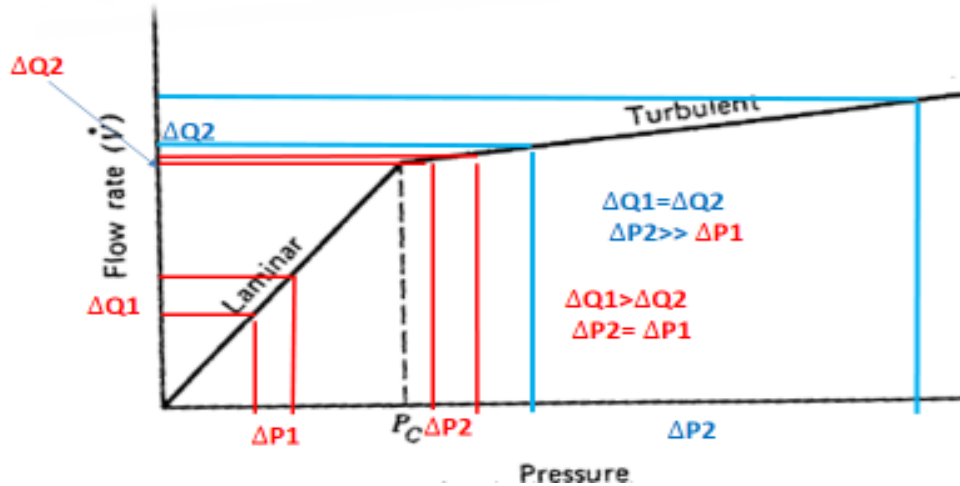
$$V_c = 0.28 \text{ m/s}$$

$$K.E = (1/2) \times (2 \times 10^{-3}) \times (0.28)^2 = 8 \times 10^{-5} \text{ joule}$$

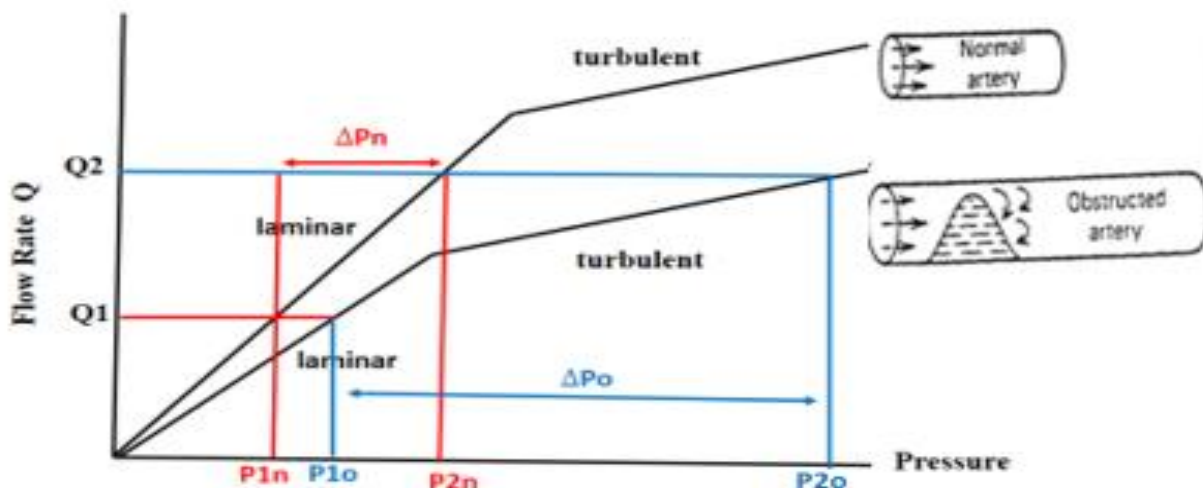
The relation between pressure exerted by the heart and rate of flow:

The higher the pressure exerted by the heart the faster blood will flow,

- Turbulence increases the pressure required to drive a given flow.
- At a given pressure, turbulence leads to a decrease in flow.



Laminar flow is more efficient than turbulent flow. In an obstructed artery the pressure needed to produce a given flow rate is greater than in normal artery of the same size.



Bernoulli's principle

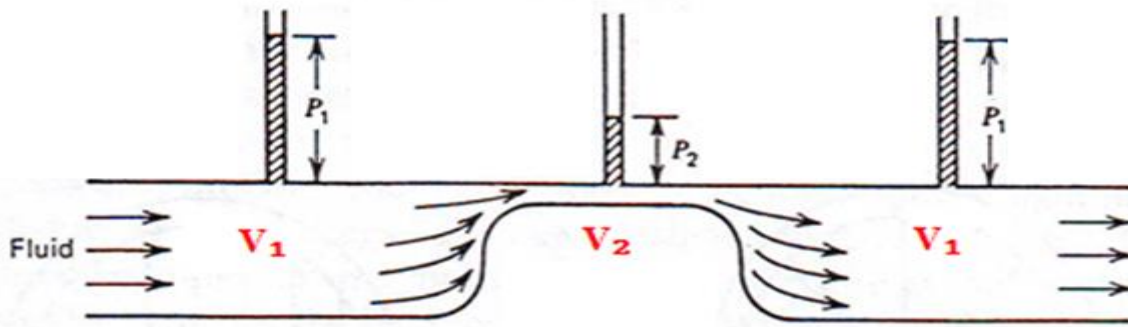
Bernoulli's principle applied to the cardiovascular system

Bernoulli's principle is based on the law of conservation of energy.

-Pressure in a fluid is a form of potential energy PE(it has the ability to perform useful work)(ρgh)

-In a moving fluid there is kinetic energy KE due to motion (energy \ volume) $(\frac{1}{2}\rho v^2)$

As shown in fig below, the velocity increases in the narrow section and the increased KE of the fluid is obtained by a reduction of the PE of the pressure in the tube.

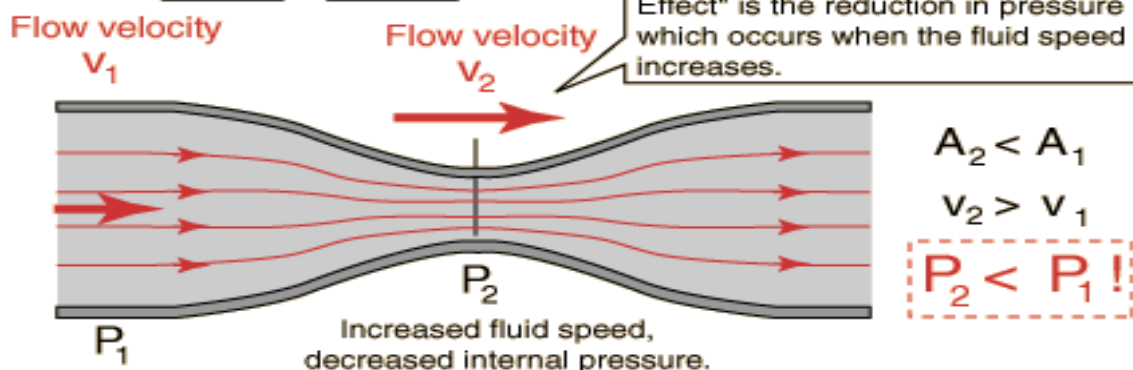


Energy per unit volume before = Energy per unit volume after

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$



The often cited example of the Bernoulli Equation or "Bernoulli Effect" is the reduction in pressure which occurs when the fluid speed increases.



If fluid is flowing through the frictionless tube shown in figure, the velocity increases in the narrow section and increased kinetic energy KE of the fluid is obtained by a reduction of the potential energy of the pressure in the tube.

As the velocity reduces again on the far side of the restriction the kinetic energy is converted back into potential energy and the pressure increases again as indicated on the manometers.

Bernoulli's equation tells us that when velocity increases, the pressure (that the fluid exerts on its walls) decreases.

When you have cholesterol buildup and arteriosclerosis, then the arteries decrease in area since the radius is smaller.

Continuity of flow equation tells us that: when the area decreases, the velocity increases in order to maintain a constant flow rate.

$$Q = A_1 V_1 = A_2 V_2$$

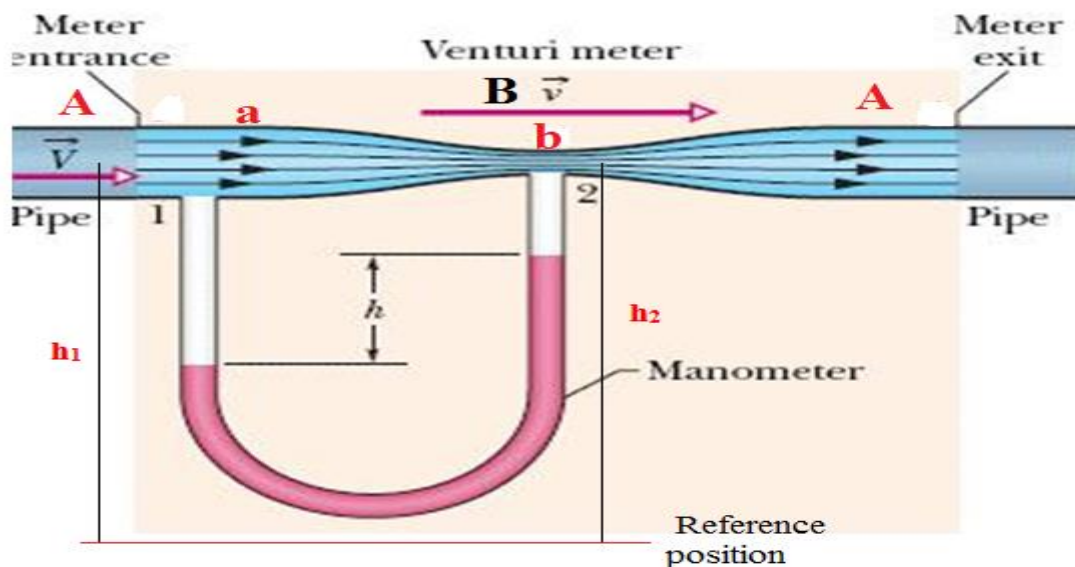
From the continuity of flow equation, the velocity of the blood must increase to maintain the same flow rate Q. This increase in velocity results in a lower pressure at that area.

Bernoulli's principle application

- **Venturimeter:**

The flow speed of incompressible fluid can be measured using a device called venturimeter. It works on Bernoulli's principle.

It consists of a tube with a broad diameter and a small constriction at the middle as shown in figure below. A manometer in the form of a U-tube is also attached to it, with one arm at the broad neck point of the tube and the other at constriction as shown in figure. The manometer contains a liquid of density ρ_m .



$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2.$$

The potential energy at reference position is :

$$\rho g h_1 = \rho g h_2$$

The speed v_1 of the liquid flowing through the tube at the broad neck area A is to be measured from equation of continuity.

Speed at the constriction becomes v_2

$$v_2 = \frac{a}{b} v_1$$

Then using *Bernoulli's* equation, we get:

$$P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_1^2 \left(\frac{a}{b}\right)^2$$

So that
$$P_1 - P_2 = \frac{1}{2} \rho v_1^2 \left[\left(\frac{a}{b}\right)^2 - 1\right]$$

This pressure difference causes the fluid in the U tube connected at the narrow neck to rise in comparison to the other arm. The differences in height h measure the pressure difference.

$$P_1 - P_2 = \rho_m g h = \frac{1}{2} \rho v_1^2 \left[\left(\frac{a}{b}\right)^2 - 1\right]$$

So that the speed of fluid at wide neck is:

$$v_1 = \sqrt{\frac{2 \rho_m g h}{\rho} \left[\left(\frac{a}{b}\right)^2 - 1\right]^{-\frac{1}{2}}}$$

ρ : fluid density

ρ_m : a density of manometer liquid

h : height difference between the fluid on the 2 sides of the "U"

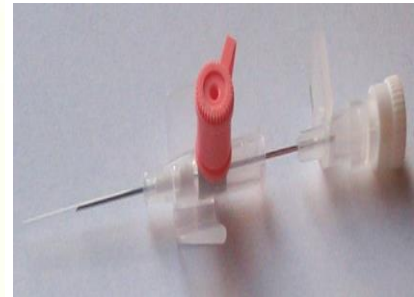
g : gravitational acceleration constant.

a & b : the cross-sectional areas at position A & B respectively,

By knowing the difference in height (h) the velocity v can be calculated.

Venturi Tube Application(principle of injection)

- Venturi tubes are the basis for Venturi injectors which may be used in providing suction or for producing diluted gas mixtures.
- **The Venturi oxygen mask** is based on the Venturi principal in that relatively rapidly moving oxygen molecules pull along (entrainment) air molecules by two processes:



1. The first process is based on the Bernoulli effect in which there is a relative reduction in pressure associated with the higher oxygen velocity
2. And the second involves friction between the high-speed oxygen molecules in the lower speed air molecules which has the effect of pulling air molecules into the higher speed stream.