Mechanics of Materials

References:

1. Mechanics of Materials I - E. J. Hearn
2. Strength of Materials - Ferdinand L. Singer and Andrew Pytel

Syllabus:

1. Simple Stress and Strain.
2. Compound Bars.
3. Shearing Force and Bending Moment Diagrams.
4. Bending Theory.
5. Shear Stress Distribution.
6. Slope and Deflection of Beams.

Chapter One

Simple Stress and Strain

1.1 Load

In any engineering structure or mechanism, the individual components will be subjected to external forces arising from the service conditions or environment in which the component works.

1.2 Types of Loads

(a) Static or **dead load**, generally caused by gravity effects.

(b) Live load, produced by cars crossing a bridge.

(c) Impact or Shock loads, caused by sudden blows.

(d) Fatigue, fluctuating or alternating loads, the magnitude and sign of the load changing with time.

1.3 Direct or Normal Stress (σ)

Is the normal force applied to the cross-sectional area.

$$Stress\ (\sigma) = \frac{Load}{Area} = \frac{P}{A}$$

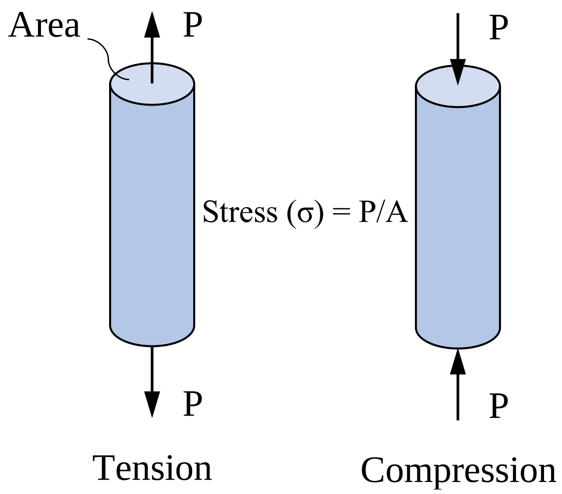

Fig. 1.1 Types of direct stress.

1.4 Direct Strain (Є)

If a bar is subjected to a direct load, and hence stress, the bar will change in length. If the bar has an original length L and changes in length by an amount $\boldsymbol{\delta L}$, the strain produced is defined as follows.

$$\boldsymbol{Strain\ (\epsilon) = \frac{Change\ in\ length}{Original\ length} = \frac{\delta L}{L}}$$

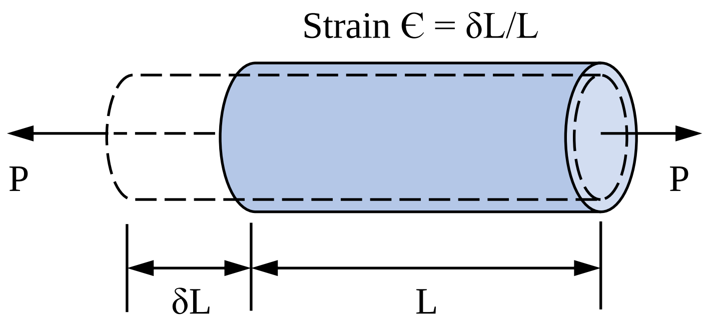

Fig. 1.2 Deformation of the material.

1.5 Elastic materials-Hooke's law

A material is said to be elastic if it returns to its original dimensions when the load is removed. Since loads are proportional to the stresses they produce, and the deformations are proportional to the strains, stress is proportional to strain. Hooke's law, in its simplest form:

$$\boldsymbol{Stress\ (\sigma) \propto Strain\ (\epsilon)}$$

$$\boldsymbol{\frac{Stress}{Strain} = Constant}$$

1.6 Modulus of Elasticity-Young's modulus

Within the elastic limits of materials, the limits in which Hooke's law applies it has a constant, this constant is termed by the symbol $\boldsymbol{E}$ and called the modulus of elasticity or Young's modulus.

$$\boldsymbol{E = \frac{Stress}{Strain} = \frac{\sigma}{\epsilon}}$$

1.7 Tensile Test

In order to compare the strengths of various materials, it is necessary to carry out some standard forms of the test to establish their relative properties. One such test is the standard tensile test in which a circular bar of the uniform cross-section is subjected to a gradually increasing tensile load until failure occurs. Measurements of the change in length of a selected gauge length of the bar are recorded throughout the loading operation by means of extensometers and a graph of load against extension or stress against strain is produced, as shown in Fig. 1.3. This shows a typical result for a test on a mild (low carbon) steel bar. Other materials will exhibit different graphs but of a similar general form.

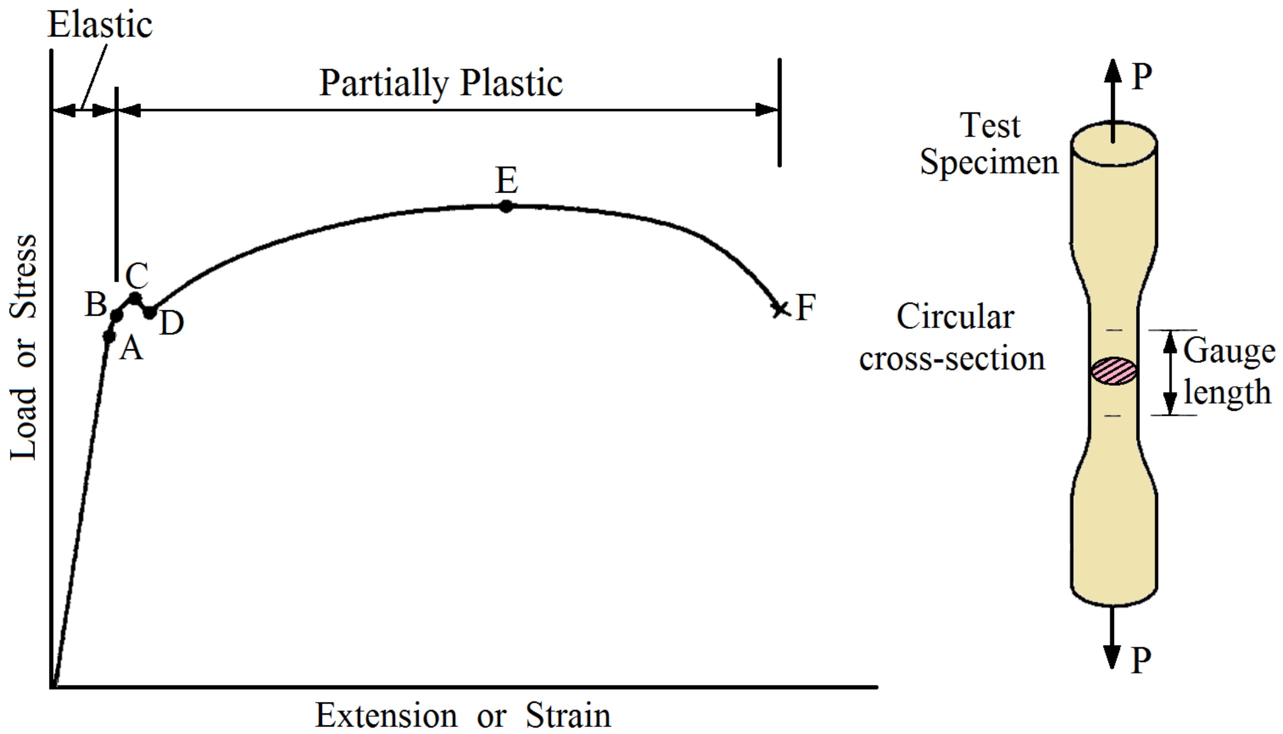

Fig. 1.3 Typical tensile test curve for mild steel.

For the first part of the test, it will be observed that Hooke's law is obeyed, i.e. the material behaves elastically and stress is proportional to strain, giving the straight-line graph indicated. Some point A is eventually reached, however, when the linear nature of the graph ceases and this point is termed the ***limit of proportionality***.

For a short period beyond this point the material may still be elastic in the sense that deformations are completely recovered when the load is removed (i.e. strain returns to zero) but Hooke's law does not apply. The limiting point B for this condition is termed the ***elastic limit***. For most practical purposes, it can often be assumed that points A and B are coincident.

Beyond the elastic limit, plastic deformation occurs and strains are not recoverable. There will thus be some permanent deformation or permanent set when the load is removed. After the point C termed the upper yield point and D the lower yield point.

For example, the certain materials of high carbon steels and non-ferrous metals, it is not possible to detect any difference between the upper and lower yield points and in some cases, no yield point exists at all. In such cases, proof stress is used to indicate the onset of plastic strain or as a comparison of the relative properties with another similar material. This involves a measure of the permanent deformation produced by a loading cycle. For example, the 0.1 % proof stress is that stress which, when removed, produces a permanent strain or "set" of 0.1 % of the original gauge length, as shown in Fig. l.4.

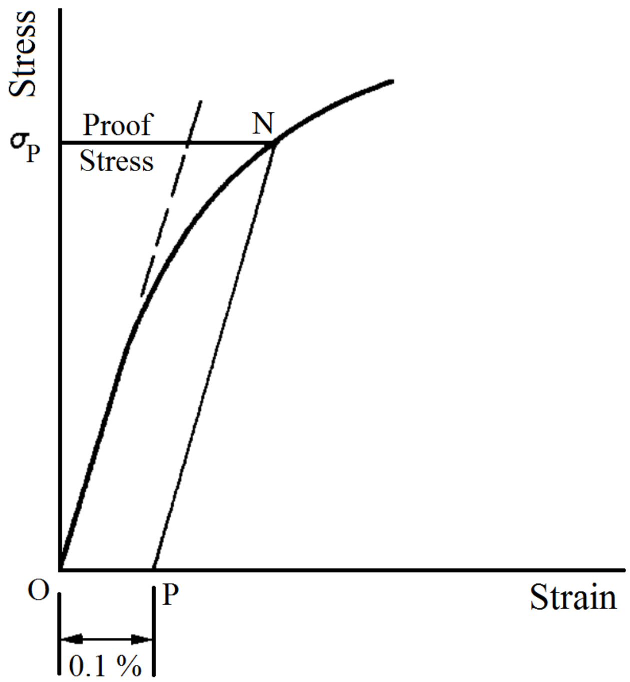

Fig. 1.4 Determination of 0.1 % proof stress.

1.8 Ductile and Brittle Materials

Ductility is the capacity of a material to allow large extensions or the ability to be drawn out plastically. While a ***brittle material*** is one which exhibits relatively small extensions to fracture so that the partially plastic region of the tensile test graph is much reduced.

$$\textit{Percentage elongation} = \frac{\textit{increase in gauge length to fracture}}{\textit{original gauge length}} \times 100\ \%$$

$$\textit{Percentage reduction in area} = \frac{\textit{reduction in cross-sectional area of necked portion}}{\textit{original area}} \times 100\ \%$$

1.9 Poisson's ratio

Consider the rectangular bar as shown in Fig. 1.5 subjected to a tensile load. Under the action of this load, the bar will increase in length by an amount δL giving a longitudinal strain in the bar of

$$\epsilon_L = \frac{\delta L}{L}$$

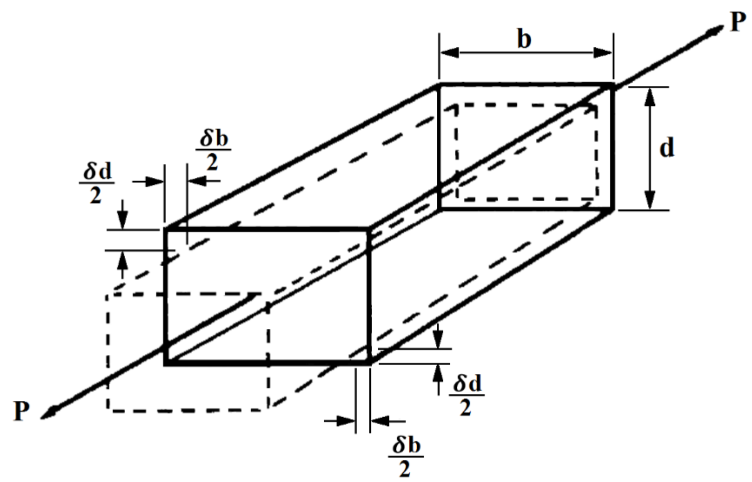

Fig. 1.5 Rectangular bar.

The bar will also exhibit, a reduction in dimensions laterally, i.e. its breadth and depth will both reduce. The associated lateral strains will both be equal, will be of opposite sense to the longitudinal strain, and given by

$$\epsilon_{lat} = -\frac{\delta b}{b} = -\frac{\delta d}{d}$$

$$\textit{Poisson's ratio } (\nu) = \frac{\textit{lateral strain}}{\textit{longitudinal strain}} = \frac{(-\delta d/d)}{\delta L/L}$$

Example: For the two-dimensional stress case:

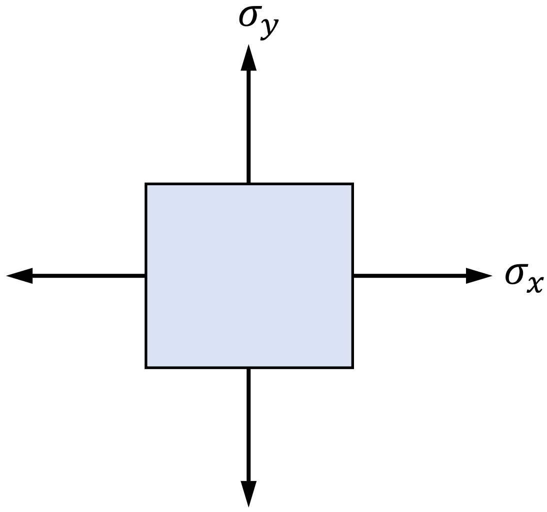

Fig. 1.6 Simple two-dimensional system of direct stresses.

The following strains will be produced

(a) In the X direction resulting from σ_x, $\epsilon_x = \sigma_x / E$,

(b) In the Y direction resulting from σ_y, $\epsilon_y = \sigma_y / E$.

(c) In the X direction resulting from σ_y, $\epsilon_x = -\nu(\sigma_y / E)$,

(d) In the Y direction resulting from σ_x, $\epsilon_y = -\nu(\sigma_x / E)$.

Strains (c) and (d) being the so-called ***Poisson's ratio strain***, opposite in sign to the applied strains, i.e. compressive.

The total strain in the X direction will therefore be given by:

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

and the total strain in the Y direction will be:

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

1.10 Shear Stress

Shear stress always tangential to the area on which it acts, and the direct stresses are always normal to the area on which they act.

$$\boldsymbol{Shear\ Strass\ (\tau) = \frac{shear\ load}{area\ resisting\ shear} = \frac{Q}{A}}$$

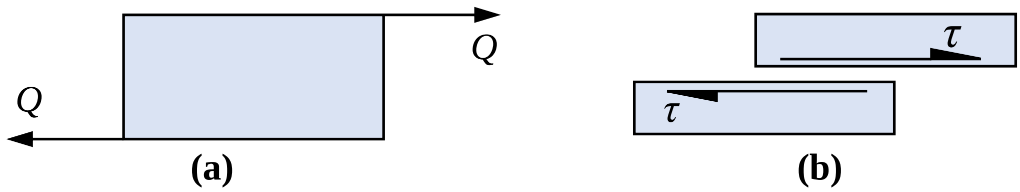

Fig. 1.7 Shear force and resulting shear stress system showing a typical form of failure by relative sliding of planes.

1.11 Shear Strain

The angle of deformation **γ** is the shear strain. Shear strain is dimensionless; it has no units and is measured in radians.

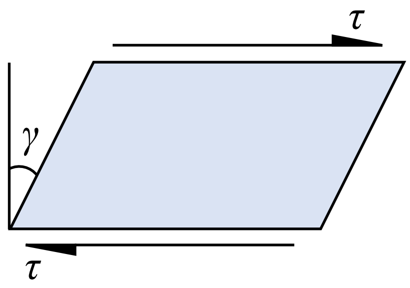

Fig. 1.8 Deformation (shear strain) produced by shear stresses.

For materials within the elastic range, the shear strain is proportional to the shear stress producing it,

$$\boldsymbol{\frac{shear\ stress}{shear\ strain} = \frac{\tau}{\gamma} = Constant = G}$$

The constant G is termed the modulus of rigidity or shear modulus and is directly comparable to the modulus of elasticity used in the direct stress application.

Example 1.1

Determine the stress in each section of the bar shown in Figure 1.9 when subjected to an axial tensile load of 20 kN. The central section is 30 mm square cross-section; the other portions are of circular section, their diameters being indicated. What will be the total extension of the bar? For the bar material E = 210 GN/m^2.

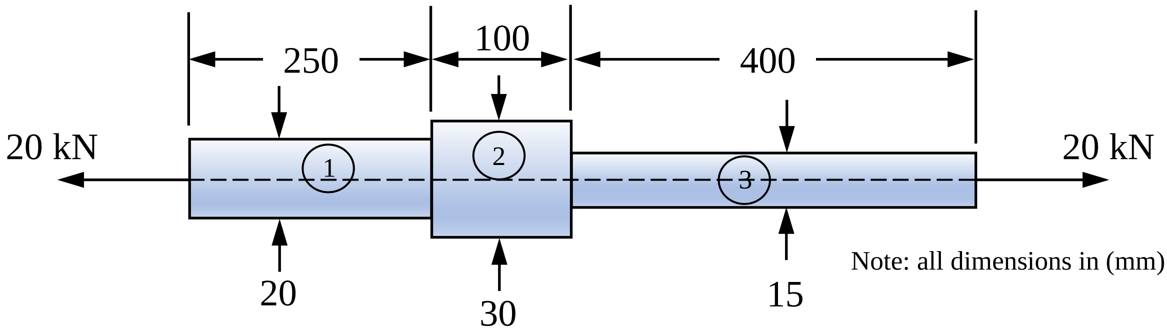

Figure 1.9 Multi section bar.

Solution:

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{P}{A}$$

$$\text{Stress in section (1)} = \frac{20\times10^3}{\frac{\pi(20\times10^{-3})^2}{4}} = \frac{80\times10^3}{\pi\times400\times10^{-6}} = 63.66\ \text{MN/m}^2$$

$$\text{Stress in section (2)} = \frac{20\times10^3}{30\times30\times10^{-6}} = 22.2\ \text{MN/m}^2$$

$$\text{Stress in section (3)} = \frac{20\times10^3}{\frac{\pi(15\times10^{-3})^2}{4}} = \frac{80\times10^3}{\pi\times225\times10^{-6}} = 113.2\ \text{MN/m}^2$$

Now the extension of a bar can always be written in terms of the stress in the bar since,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\delta / L}$$

$$\delta = \frac{\sigma L}{E}$$

$$\text{Extension of section (1)} = 63.66\times10^6\times\frac{250\times10^{-3}}{210\times10^9} = 75.8\times10^{-6}\ \text{m}$$

$$\text{Extension of section (2)} = 22.2 \times 10^6 \times \frac{100 \times 10^{-3}}{210 \times 10^9} = 10.6 \times 10^{-6} \text{ m}$$

$$\text{Extension of section (3)} = 113.2 \times 10^6 \times \frac{400 \times 10^{-3}}{210 \times 10^9} = 215.6 \times 10^{-6} \text{ m}$$

$$\text{Total extension} = (75.8 + 10.6 + 215.6)10^{-6}$$

$$= 302 \times 10^{-6}\ m$$

$$= \mathbf{0.302\ mm}$$

Example 1.2

(a) A 25 mm diameter bar is subjected to an axial tensile load of 100 kN. Under the action of this load, a 200 mm gauge length is found to extend 0.19×10^{-3} mm. Determine the modulus of elasticity for the bar material.

(b) In order to reduce weight whilst keeping the external diameter constant, the bar is bored axially to produce a cylinder of uniform thickness, what is the maximum diameter of bore possible given that the maximum allowable stress is 240 MN/m^2? The load can be assumed to remain constant at 100 kN.

(c) What will be the change in the outside diameter of the bar under the limiting stress quoted in (b)? (E = 210 GN/m^2 and v = 0.3).

(a) Young's modulus $E = \dfrac{PL}{A\ \delta L}$

$$= \frac{100 \times 10^3 \times 200 \times 10^{-3}}{\frac{\pi(25 \times 10^{-3})^2}{4} \times 0.19 \times 10^{-3}} = \mathbf{214\ GN/m^2}$$

(b) Let the required bore diameter (d) in (mm); the cross-sectional area of the bar will be reduced to

$$A = \left[\frac{\pi \times 25^2}{4} - \frac{\pi\, d^2}{4}\right] \times 10^{-6} = \frac{\pi}{4}\left(25^2 - d^2\right)10^{-6} \text{ m}^2$$

$$\text{stress in bar} = \frac{P}{A} = \frac{4 \times 100 \times 10^3}{\pi\left(25^2 - d^2\right)10^{-6}}$$

However, this stress is restricted to a maximum allowable value of 240 MN/m^2.

$$240 \times 10^6 = \frac{4 \times 100 \times 10^3}{\pi(25^2 - d^2)10^{-6}}$$

$$25^2 - d^2 = \frac{4 \times 100 \times 10^3}{240 \times 10^6 \times \pi \times 10^{-6}} = 530.5$$

$$d^2 = 94.48 \quad \text{and} \quad \mathbf{d = 9.72 \ \ mm}$$

The maximum possible bore is **9.72 mm.**

(c) The change in the outside diameter of the bar will be obtained from the lateral strain,

$$\text{lateral strain} = \frac{-\,\delta d}{d}$$

$$\text{Poisson's ratio } v = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{(-\,\delta d/d)}{\delta L/L}$$

$$\text{longitudinal strain } (\epsilon_L) = \frac{\delta L}{L} = \frac{\sigma}{E} = \frac{240 \times 10^6}{210 \times 10^9}$$

$$\frac{\delta d}{d} = -\,v\frac{\sigma}{E} = -\,\frac{0.3 \times 240 \times 10^6}{210 \times 10^9}$$

$$\text{change in outside diameter} = -\,\frac{0.3 \times 240 \times 10^6}{210 \times 10^9} \times 25 \times 10^{-3}$$

$$\boldsymbol{\delta d = -\,8.57 \times 10^{-6} \ \ \mathrm{m}} \ \text{(a reduction)}$$

Example 1.3

The coupling shown in Fig.1.10 is constructed from steel of rectangular cross-section and is designed to transmit a tensile force of 50 kN. If the bolt is 15 mm diameter calculate:

(a) The shear stress in the bolt.

(b) The direct stress in the plate.

(c) The direct stress in the forked end of the coupling.

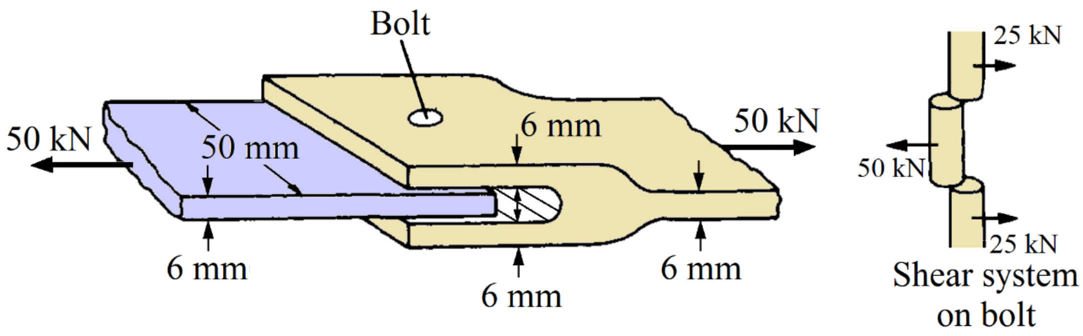

Figure 1.10 Steel rectangular coupling.

Solution:

(a) The bolt is subjected to double shear, tending to shear. There is thus twice the area of the bolt resisting the shear.

$$\text{shear stress in bolt} = \frac{P}{2A} = \frac{50 \times 10^3 \times 4}{2 \times \pi (15 \times 10^{-3})^2}$$

$$= \frac{100 \times 10^3}{\pi (15 \times 10^{-3})^2} = \mathbf{141.5 \ MN/m^2}$$

(b) The plate will be subjected to a direct tensile stress given by.

$$\text{tensile stress } \sigma = \frac{P}{A} = \frac{50 \times 10^3}{50 \times 6 \times 10^{-6}} = \mathbf{166.7 \ MN/m^2}$$

(c) The force in the coupling is shared by the forked end pieces, each being subjected to direct stress.

$$\text{direct stress } \sigma = \frac{P}{A} = \frac{25 \times 10^3}{50 \times 6 \times 10^{-6}} = \mathbf{83.3 \ MN/m^2}$$

Problems

1.1 A 25 mm square cross-section bar of length 300 mm carries an axial compressive load of 50 kN. Determine the stress set up in the bar and its change of length when the load is applied. For the bar material E = 200 GN/m^2.

[80 MN/m^2; 0.12 mm]

1.2 A steel tube, 25 mm outside diameter and 12 mm inside diameter, carries an axial tensile load of 40 kN. What will be the stress in the bar? What further increase in load is possible if the stress in the bar is limited to 225 MN/m^2?

[l06 MN/m^2; 45 kN]

1.3 Define the terms shear stress and shear strain, illustrating your answer by means of a simple sketch. Two circular bars, one of brass and the other of steel, are to be loaded by a shear load of 30 kN. Determine the necessary diameter of the bars (a) in single shear, (b) in double shear, if the shear stress in the two materials must not exceed 50 MN/m^2 and 100 MN/m^2 respectively.

[27.6, 19.5, 19.5, 13.8 mm]

1.4 Two fork-end pieces are to be joined together by a single steel pin of 25 mm diameter and they are required to transmit 50 kN. Determine the minimum cross-sectional area of material required in one branch of either fork if the stress in the fork material is not to exceed 180 MN/m^2. What will be the maximum shear stress in the pin?

[1.39×10^{-4} m^2; 50.9 MN/m^2]

1.5 A simple turnbuckle arrangement is constructed from a 40 mm outside diameter tube threaded internally at each end to take two rods of 25 mm outside diameter with threaded ends. What will be the nominal stresses set up in the tube and the rods, ignoring thread depth, when the turnbuckle carries an axial load of 30 kN? Assuming a sufficient strength of thread, what maximum load can be transmitted by the turnbuckle if the maximum stress is limited to 180 MN/m^2?

[39.2, 61.1 MN/m^2, 88.4 kN]

1.6 An I-section girder is constructed from two 80 mm × 12 mm flanges joined by an 80 mm × 12 mm web. Four such girders are mounted vertically one at each corner of a horizontal platform which the girders support. The platform is 4 m above ground level and weighs 10 kN. Assuming that each girder supports an equal share of the load, determine the maximum compressive stress set up in the material of each girder when the platform supports an additional load of 15 kN. The weight of the girders may not be neglected. The density of the cast iron from which the girders are constructed is 7470 kg/m^3.

[2.46 MN/m^2]

1.7 A bar ABCD consists of three sections: AB is 25 mm square and 50 mm long, BC is of 20 mm diameter and 40 mm long and CD is of 12 mm diameter and 50 mm long. Determine the stress set up in each section of the bar when it is subjected to an axial tensile load of 20 kN. What will be the total extension of the bar under this load? For the bar material, E = 210 GN/m^2.

[32, 63.7, 176.8 MN/m^2, 0.062 mm]

1.8 A steel bar ABCD consists of three sections: AB is of 20 mm diameter and 200 mm long, BC is 25 mm square and 400 mm long, and CD is of 12 mm diameter and 200 mm long. The bar is subjected to an axial compressive load, which induces a stress of 30 MN/m^2 on the largest cross-section. Determine the total decrease in the length of the bar when the load is applied. For steel E = 210 GN/m^2. [0.272 mm]

1.10 Figure 1.24 shows a special spanner used to tighten screwed components. A torque is applied at the tommy-bar and is transmitted to the pins, which engage into holes located into the end of a screwed component.

(a) Using the data given in Fig. 1.24 calculate:

(i) the diameter D of the shank if the shear stress is not to exceed 50 N/mm^2.

(ii) the stress due to bending in the tommy-bar.

(iii) the shear stress in the pins.

(b) Why is the tommy-bar a preferred method of applying the torque?

[9.14 mm; 254.6 MN/m^2; 39.8 MN/m^2]

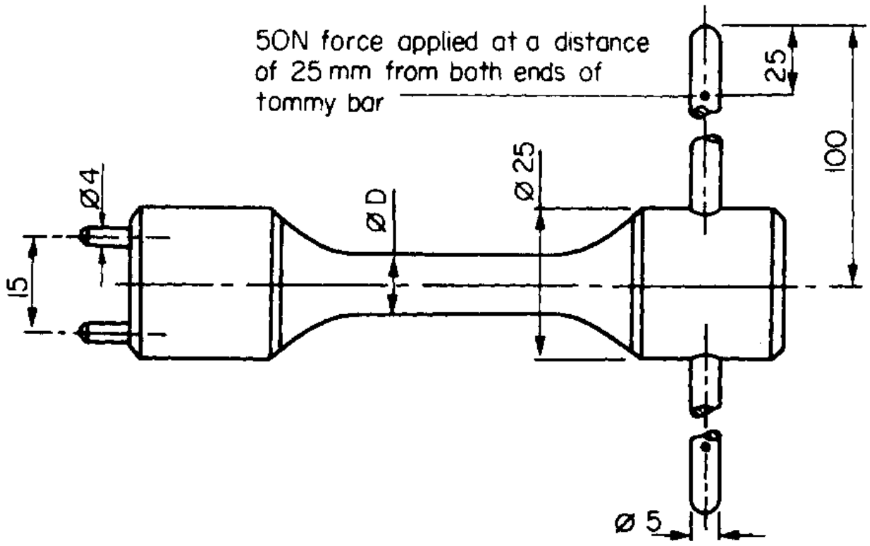
50N force applied at a distance of 25 mm from both ends of tommy bar
25
100
Ø 25
Ø D
Ø 4
15
Ø 5

Fig. 1.24