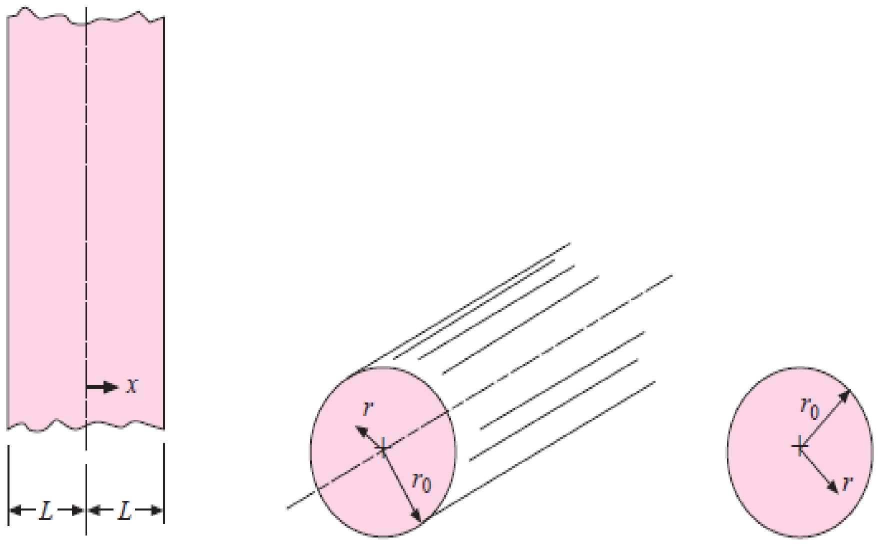
Temperatures charts (Heisler charts)

These diagrams are used to infinite bodies to find temperatures distribution and heat flow through plates, cylinders and spheres.

Nomenclature for one-dimensional solids suddenly subjected to convection environment at T_∞: (*a*) infinite plate of thickness $2L$; (*b*) infinite cylinder of radius r_0; (*c*) sphere of radius r_0.



T_0 = centerline temperature (*a*)
T_0 = centerline axis temperature (*b*)
T_0 = center temperature (*c*)

These bodies are exposed to a convection condition at T∞ and center temperature T_0.

The Biot number and Fourier number.

$$\text{Fourier number} = \text{Fo} = \frac{\alpha\tau}{s^2} = \frac{k\tau}{\rho c s^2}$$

For plates s = L ⟶ $\text{Fo} = \frac{\alpha\, t}{L^2}$ ⟶ $\text{Bi} = \frac{h\, L}{K}$

For cylinders and spheres s= r° ⟶ $\text{Fo} = \frac{\alpha\, t}{r^{\circ\, 2}}$ ⟶ $\text{Bi} = \frac{h\, r^\circ}{K}$

Figures (4-7)to(4-12) are the temperatures of solid as a function to the time.

Where: θ = (T - T∞) ⟶

θ = (T(x , t) - T∞) (For plates)

θ = (T(r , t) - T∞) (For cylinders and spheres)

θi = (Ti - T∞)

θ_0 = (T_0 - T∞)

Figure 4-7 | Midplane temperature for an infinite plate of thickness 2*L*: (*a*) full scale.

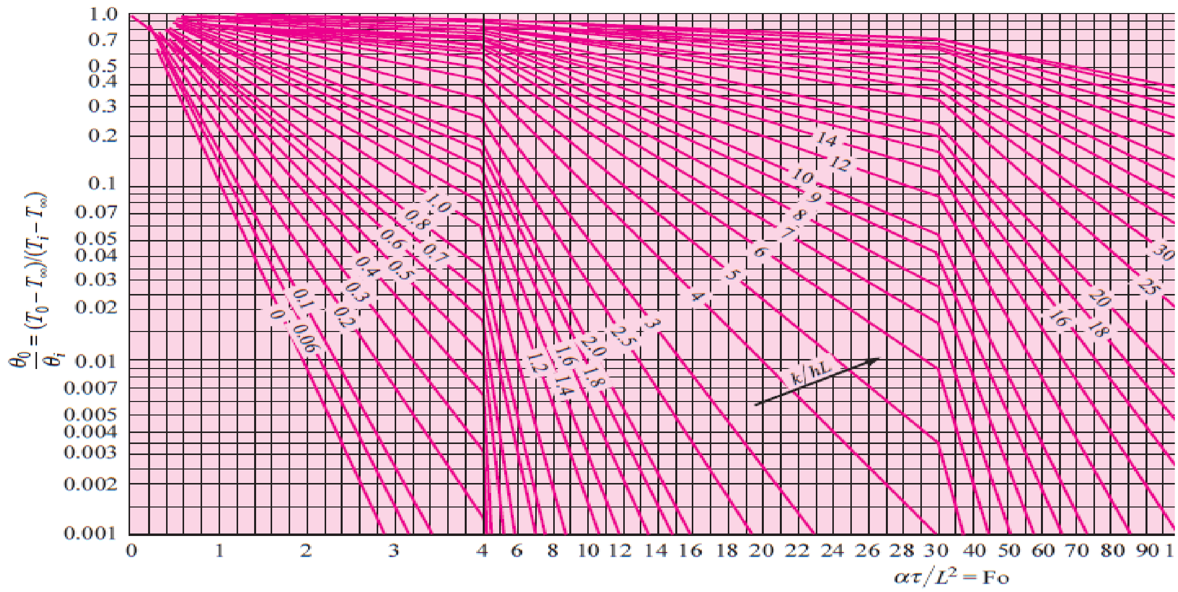


Figure 4-8 | Axis temperature for an infinite cylinder of radius r_0: (*a*) full scale.

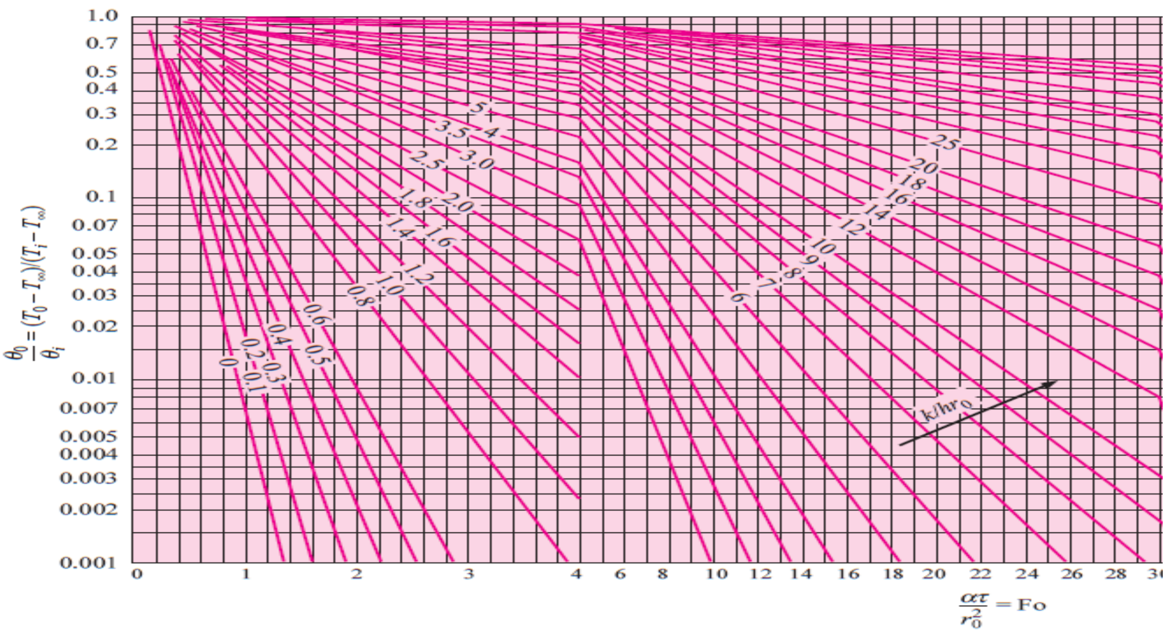


Figure 4-9 | Center temperature for a sphere of radius r_0: (a) full scale.

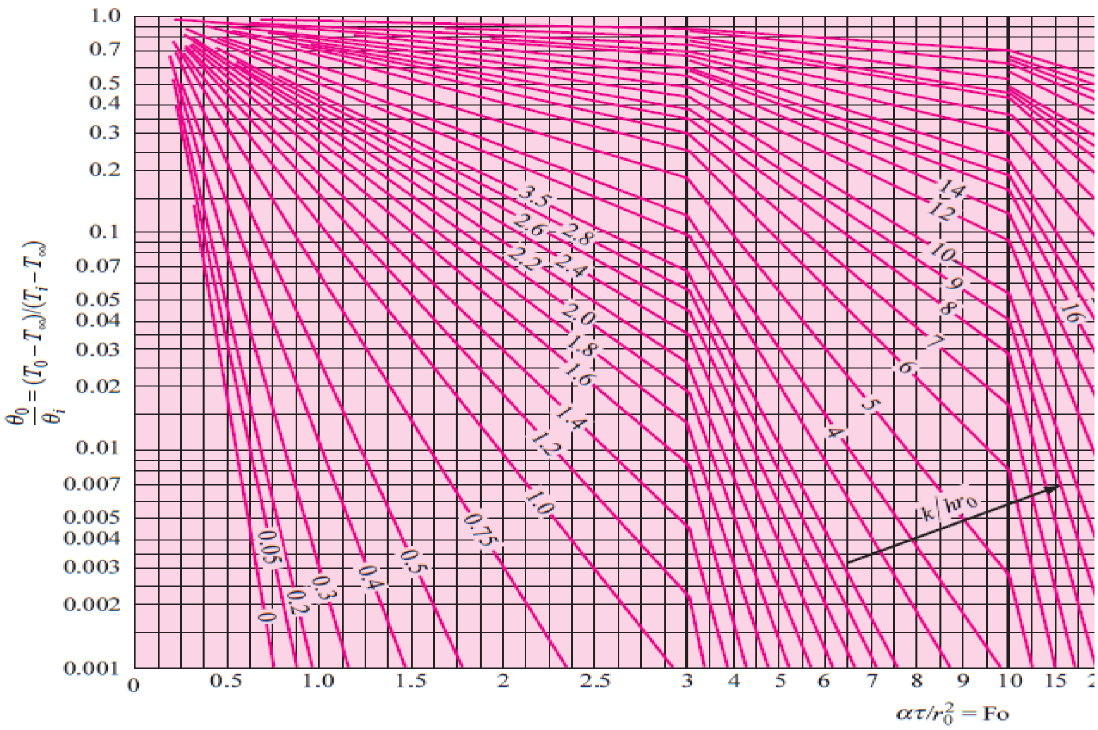


Figure 4-10 | Temperature as a function of center temperature in an infinite plate of thickness $2L$, from Reference 2.

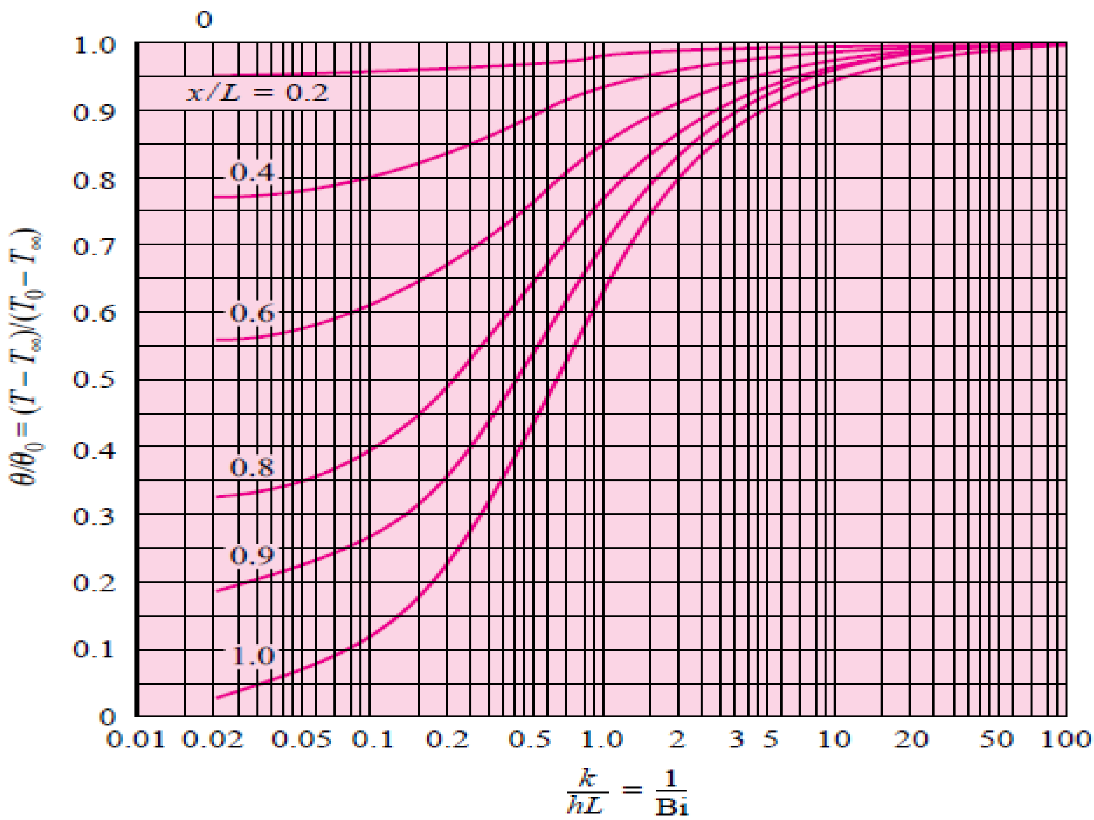


Figure 4-11 | Temperature as a function of axis temperature in an infinite cylinder of radius r_0, from Reference 2.

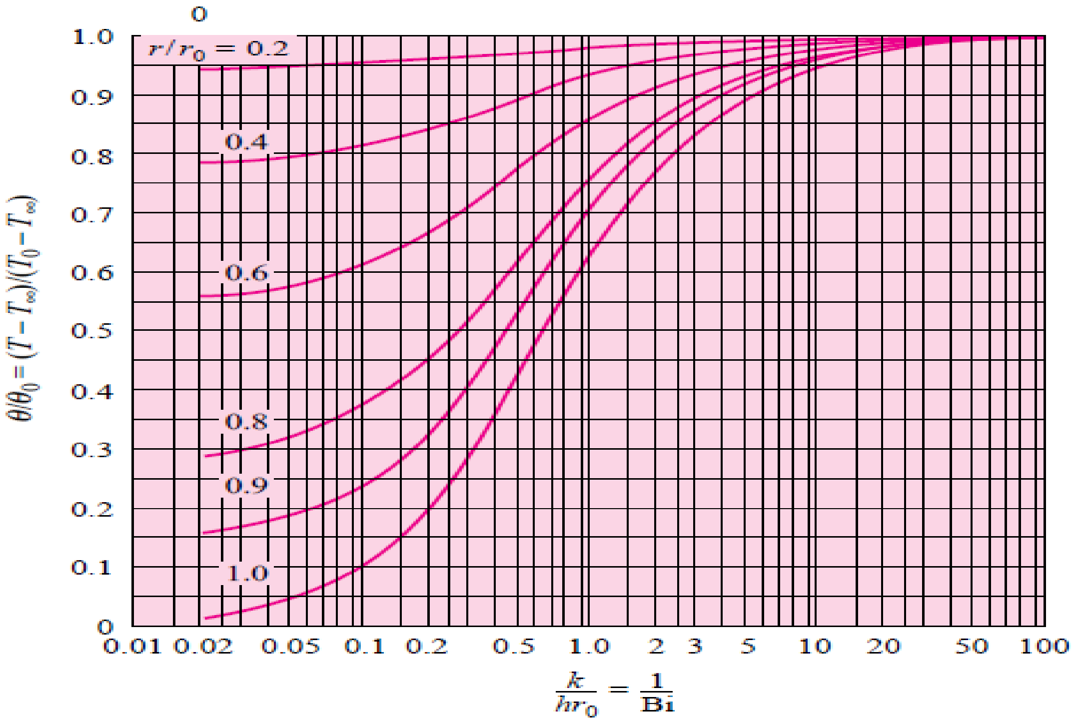
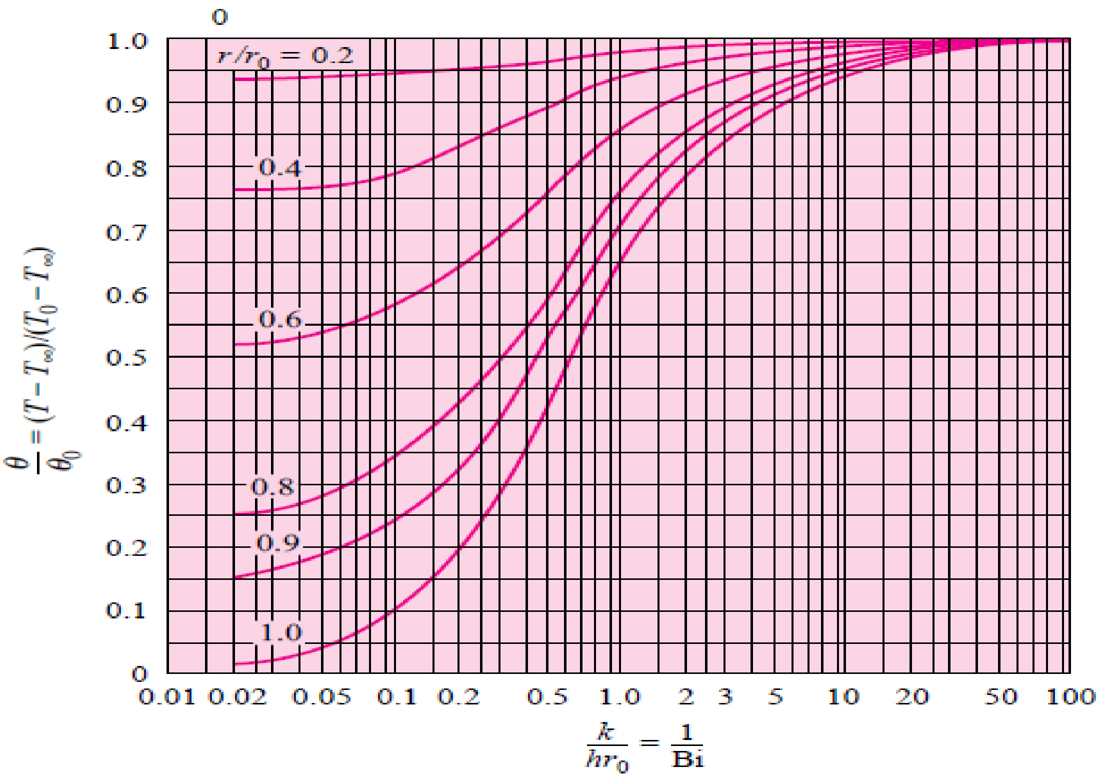


Figure 4-12 | Temperature as a function of center temperature for a sphere of radius r_0, from Reference 2.



The heat loss for the infinite bodies is given in figures (4-14) to (4-16).

Where: Q_0 represents the initial internal energy content of the body.

$Q_0 = \rho\ Cp\ v\ (Ti - T\infty) = \rho\ Cp\ v\ \theta_i$

While Q is the actual heat lost by the body

Figure 4-14 | Dimensionless heat loss Q/Q_0 of an infinite plane of thickness $2L$ with time,

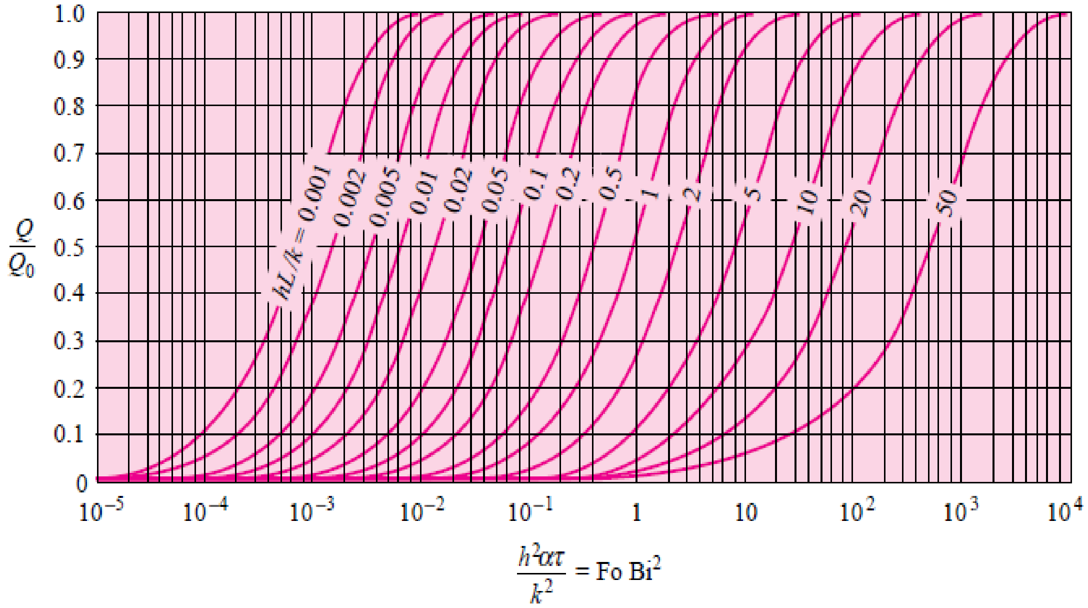


Figure 4-15 | Dimensionlesss heat loss Q/Q_0 of an infinite cylinder of radius r_0 with time,

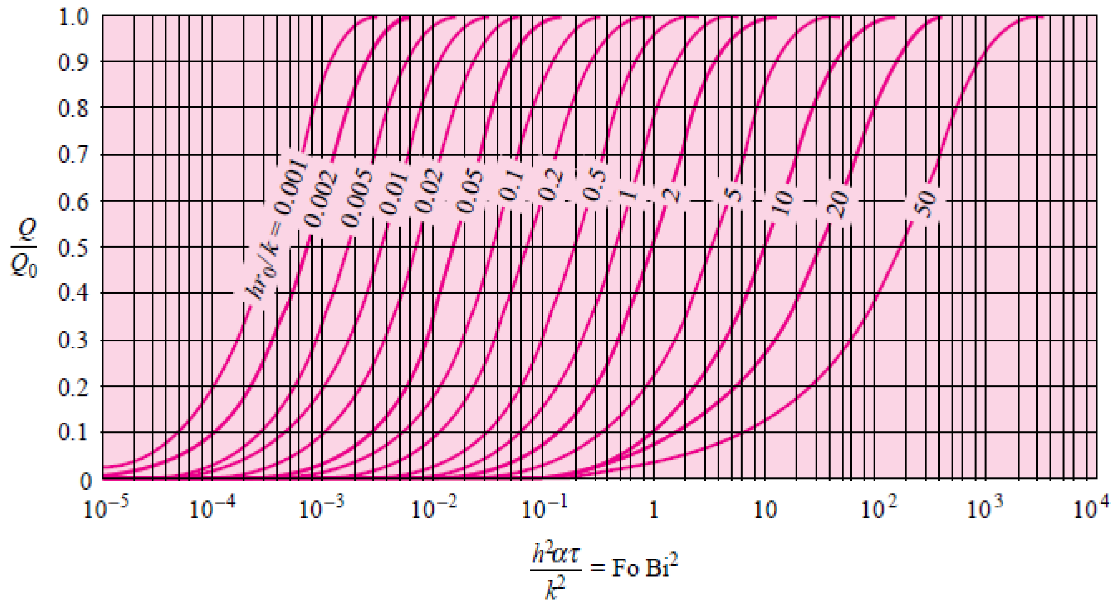
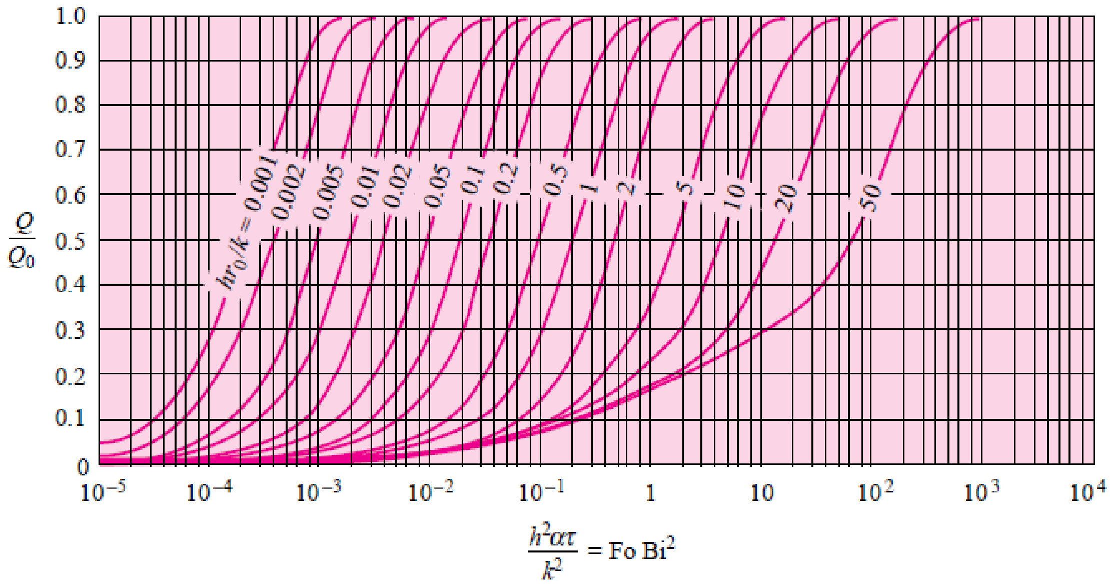


Figure 4-16 | Dimensionless heat loss Q/Q_0 of a sphere of radius r_0 with time



Notes

- Figures (4-7) , (4-8) , (4-9) → T_0 ,Ti
- Figures (4-10) ,(4-11) , (4-12) → T ,T_0
- Figures (4-14) , (4-15) , (4-16) → Q,Q_0

Applicability of the Heisler charts

The calculations for the Heisler charts were performed by truncating the infinite series solutions for the problems into a few terms. This restricts the applicability of the charts to values of the Fourier number greater than 0.2.

$$Fo = \frac{\alpha\tau}{s^2} > 0.2$$

Example:

An infinite stainless steel cylinder is heated to uniform temperature of 200 °C and then allowed to a cool environment where air temperature is 30 °C and heat transfer coefficient is 200 W/m^2.°C .The cylinder has diameter of 10 cm

a- Calculate the temperature of center after 10 min.

b- Heat loss per meter area.

$\alpha = 0.444 * 10^{-5}$ m²/sec , $\rho = 7817$ kg/m³ , Cp = 460 J/kg.°C ,
K=16.3 W/m.°C

Solution

Test Bi number

$$Bi = \frac{h*\left(\frac{v}{A}\right)}{K} < 0.1$$

$$\frac{v}{A} = \frac{\frac{\pi}{4}* d^2 *L}{\pi\, dL} = \frac{d}{4}$$

$$Bi = \frac{h*\left(\frac{v}{A}\right)}{K} = \frac{200*\left(\frac{0.1}{4}\right)}{16.3} = 0.3 > 0.1 \longrightarrow \text{No lumped heat capacity}$$

$$Fo = \frac{\alpha t}{r^{\circ 2}} = \frac{0.444 * 10^{-5}*600}{0.05^2} = 1.066$$

$$\frac{1}{Bi} = \frac{K}{h\, r^{\circ}} = \frac{16.3}{200*0.05} = 1.63$$

→ Figure (4-8) ↓

$$\frac{\theta 0}{\theta i} = 0.406 = \frac{T0-T\infty}{Ti-T\infty}$$

$$\frac{T0-30}{200-30} = 0.406 \longrightarrow T_0 = 99.02\ °C$$

$$Fo.Bi^2 = \frac{h^2 \alpha t}{K^2} = \frac{200^2 0.444 * 10^{-5}*600}{16.3^2} = 0.4$$

$$Bi = \frac{h\, r^{\circ}}{K} = \frac{200*0.05}{16.3} = 0.613$$

→ Figure (4-15) ↓

$$\frac{Q}{Q0} = 0.55$$

$$\frac{Q0}{A} = \rho\, Cp\, \frac{V}{A}\,(Ti - T\infty)$$

$$= 7817 * 460 * \frac{0.1}{4} * (200 - 30) = 15282235\ W/m^2$$

$$\frac{Q}{A} = \frac{Q0}{A} * 0.55 = 840\,5229.25\ W/m^2$$

Example:

A large plate of aluminum 5.0 cm thick and initially at 200◦ C is suddenly exposed to the convection environment at 70°C and heat transfer coefficient is 525 W/m^2. °C. Calculate the temperature at a depth of 1.25 cm and 1 min from one of the faces.

$\alpha = 8.4 * 10^{-5}$ m^2/sec , K=215 W/m.°C

Solution

2L = 5 cm = thckness ⟶ L = 2.5 cm

$$Fo = \frac{\alpha t}{L^2} = \frac{8.4 * 10^{-5} * 60}{0.025^{\,2}} = 8.064$$

$$\frac{1}{Bi} = \frac{K}{h\,L} = \frac{215}{525 * 0.025} = 16.38$$

⟶ Figure (4-7) ⟶

$$\frac{\theta 0}{\theta i} = 0.61 = \frac{T0 - T\infty}{Ti - T\infty}$$

x= 2.5 - 1.25 = 1.25 cm

$$\frac{x}{L} = \frac{1.25}{2.5} = 0.5$$

$$\frac{1}{Bi} = \frac{K}{h\,L} = \frac{215}{525 * 0.025} = 16.38$$

⟶ Figure (4-10) ⟶

$$\frac{\theta}{\theta 0} = 0.98 = \frac{T - T\infty}{T0 - T\infty}$$

$$\frac{\theta 0}{\theta i} = \frac{T0 - T\infty}{Ti - T\infty} = 0.61 \longrightarrow \frac{T0 - T\infty}{200 - 70} = 0.61 \longrightarrow \frac{T0 - T\infty}{130} = 0.61$$

$T_0 - T\infty = \theta_0 = 130 * 0.61 = 79.3$

$$\frac{\theta}{\theta 0} = 0.98 \longrightarrow \theta = 79.3 * 0.98 = 77.714$$

$\theta = T - T\infty \longrightarrow 77.714 = T - 70 \longrightarrow T = 147.714$ °C **Or**

$$\frac{\theta 0}{\theta i} * \frac{\theta}{\theta 0} = \frac{\theta}{\theta i} \longrightarrow 0.61 * 0.98 = 0.5978$$

$$\frac{\theta}{\theta i} = \frac{T - T\infty}{Ti - T\infty} \longrightarrow 0.5978 = \frac{T-70}{200-70}$$

$$T - 70 = 77.714 \longrightarrow T = 147.714\ °C$$

H.W

How much energy has been removed per unit area from the plate in this time?

ρ=2700kg/m^3, Cp = 0.9 kJ/kg · ° C

Example:

A long aluminum cylinder 5.0 cm in diameter and initially at 200◦ C is suddenly exposed to a convection environment at 70◦ C and h = 525 W/m^2 ·°C. Calculate the temperature at a radius of 1.25 cm after 1 min.

α= 8.4 * 10^{-5} m^2/sec , K=215 W/m.°C

Solution

$$Fo = \frac{\alpha t}{r°^2} = \frac{8.4 * 10^{-5} * 60}{0.025^2} = 8.064$$

$$\frac{1}{Bi} = \frac{K}{h\,r°} = \frac{215}{525 * 0.025} = 16.38$$

→ Figure (4-8) →

$$\frac{\theta 0}{\theta i} = 0.38 = \frac{T0 - T\infty}{Ti - T\infty}$$

$$\frac{r}{r°} = \frac{1.25}{2.5} = 0.5$$

$$\frac{1}{Bi} = \frac{K}{h\,r°} = \frac{215}{525 * 0.025} = 16.38$$

→ Figure (4-11) →

$$\frac{\theta}{\theta 0} = 0.98 = \frac{T - T\infty}{T0 - T\infty}$$

$$\frac{\theta 0}{\theta i} * \frac{\theta}{\theta 0} = \frac{\theta}{\theta i} \longrightarrow 0.38 * 0.98 = 0.372$$

$$\frac{\theta}{\theta i} = \frac{T - T\infty}{Ti - T\infty} \longrightarrow 0.372 = \frac{T-70}{200-70}$$

$$T - 70 = 48.4 \longrightarrow T = 118.4\ °C$$

Example:

A large slab of aluminum has a thickness of 10 cm and is initially uniform in temperature at 400◦ C. Suddenly it is exposed to a convection environment at 90◦ C with h = 1400 W/m^2·◦C. How long does it take the centerline temperature to drop to 180◦ C.

Solution

$\alpha = 8.418 * 10^{-5}$ m^2/sec , K=204 W/m.°C

2L = 10 cm = thckness ⟶ L = 5 cm

From figure (4-7) $Fo = \frac{\alpha t}{L^2} = \frac{8.418 * 10^{-5} * t}{0.05^{\ 2}}$

$\frac{\theta 0}{\theta i} = \frac{T0 - T\infty}{Ti - T\infty} = \frac{180-90}{400-90} = 0.29$

$\frac{1}{Bi} = \frac{K}{h\,L} = \frac{204}{1400*0.05} = 2.2$

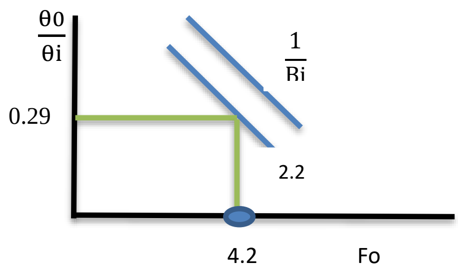


Figure (4-7)

Fo = 4.2

$4.2 = \frac{\alpha t}{L^2} = \frac{8.418 * 10^{-5} * t}{0.05^{\ 2}}$ ⟶ t =125 sec

Example:

A short aluminum cylinder 5cm in diameter and 10 cm thickness is initially at a uniform temperature of 200◦ C. It is suddenly subjected to a convection environment at 70◦ C, and h = 525 W/m^2 . ◦ C. Calculate the temperature at a radial position of 1.25 cm and a distance of 0.625 cm from one end of the cylinder 1 min after exposure to the environment.

$\alpha = 8.4 * 10^{-5}$ m^2/sec , K=215 W/m.°C

Solution

1- For plate

Thickness = 10 cm ⟶ 2L =10 ⟶ L = 5 cm

x= 5 – 0.625= 4.375 cm

$$Fo = \frac{\alpha t}{L^2} = \frac{8.4*10^{-5}*60}{0.05^{2}} = 2.016$$

$$\frac{1}{Bi} = \frac{K}{h\,L} = \frac{215}{525*0.05} = 8.19$$

→ Figure (4-7) →

$$\frac{\theta 0}{\theta i} = 0.75 = \frac{T0-T\infty}{Ti-T\infty}$$

$$\frac{x}{L} = \frac{4.375}{5} = 0.875$$

$$\frac{1}{Bi} = \frac{K}{h\,L} = \frac{215}{525*0.025} = 8.19$$

→ Figure (4-10) →

$$\frac{\theta}{\theta 0} = 0.95 = \frac{T-T\infty}{T0-T\infty}$$

$$\left(\frac{\theta}{\theta i}\right)plate = \left(\frac{\theta 0}{\theta i}\right)plate * \left(\frac{\theta}{\theta 0}\right)plate$$

$$\left(\frac{\theta}{\theta i}\right)plate = 0.75 * 0.95 = 0.7125$$

2- for cylinder

$$Fo = \frac{\alpha t}{r^{\circ 2}} = \frac{8.4*10^{-5}*60}{0.025^{2}} = 8.064$$

$$\frac{1}{Bi} = \frac{K}{h\,r^{\circ}} = \frac{215}{525*0.025} = 16.38$$

→ Figure (4-8) →

$$\frac{\theta 0}{\theta i} = 0.38 = \frac{T0-T\infty}{Ti-T\infty}$$

$$\frac{r}{r^{\circ}} = \frac{1.25}{2.5} = 0.5$$

$$\frac{1}{Bi} = \frac{K}{h\,r^{\circ}} = \frac{215}{525*0.025} = 16.38$$

→ Figure (4-11) →

$$\frac{\theta}{\theta 0} = 0.98 = \frac{T-T\infty}{T0-T\infty}$$

$(\frac{\theta}{\theta i})$cylinder= $(\frac{\theta 0}{\theta i})$ cylinder *$(\frac{\theta}{\theta 0})$ cylinder

$(\frac{\theta}{\theta i})$cylinder = 0.38 * 0.98 = 0.3724

$(\frac{\theta}{\theta i})$body= $(\frac{\theta}{\theta i})$ plate *$(\frac{\theta}{\theta i})$ cylinder

$(\frac{\theta}{\theta i})$body = 0.7125 * 0.3724 = 0.265

$\frac{T-T\infty}{Ti-T\infty}$ = 0.265

$\frac{T-70}{200-70}$ = 0.265

T = 104.5 °C

Note: as shown in Figures 4-14, 4-15, and 4-16, to obtain the heat for a multidimensional body. The results of this analysis for intersection of two bodies is

$$\left(\frac{Q}{Q_0}\right)_{total} = \left(\frac{Q}{Q_0}\right)_1 + \left(\frac{Q}{Q_0}\right)_2 \left[1 - \left(\frac{Q}{Q_0}\right)_1\right]$$