Convection resistance

$q_{Convection} = h\,A\,(Tw - T\infty)$

$q_{Convection} = \frac{(Tw - T\infty)}{\frac{1}{h\,A}}$, $\frac{1}{h\,A}$ terms becomes the convection resistance

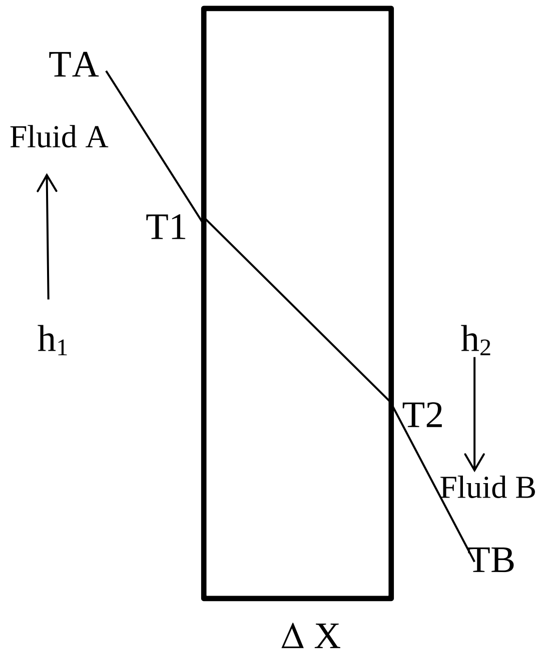
The overall heat transfer coefficient

Consider the plane wall shown in figure to a hot fluid A on one side and cooler fluid B on other side:

$q = h_1 A\,(TA - T1)$

$q = \frac{(T1 - T2)}{\frac{\Delta X}{KA}}$

$q = h_2\,A\,(T2 - TB)$



$q = \frac{(TA - TB)}{\frac{1}{h1\,A} + \frac{\Delta X}{K\,A} + \frac{1}{h2\,A}}$

The overall heat transfer by combined conduction and convention is expressed in terms at an overall heat transfer coefficient (U) defined by :

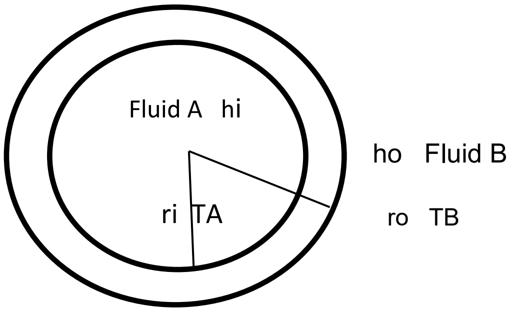
$q = U\,A\,\Delta T_{(overall)}$

$q = U\,A\,(TA - TB)$

$U\,A = \frac{1}{\frac{1}{h1\,A} + \frac{\Delta X}{K\,A} + \frac{1}{h2\,A}} \longrightarrow U = \frac{1}{\frac{1}{h1} + \frac{\Delta X}{K} + \frac{1}{h2}} = \frac{1}{R\ total}$

For a hollow cylinder exposed to convection on its inner and outer surface as shown in figure below the heat rate (q) is calculated:

$$q = \frac{(TA - TB)}{\frac{1}{hiAi} + \frac{\ln\left(\frac{ro}{ri}\right)}{2\pi K L} + \frac{1}{hoAo}}$$



The area of convection is not the same, the terms Ai, Ao represent the inside and outside surface area of the tube .The overall heat transfer coefficient may be based on either inside or outside area of the tube.

$$Ui = \frac{1}{\frac{1}{hi} + \frac{Ai \ln\left(\frac{ro}{ri}\right)}{2\pi K L} + \frac{Ai}{Ao}\frac{1}{h0}}$$ (Inside overall heat transfer coefficient)

$$UO = \frac{1}{\frac{Ao}{Ai}\frac{1}{hi} + \frac{Ao \ln\left(\frac{ro}{ri}\right)}{2\pi K L} + \frac{1}{h0}}$$ (Outside overall heat transfer coefficient)

Note: $Ai = 2\pi\, ri\, L$,

$Ao = 2\pi\, ro\, L$

Example:

A 0.5 ft thick of concrete wall having k = 0.5 Btu /h.ft. °F is exposed to air at 70 °F (hi= 2 Btu /h.ft^2. °F) and air at 20 °F (ho= 10 Btu /h.ft^2. °F)on the opposite side. Determine the heat transfer rate pet unit area.

Solution

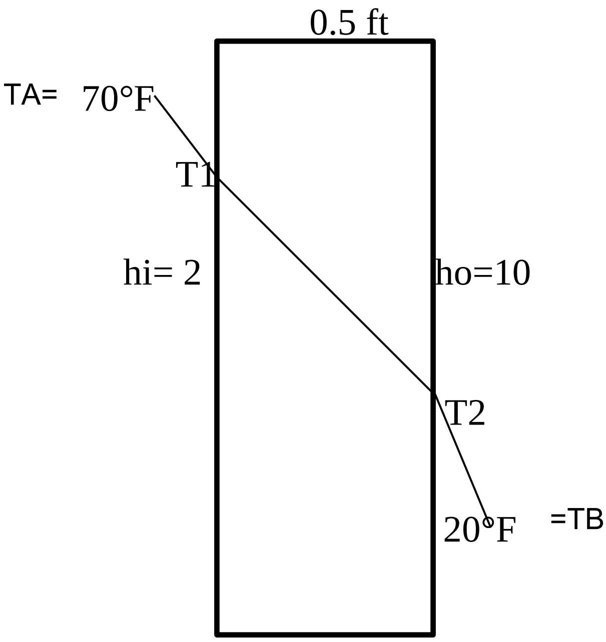
q = U A (TA –TB)

q /A = U (TA –TB)

$$q/A = \frac{1}{\frac{1}{hi} + \frac{\Delta X}{K} + \frac{1}{ho}} (TA - TB)$$

$$q/A = \frac{1}{\frac{1}{2} + \frac{0.5}{0.5} + \frac{1}{10}} (70 - 20)$$

=31.25 Btu/h.ft^2



Example:

Water flows at 50°C inside a 2.5-cm-inside-diameter tube such that hi = 3500 W/m^2· °C. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m · °C. The outside of the tube loses heat by free convection with ho = 7.6 W/m^2 · °C. Calculate the overall heat-transfer coefficient in the outside tube and heat loss to surrounding air at 20°C.Take the length is 1 m

Solution.

ri = 1.25 cm = 0.0125 m

ro = ri + 0.8 × 10 $^{-3}$= 0.0133 m

q = Uo A (TA –TB)

$$Uo = \frac{1}{Ao * R\ total}$$

$$R1 = \frac{1}{hiAi}$$

$$= \frac{1}{3500*2*\pi*0.0125*1} = 0.00364$$

$$R2 = \frac{\ln\left(\frac{ro}{ri}\right)}{2\pi\ K\ L}$$

$$= \frac{\ln\left(\frac{0.0133}{0.0125}\right)}{2*\pi*16*1} = 0.00062$$

$$R3 = \frac{1}{hoAo}$$

$$= \frac{1}{7.6*2*\pi*0.0133*1} = 1.575$$

$$Uo = \frac{1}{Ao\ \Sigma\ R} = \frac{1}{(2*\pi*0.0133*1)(0.00364+0.00062+1.575)}$$

= 7.577 W/m^2.°C

q= Uo Ao (TA-TB)

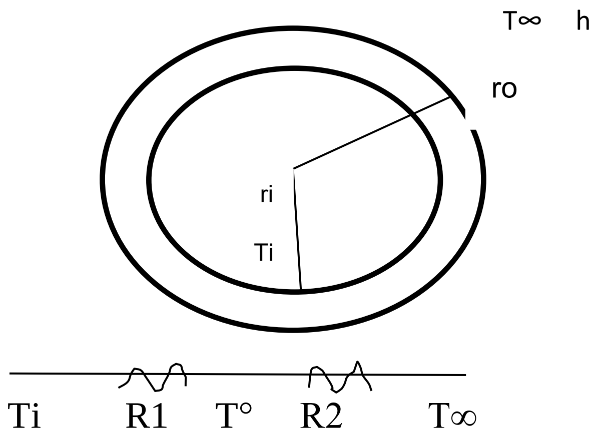
= 7.577* 2* π *0.0133*1*(50 -20)= 19 W

Critical thickness of insulation

The wall insulation is required in various process equipment, reactors, pipelines …etc to minimize heat loss between the system and environment. Consider a layer of insulation which might be installed around circular pipe as shown in figure the inner temperature of the insulation is Ti and the outer surface is exposed to a convection environment T∞

$$q=\frac{(Ti-T\infty)}{\frac{\ln\left(\frac{ro}{ri}\right)}{2\pi\, K\, L}+\frac{1}{2\pi\, ro\, L\, h}} \quad \text{or}$$

$$q=\frac{2\pi L(Ti-T\infty)}{\frac{\ln\left(\frac{ro}{ri}\right)}{K}+\frac{1}{ro\, h}}$$



To determine the outer radius of insulator ro, which will maximize the heat transfer. The maximization condition is

$$\frac{dq}{dro}=\frac{-\,2\pi L(Ti-T\infty)\frac{1}{K\,ro}+\frac{1}{hro\text{^}2}}{\left\{\frac{\ln\left(\frac{ro}{ri}\right)}{K}+\frac{1}{ro\, h}\right\}\text{^}2} \longrightarrow$$

$$ro=\frac{k}{h}= rc \text{ (critical radius)}$$

Example:

Calculate the critical radius of insulation for asbestos [k = 0.17 W/m ·◦C] surrounding a pipe and exposed to room air at 20◦C with h = 3 W/m^2· ◦C. Calculate the heat loss per unit length from a 200◦C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solution:

With insulation

ro = $\frac{k}{h}$

= $\frac{0.17}{3}$ = 0.0567 m = 5.67 cm

$$\frac{q}{L} = \frac{2\pi(Ti - T\infty)}{\frac{\ln\left(\frac{ro}{ri}\right)}{K} + \frac{1}{ro\ h}}$$

$$\frac{q}{L} = \frac{2\pi(200 - 20)}{\frac{\ln\left(\frac{0.0567}{0.025}\right)}{0.17} + \frac{1}{3 * 0.0567}} = 105.7$$

Without insulation

$\frac{q}{L}$ = h A (Tw-T∞)

= h * 2* π* ri *(Tw-T∞)

= 3*2*π* 0.025*(200-20) = 84.8w/m