University of Basrah
College of Engineering
Department of Electrical Engineering

Basics of Electrical Engineering I

First year

Lecturer: Dr. Basim T. Kadhem

University of Basrah College of Engineering Department of Electrical Engineering Class: First year Subject: Basic of Electrical Engg. I Lecturer: Dr.Basim Talib Kadhem	Theoretical: 3 hr/wk Tutorial: 1 hr/wk Practical:- -- hr/wk

Content

Chapter One
Basic Concept and Units

1.1 Modern Electron Theory:

According to this theory, all matter whether solid, liquid, or gaseous consists of Minute particles called molecules. Which are themselves made up of still minute particles known as atoms. Those substances whose molecules consist of similar atoms are known as elements.

An atom is the smallest particles of matter that can take part in a chemical change.

Whose molecules consist of dissimilar atoms are called compound.

The atoms consist of three basic parts:

1- Electrons negative charge

2- Protons positive charge

3- Neutrons neutral

(Protons and Neutrons: Nucleus)

An atom is taken to consist of the following:

(i) It has a hard central core known as nucleus. It contains two types of particles, one is known as proton which carries positive charge, the other is neutron.

(ii) A revolving particles around the relatively massive nucleus, in one or more elliptical orbits are infinitesimally small particles known as electrons.

Mass of proton = 1836 times that of electron.

The maximum number of electrons that can accommodate in an orbit is given by $2*n^2$ where n is the number of orbit.

Orbit No.	No. of electrons
1	$2*1^2=2$
2	$2*2^2=8$
3	$2*3^2=18$
4	$2*4^2=32$

1.2 Electric Charge:

Whether a given body exhibits a charge or not depends upon the relative no. of protons and electrons.

a)- if the no. of protons is equal to the no. of electrons in a body, the body will be neutrally charged.

b)- if some electrons are removed from a neutralized body, then the body will be positively charged.

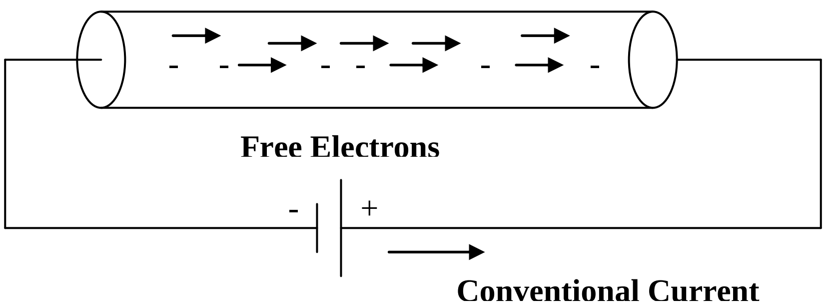
c)- if a neutralized body is supplied with electrons then the body will be negatively charged.

In practice, Coulomb is used as the unit of charge.

1 Coulomb = charge on $6.28*10^{18}$ electrons.

or charge of electron = $1/6.28*10^{18} = 1.6 *10^{-19}$ C

1.3 Movement of electrons and electric current:



They are also called free electrons and they move at random from one atom to another in materials. The flow of free electrons is called electric current.

The conventional current flows in the opposite direction to that of electron current.

The strength of electric current I is the rate of change of electric charge in time.

$$I = \frac{dQ}{dt}$$

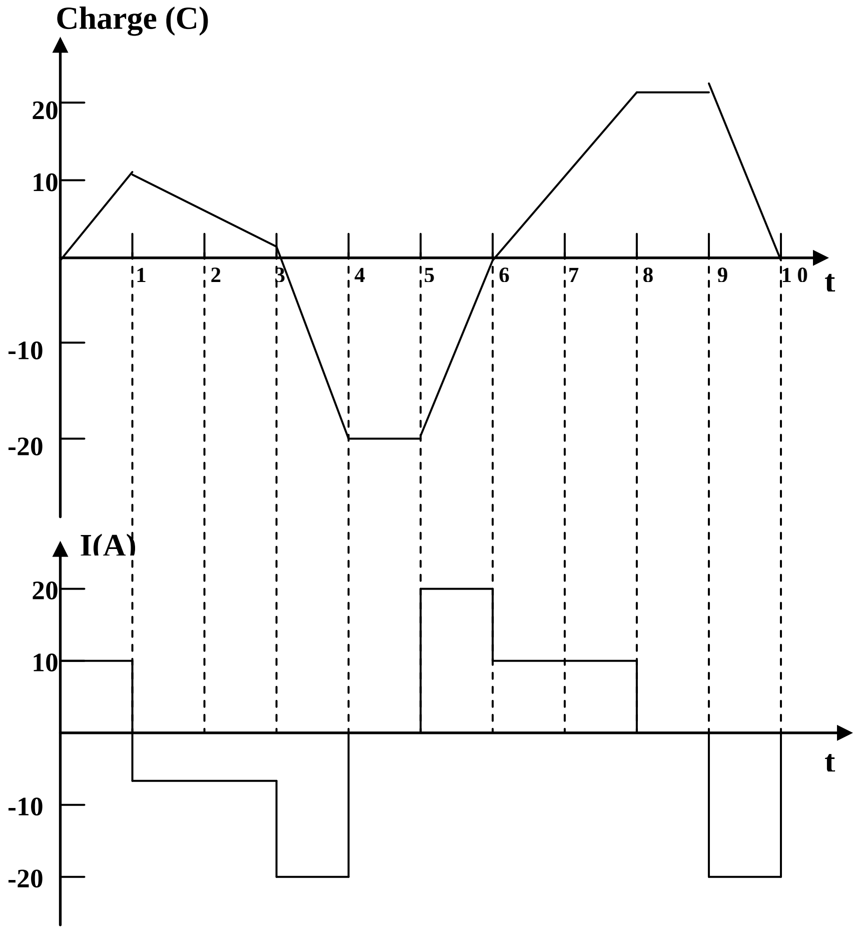
Where ***I*** =Current in Ampere (A)

Q = Charge in Coulomb (C)

t = Time in seconds (sec.)

Example: Fig. 1 represents a graph of the flow charge that has passed a given reference point. Sketch the current waveform.

1) for 0<=t<1



Conditions of continuous current flow:

1- There must be a complete circuit around which the electrons may move.

2- There must be a source which caused the current.

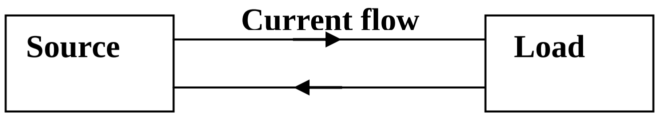


Fig. 2 Simple Circuit.

The electric circuit consists of four main parts:

Source, load unit, transmission system, and control units. Fig. 3 shows the different parts of an electric circuit.

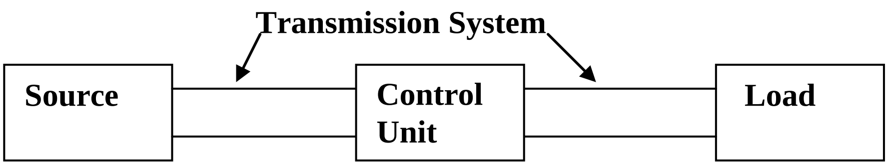


Fig. 3 Parts of an electric circuit.

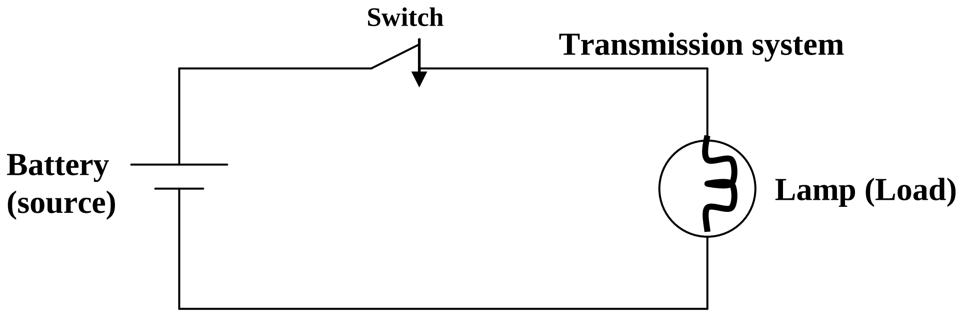


Fig. 4 An example of basic electrical circuit.

1.4 Electrical Potential, Potential Difference and Electromotive Force:

- Electric Potential: is the capacity of a charge to do work.

- Or the absolute potential of a point was defined as the amount of work done to bring a unit positive charge from infinity to the point.

Electrical potential = work/charge (Joule/Coulomb)

- Potential Difference (PD): is the difference in the potential between two charged bodies. It is the energy produced due to transferring the unit charge between two points in a circuit.
- Electromotive force (emf): represents the energy dissipated due to passing of a unit charge through the source.

1.5 **Work and Power;**

Potential difference in volt between any two points has been defined as equal to the work done (in joules) in moving a charge of one coulomb from one point to another.

If q coulombs of charge, the work done will be:

W =V q (joules)

The power (P) is the rate of work done.

$$P = \frac{dw}{dt} = \frac{d(vq)}{dt} = v\frac{dq}{dt} = vi$$

Example: Find the charge in coulomb that must pass a fixed point in one hour in a 100w light bulb at 120 volts. Assume direct current.

Solution: I=P/V =100/120 = 0.833 Amp.

I=Q/t

Q= I * t =0.833*3600=3000 C.

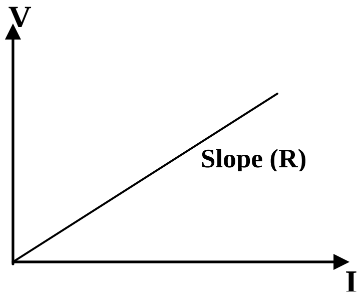
1.6 Principle of Ohm's law

The ratio of the PD between the ends of a conductor to the current flowing between them is constant provided that the physical conditions do not change.

$$\frac{V}{I} = R \quad = \text{constant}$$

Where R : the resistance of the conductor between the two points. The unit of the resistance is ohms (Ω).

If ohm's low expressed graphically (taking voltage along Y-axis and current along X-axis) the graph will be a straight line passing through the origin. The slop of the line will represent the resistance.



1.7 <u>The SI system of units</u> [Standard International Units]

The SI system of units selects six physical quantities as basis:

Mass, Length, Time, Electric current, Absolute Temperature, and luminous intensity.

All other units are derived units and related to the basic units by definition.

Quantity	Unit	Symbol
Mass	Kilogram	Kg
Length	Meter	m
Time	Second	Sec.
Electric Current	Ampere	A – (Amp.)
Absolute Temperature	Kelvin	K
Luminous intensity	Candela	Cd

The SI units has the advantages of grouping units in multiples and sub multiples.

multiple	prefix	Symbol
10	peca	da
10^2	Hecto	h
10^3	Kilo	K
10^6	mega	M
10^9	Giga	G
10^{12}	Tera	T

Fraction	**prefix**	Symbol
10^{-1}	deci	d

10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Derived Units:

1- Area A m^2
2- Volume V m^3
3- Velocity v m/s
4- Acceleration a m/s^2
5- Angular velocity ω rad/sec.
6- Work or energy W Joule
   $W=F\,l = N.\,m = kg\ m^2/sec^2$
7- Force F Newton=Kg m/sec^2
8- Torque T N.m
9- Power P watt=J/sec.
10- potential V volt= J/C
11- Charge C A.sec.
12- Resistance R Ω=ohm=V/A
13- Energy Kwh: it is unit of energy which is used commercially (kilowatt hour). It represents the work done by working at the rate of one kilowatt for a period of one hour.

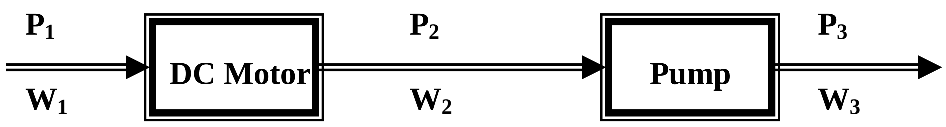
1 kwh = 1000 wh
= 1000*60 w.min
=1000*60*60 w.sec.
$=3.6*10^6$ J= 3.6 MJ

14- Efficiency η (unitless)

$$\eta = \frac{output\ power}{input\ power} * 100\%$$

15- hours power hp
   1 hp= 746 watts.

1.8 D.C. Motor driving a pump:



P_1 = input electrical power to the motor = VI (watt)

W_1 = total input electrical energy = (kwh)

P_2 = Output mechanical power = $T.\omega$

= T. 2*π n , where n=no. of revolutions/sec.

$W_2 = P_2$ * time of operation (J)

$P_3 = W_3$/time of operation (watt)

W_3 = m. g. h

Where m: mass, g: gravity, and h: height

Example: A 500 V dc motor of efficiency 95% derives a pump of efficiency 80%. The pump raises 1200 m^3 of water per hour against a head of 25m. Calculate the input current to motor given that 1 m^3 of water has mass of 1000 kg.

Solution: the output power P_3 of the pump is the work to raise the water to 25 m height at the given period of time.

$P_3 = W_3/t = (m. g. h)/t = [(1200*1000)*9.81*25]/3600$

=81750 watt

$\eta_{pump} = P_3/P_2$

P_2 = 81750/0.8 = 102187.5 watt = Mech. Output of the motor.

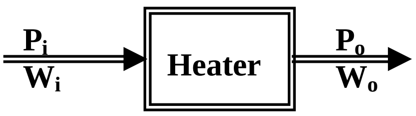
$\eta_{motor} = P_2/P_1$

P_1 =102187.5/0.95 = 107565.78 watts

= VI = 500*I

I = 215.13 Amp.

1.9 Electric Heater:



P_i = electric input power (watt)

=VI

W_i = electric input energy (kwh)

= P_i *time of operation.

P_o = output power (w)

W_o = output energy

= mass of water * specific heat * $(t_2\text{-}t_1)$

(caloric)

Example: It is desired to raise the temperature of $453.6*10^5$ grams of water from 25^oC to 95^oC in 10 hours. Find the rating of the heater in Kw assuming the overall efficiency of the heater to be 92%.

Solution: Energy required to heat the water from 25^oC to 95^oC = W_o

W_o= mass of water *specific heat* $(t_2\text{-}t_1)$

$= 453.6*10^5*1*(95-25)$

$= 3175.2*10^6$ Calories

[1 Caloric = 4.2 Joules]

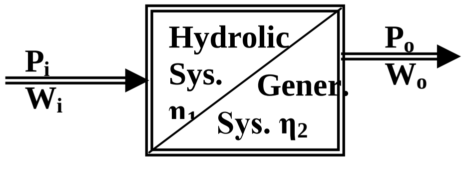
$W_o=3175.2*10^6*4.2= 1.33358*10^{10}$ Joules.

$\eta = W_o/W_i$

$W_i = W_o/\eta = 1.33358*10^{10}/0.92 =1.449547*10^{10}$ J (input energy to the heater)

Input power = $1.449547*10^{10}/(10*3600*1000)$ = 403 Kw.

1.10 Hydro electric power station:



P_o= electrical output power (w)

W_o= output electrical energy (kwh)

W_i= input mech. Energy.

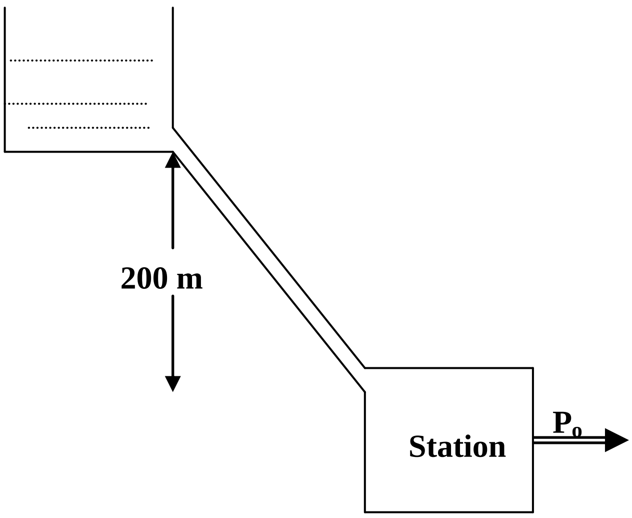
= mass of water*9.81*height (J)

$P_i= W_i$/(time of operation)

$\eta_{total}= \eta_1\ \eta_2$.

Example: A 100Mw hydro-electric station is supplying full load for 10 hours in a day. Calculate the volume of water which has been used. Assume effective head of water as 200 m and efficiency of the station is 80%.

Solution:



W_o= Total energy supplied

= 100Mw*10 hours

$=100*10^3*10 = 10^6$kwh

$\eta=W_o/W_i$

$W_i= 10^6$kwh/o.8

$=1.25*10^6$ kwh

1 kwh = 3.6 MJ

$W_i = 1.25*10^6*3.6*10^6= 4.5*10^{12}$ J

= m*9.81*h = m*9.81*200

$m= 2.293*10^9$ kg

since $1m^3$=1000 kg

volume of the used water = $2.293*10^9/1000 =2.293*10^6$ m^3

Chapter Two

Analysis of D.C. Circuit

2.1 Introduction :

An electric circuit is a closed path or combination of baths through which the current can flow. Fig. 2.1 shows a simple dc circuit. The direct current starts from positive terminal of the battery and comes back to the starting point Via the load.

Fig. 2.1 Simple dc Circuit.

2.2 Resistance and resistivity:

The resistance that any piece of material possesses may be shown experimentally to depend on:

a- It is directly proportional to the length and inversely proportional to the cross sectional area of the material.

b- It depends upon the type of material of which the conductor made of.

c- It depends upon the ambient physical factors such as temperature.

$$R \alpha \frac{l}{A} \qquad\qquad R = \rho \frac{l}{A}$$

Where ρ is an appropriate constant of proportionality for the material at the given ambient conditions, it is known as the resistivity of the material. It's unit is Ω.m.

2.3 Conductance and conductivity:

The reciprocal of the resistance is called as the conductance and it represented by [G].

$$G = \frac{1}{R} = \frac{a}{\rho l}$$ ℧ (mho) Simens (s)

$$\frac{1}{\rho} = \sigma$$ mho/m

The reciprocal of the resistivity is called the conductivity and its unit is mho/m.

$$G = \sigma \frac{a}{l}$$

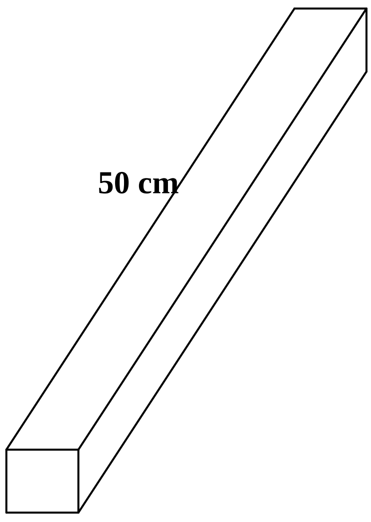
<u>Example:</u> A rectangular carbon block has the following dimensions:

1.0 cm*1.0 cm* 50 cm.

a) What is the resistance measured between the two square ends?

b) What is the resistance measured between the two opposing rectangular faces?

Resistivity of carbon at 20°C is $3.5*10^{-5}\ \Omega m$



<u>Solution:</u> a) l=50 cm =0.5 m

a= 1*1=1cm^2= 10^{-4} m^2

$$R = \rho \frac{l}{A}$$

$= 3.5*10^{-5}*0.5/10^{-4}=0.175\ \Omega$

c) l=1 cm=0.01 m

a=1*50=50 cm^2=50*10^{-4} m^2

R= $3.5*10^{-5}*0.01/(50*10^{-4})=7*10^{-5}\ \Omega$

<u>Example:</u> A piece of silver wire has a resistance of 1 Ω. What will be the resistance of manganese wire of one-third the length and one-third the diameter of the silver wire. The specific resistance of manganese is 30 times that of silver.

<u>Solution:</u>

$$R_1 = \rho_1 \frac{l_1}{A_1} \qquad\qquad R_2 = \rho_2 \frac{l_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} * \frac{l_2}{l_1} * \frac{a_1}{a_2}$$

$a_{1=}\ \pi d_1^2/4$, $a_{2=}\ \pi d_2^2/4$

$$\frac{a_1}{a_2} = \frac{d_1^{\,2}}{d_2^{\,2}}$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} * \frac{l_2}{l_1} * \frac{d_1^{\,2}}{d_2^{\,2}}$$

$R_2 = 90\ \Omega$

2.4 Effect of temperature on resistance: For metals the resistance increases with increasing temperature, while for other materials, including semi-conductors and some insulators, it decreases with increasing temperature. The resistance can be shown to change linearly with temperature, see Fig. 2.2.

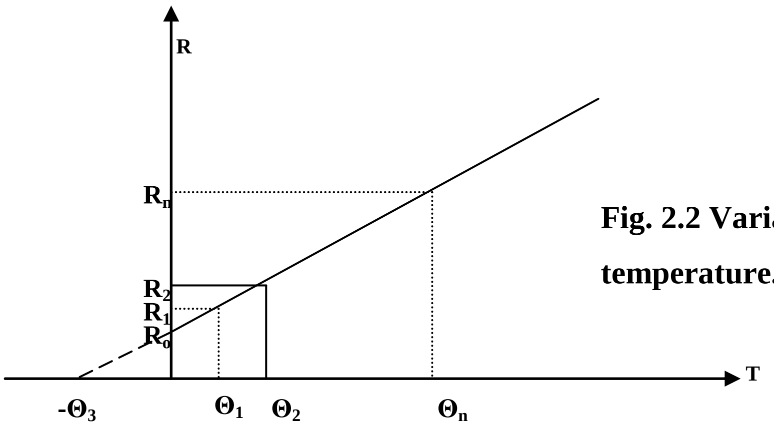


Fig. 2.2 Variation of resistance with temperature.

In order to estimate the change of resistance due to the changes of temperature a new coefficient α is introduced.

$$\alpha_1 = \frac{\dfrac{R_2 - R_1}{\theta_2 - \theta_1}}{R_1} \qquad \ldots\ldots (1)$$

Where α_1 is the temperature coefficient at temperature θ_1. Similarly at temperature θ_2:

$$\alpha_2 = \frac{\dfrac{R_2 - R_1}{\theta_2 - \theta_1}}{R_2} \qquad \text{...... (2)}$$

Re-arranging equation of α_1 gives:

$\alpha_1 R_1 (\theta_2 - \theta_1) = R_2 - R_1$

$R_2 = R_1 [1+\alpha_1 (\theta_2 - \theta_1)] = R_1 [1+\alpha_1 (\Delta t)]$

Usually the temperature coefficient is measured at 0^oC. If R_o is the corresponding resistance, then:

$R_1 = R_o [1+\alpha_o (\theta_1 - 0)] = R_o [1+\alpha_o t_1]$

$R_2 = R_o [1+\alpha_o (\theta_2 - 0)] = R_o [1+\alpha_o t_2]$

$$\frac{R_1}{R_2} = \frac{R_o[1+\alpha_o\theta_1]}{R_o[1+\alpha_o\theta_2]} = \frac{1+\alpha_o\theta_1}{1+\alpha_o\theta_2}$$

The temperature coefficient of resistance (α_n) at any other temperature θ_n can be determined as follows:

$$\alpha_n = \frac{\dfrac{R_n - 0}{\theta_n + \theta_3}}{R_n} = \frac{1}{\theta_n + \theta 3} \qquad \text{....... (3)}$$

From the slope of the straight line:

$$\frac{R_2 - R_1}{\theta_2 - \theta_1} = \frac{R_1}{\theta_1 + \theta_3}$$

Substituting in equation (1) gives:

$$\alpha_1 = \frac{\dfrac{R_1 - 0}{\theta_1 + \theta_3}}{R_1} = \frac{1}{\theta_1 + \theta 3}$$

$$\theta_3 = \frac{1 - \alpha_1\theta_1}{\alpha_1}$$

Substituting in eq. (3) gives:

$$\alpha_n = \frac{1}{\theta_n + \dfrac{1-\alpha_1\theta_1}{\alpha_1}} = \frac{\alpha_1}{1+(\theta_n-\theta_1)\alpha_1}$$

$$\alpha_1 = \frac{\alpha_o}{1+\alpha_o\theta_1}$$

And $\alpha_2 = \dfrac{\alpha_1}{1+\alpha_1(\theta_2-\theta_1)}$

or $\alpha_2 = \dfrac{\alpha_o}{1+\alpha_o\theta_2}$

Example: The field winding of a generator has a resistance of 12.9 Ω at 18°C and 14.3 Ω at 50°C. Find : 1- temperature coefficient at 0°C.
2- resistance at 0°C. 3- temperature coefficient at 18°C.

<u>Solution:</u>

$R_1=R_o[1+\alpha_o\, t_1]$ $R_2=R_o[1+\alpha_o t_2]$

$$\frac{R_1}{R_2} = \frac{1+\alpha_o t_1}{1+\alpha_o t_2}$$

$R_1=R_o[1+\alpha_o t_1]$

$\alpha_1=\alpha_o/(1+\alpha_o\, \theta_1)$

<u>Example</u>: The filament of a 240 V lamp is to be manufactured from a wire having a diameter of 0.002 mm and a resistivity of 4.3 μΩ.cm at 20 °C. If α=0.005/°C, what l ength of filament is necessary if the lamp is to dissipate 60 watts at a filament temperature of 2420°C.

Circuit Analysis

There are two general approaches to network analysis:

Direct methods

1- Kirchhoff's low

2- loop analysis

3- Nodal analysis

4- Super-position theorem

Reduction methods

1- Delta-star conversion

2- star-delta conversion

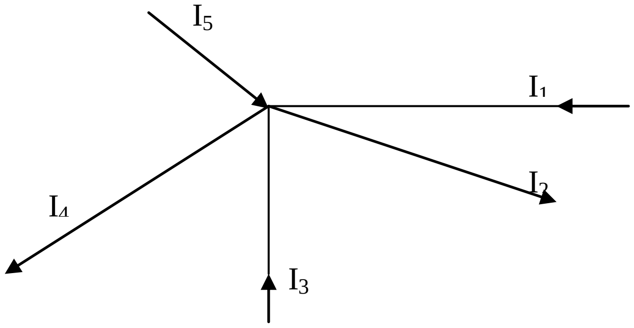
3- Thevenin's theorem

4- Norton's theorem

2.5: Kirchhoff's lows: "First law"(Current law): At any instant, the algebraic sum of the currents at a junction in a network is zero. The currents flowing towards the junction have been considered positive whilst those flowing from the junction negative.

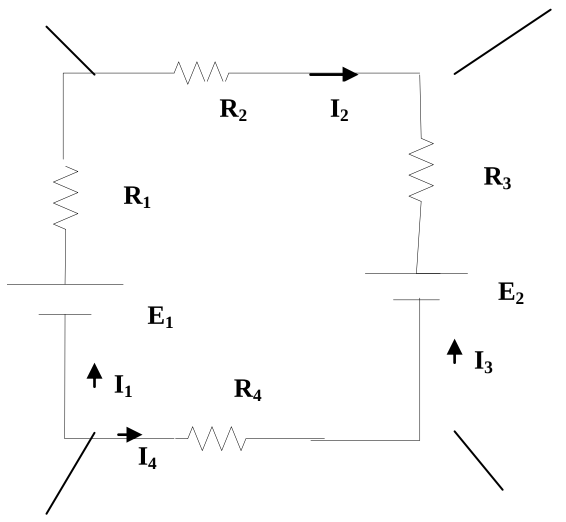
$$\mathbf{I_1 - I_2 + I_3 - I_4 + I_5 = 0}$$

$$\mathbf{I_1 + I_3 + I_5 = I_2 + I_4}$$

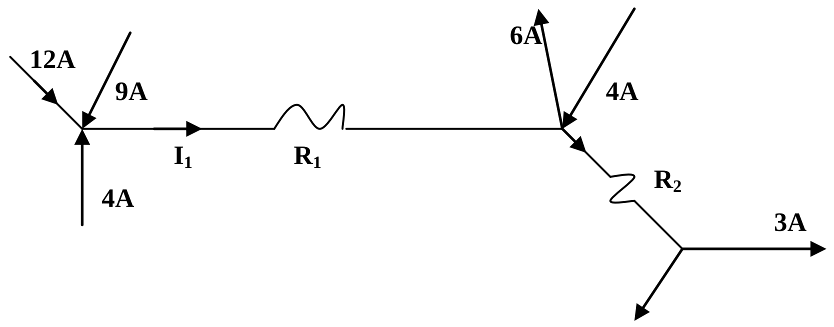


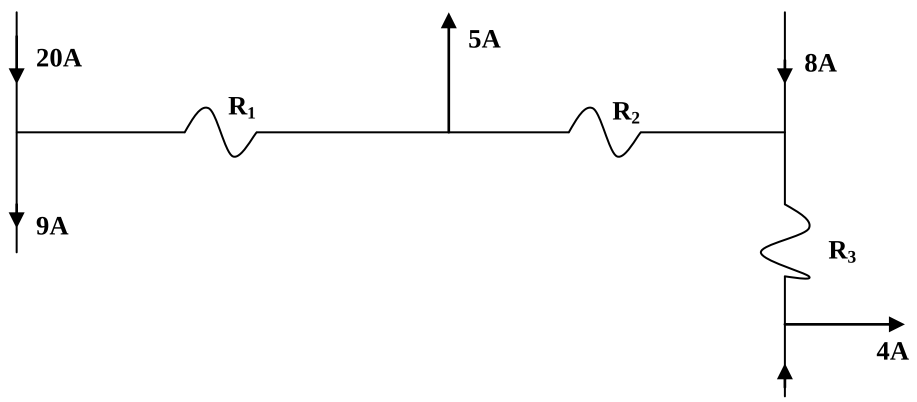
Second Law (Voltage law): At any instant in a closed loop, the algebraic sum of the emf's acting round the loop is equal to the algebraic sum of the potential drops round the loop.

$E_1 - E_2 = I_1 R_1 + I_2 R_2 - I_3 R_3 - I_4 R_4$

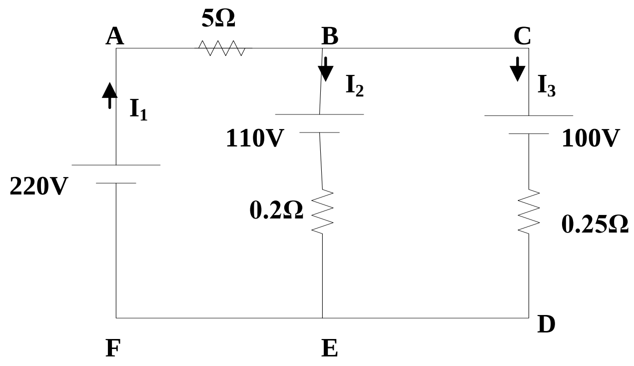


<u>Example:</u> Find the unknown currents and their directions in the circuit shown below:



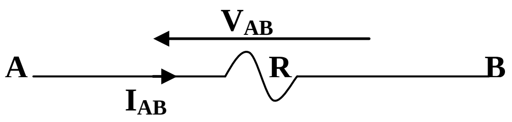


Example: For the circuit shown below, find I_1, I_2 and I_3.



2.6 Double subscript notation: In the figure shown, the potential difference between A and B can be specified by V_{AB}, which means the potential of A with respect to B.

With a current flowing in the direction shown the potential of A must be positive w.r.t. that of B.



$V_{AB}=V_A-V_B$

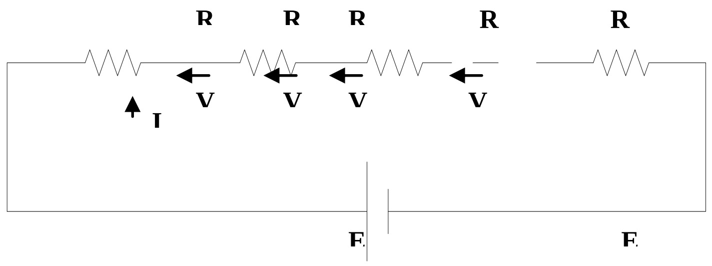
Where V_A and V_B are the potentials of A and B w.r.t. ground.

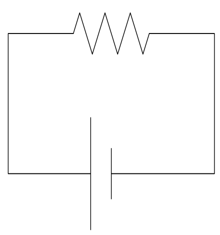
$V_{BA}=-V_{AB}$

2.7 <u>Types of DC circuits:</u> DC circuit can be classified as:

1. **1-** Series circuits
2. **2-** Parallel circuits
3. **3-** Series – parallel circuits.

1- Series circuits:





$V_1+V_2+V_3+ \ldots +V_n=E$

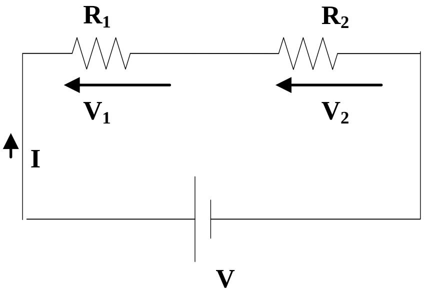
$I_1 R_1+I_2 R_2+I_3 R_3+ \ldots + I_n R_n =E$

$I_1 = I_2 = I_3 = \ldots =I_n$

$E= I (R_1+ R_2+ R_3+ \ldots + R_n)$

$E/I = R_1 + R_2 + R_3 + \ldots\ldots + R_n = R_{eq}$

Special case of series circuits [Potential dividers]:



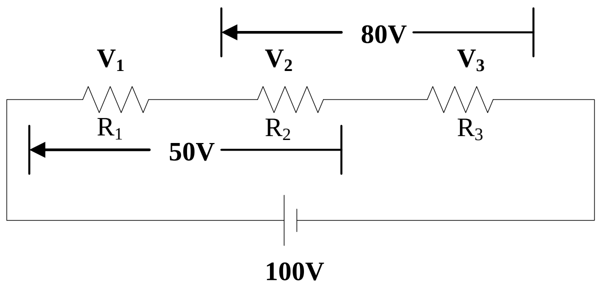
$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} * R_1$$

Or $$V_1 = V\left(\frac{R_1}{R_1 + R_2}\right)$$

Similarly $$V_2 = V\left(\frac{R_2}{R_1 + R_2}\right)$$

Example: For the circuit shown below, find R_1, R_2 and R_3 if $R_{eq} = 50\Omega$.



$100 = V_1 + V_2 + V_3$

$V_1 + V_2 = 50$

$100 = 50 + V_3$ $\qquad$ $V_3 = 50$ V

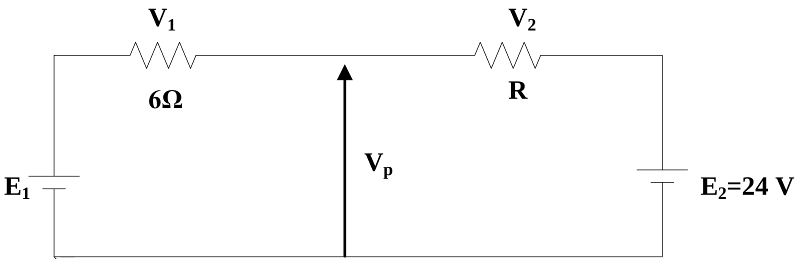
$V_1 = 20$ V $\qquad$ $V_2 = 30$ V

$I = 100/R_{eq} = 2$ A

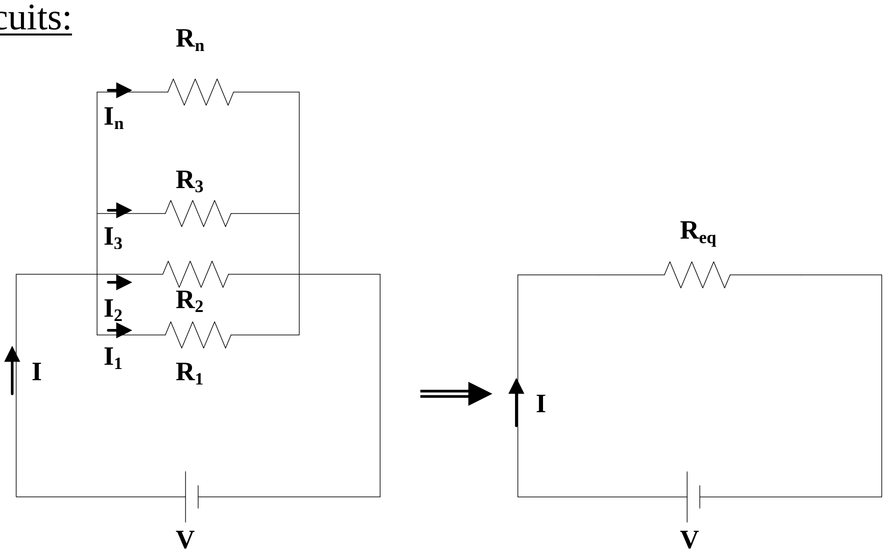
$R_1 = V_1/I = 10\Omega$

$R_2 = V_2/I = 15\Omega$ $\qquad$ $R_3 = V_3/I = 25\Omega$

Example: For the circuit shown below, if V_p=26 V and resistor R dissipates 345.6 kJ in 24 h. Find E_1 and R.



2- Parallel Circuits:



$$I_1 = \frac{V}{R_1} \ , \ I_2 = \frac{V}{R_2} \ , \ I_3 = \frac{V}{R_3} \ , \ \ldots\ldots , \ I_n = \frac{V}{R_n} \ , \ I = \frac{V}{R_{eq}}$$

$I=I_1+I_2+I_3+ \ldots\ldots +I_n$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots\ldots + \frac{V}{R_n}$$

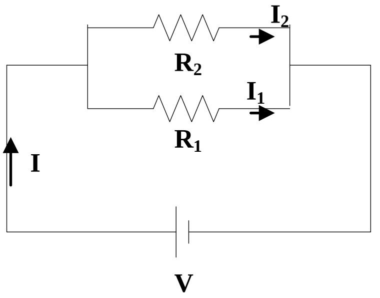
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...+ \frac{1}{R_n}\right)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...+ \frac{1}{R_n}$$

Or $G_{eq}=G_1+G_2+G_3+\ldots\ldots.+G_n$

<u>Special case of parallel circuit [Current Divider]:</u>

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



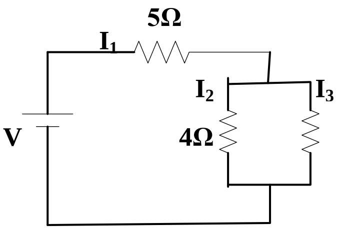
$$I_1 = \frac{V}{R_1} \quad , \quad I_2 = \frac{V}{R_2} \quad , \quad I = \frac{V}{R_{eq}}$$

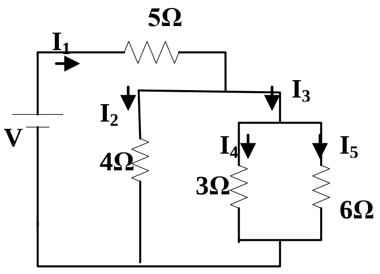
$V= I\ R_{eq}$

$$I_1 R_1 = I \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = I \frac{R_2}{R_1 + R_2} \quad \text{Similarly,} \quad I_2 = I \frac{R_1}{R_1 + R_2}$$

<u>Example:</u> For the circuit shown below, if I_1=5 A determine I_2, I_3, I_4 and I_5.

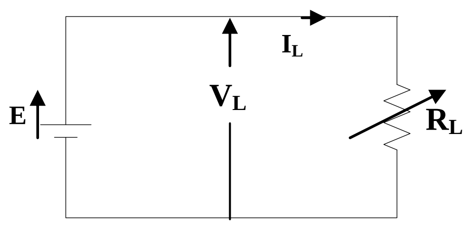


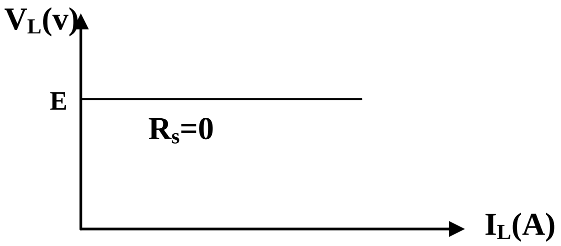


2.8 <u>Sources of Energy:</u>

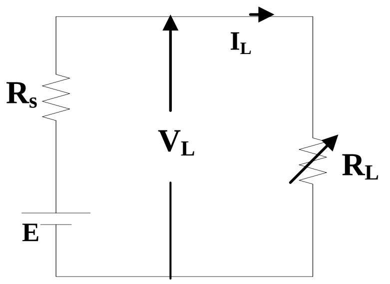
a- <u>Voltage source:</u>

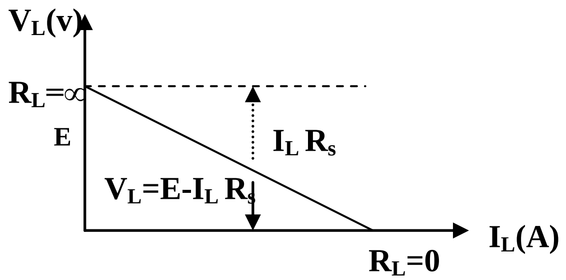
1- Ideal Voltage Source: it is a device that provides a constant voltage across its terminals (V_L=E) whatever the drown current is. It does not exist practically.





2- Practical voltage source: It is a device with an internal resistance, and can be represented by a voltage source in series with an internal resistance as shown below:

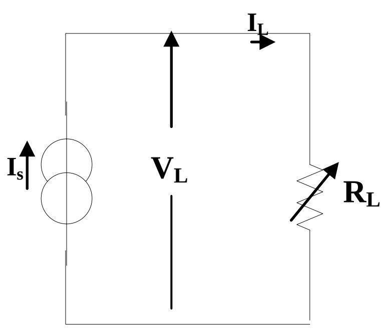


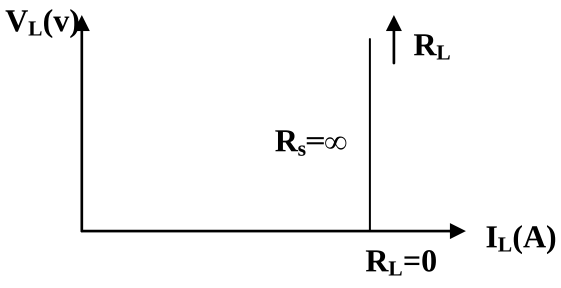


I_L= E/(R_s +R_L)

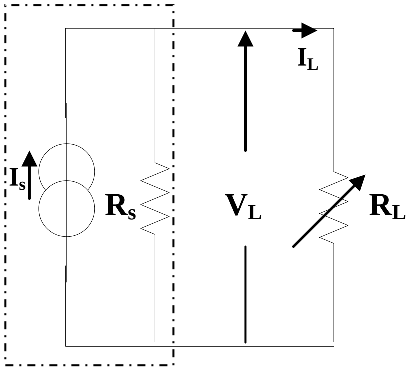
b: <u>Current source :</u>

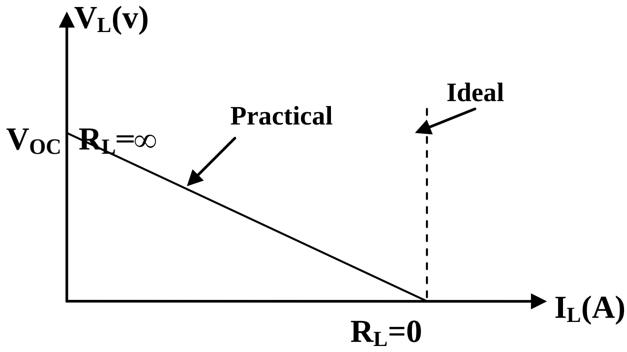
1- Ideal Current Source: it is a device that provides a constant current (I_L=I_s) to any load resistance connected across it independent to the voltage across its terminal.





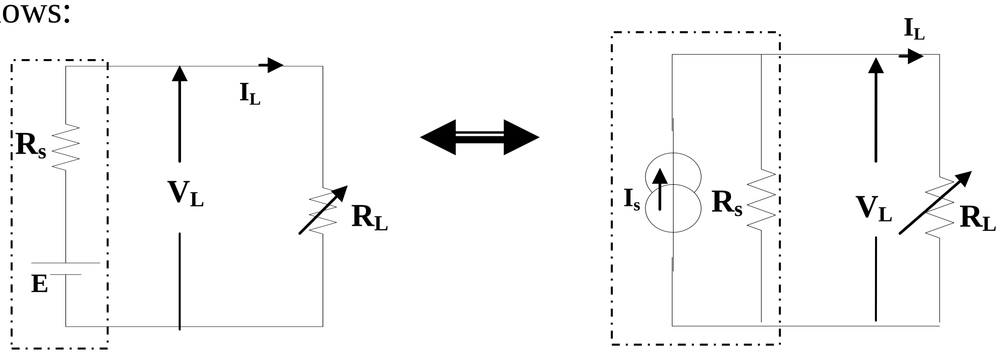
2- Practical current source:





2.9: <u>Transformation of energy sources:</u>

Voltage source can be transformed to current source and vice versa as follows:



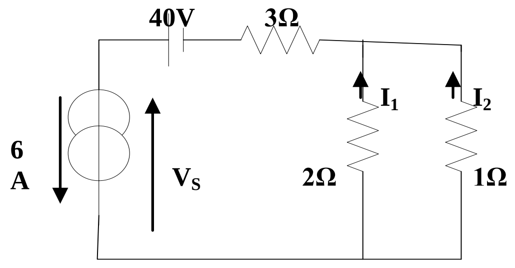
$$I_L = \frac{E}{R_s + R_L} = \frac{E}{R_s} * \frac{R_s}{R_s + R_L}$$

$$I_L = I_s * \frac{R_s}{R_s + R_L}$$

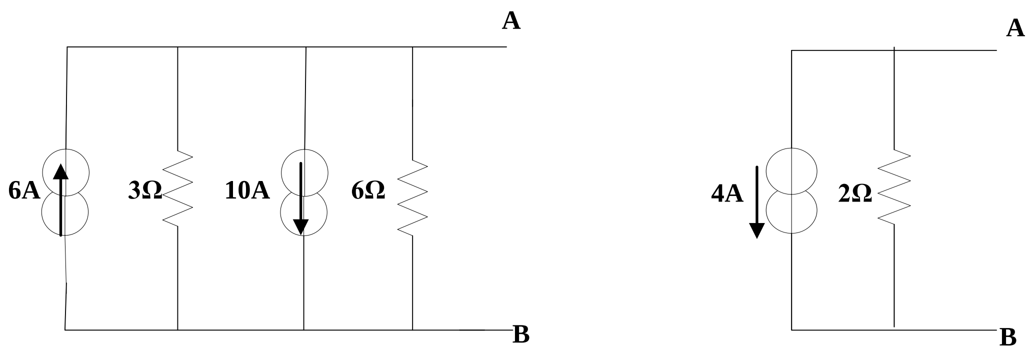
I_s :short circuit current of the voltage source,

$R_s/(R_s+R_L)$: current dividing ratio.

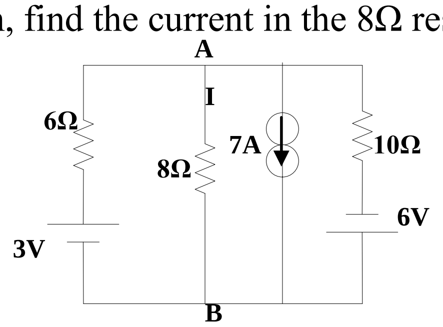
Example: : For the circuit shown below, find I_1, I_2, and V_s.



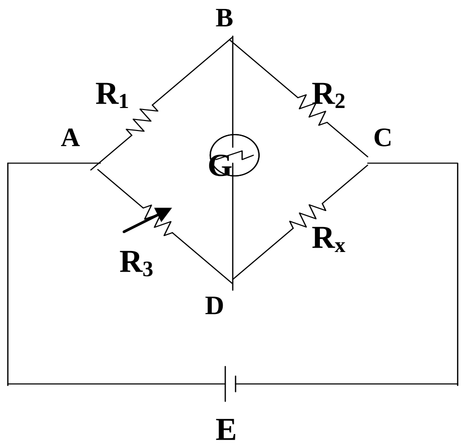
Note: If two or more current sources are connected in parallel, they may all be replaced by one equivalent current source having the magnitude and direction of the resultant.



Example: Using source transformation, find the current in the 8Ω resistor.



Example: For the circuit shown below, find the value of the unknown resistance R_x in terms of R_1, R_2 and R_3 when the current passing through the galvanometer [G] is equal to zero.

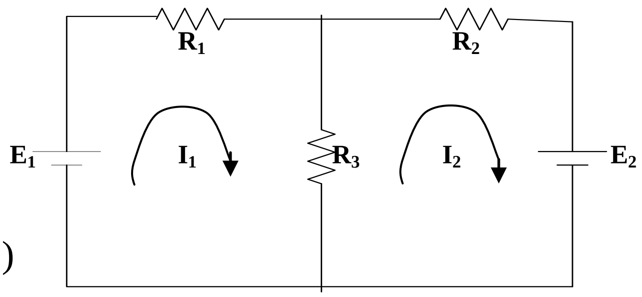


2.10: Network analysis by Maxwell's circulating currents:

The following procedures must be carried to apply this method:

1- currents are assumed to flow clockwise around the loop through the parameters of the loop without splitting at the junctions.
2- Kirchhoff's voltage law is applied around each loop.
3- Solve the resultant simultaneous linear equations for the assumed currents.

For loop (1)



$E_1-I_1 R_1 -(I_1-I_2) R_3 =0$

$E_1=I_1 (R_1 -R_3)$

$-I_2 R_3 \quad . . . . (1)$

For loop 2:

$(I_2-I_1) R_3 + I_2 R_2 + E_2=0$

$-I_1 R_3 + I_2 (R_2 +R_3)= - E_2 \quad . . . (2)$

For a circuit with n loops, the equations will always be of the form:

$R_{11} I_1 - R_{12} I_2 - R_{13} I_3 - . . . . . \quad . . - R_{1n} I_n = E_1$

$-R_{21} I_1 + R_{22} I_2 - R_{23} I_3 - . . . . . \quad . . - R_{2n} I_n = E_2$

$-R_{31} I_1 - R_{32} I_2 + R_{33} I_3 - . . . . . \quad . . - R_{3n} I_n = E_3$

.

.

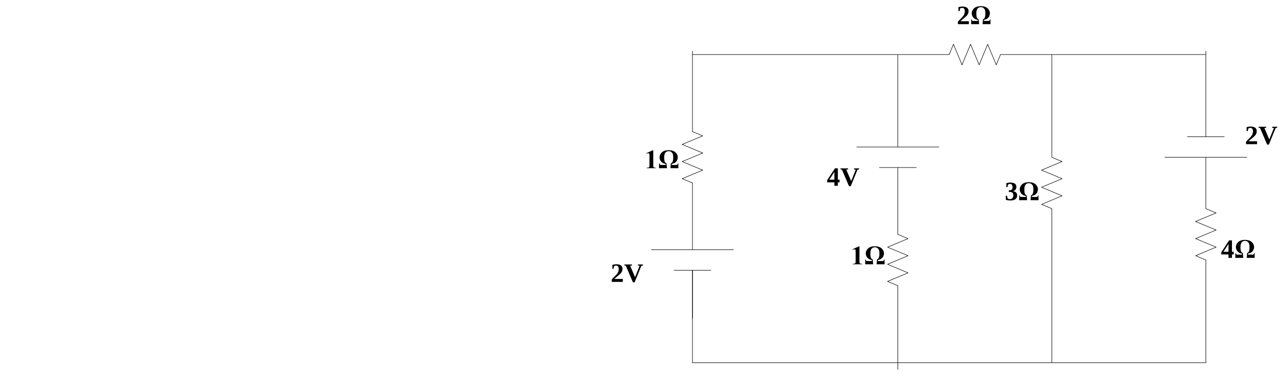
$-R_{n1} I_1 - R_{n2} I_2 - R_{n3} I_3 - . . . . . \quad . . + R_{nn} I_n = E_n$

Where E_1 : the algebraic sum of emf's in loop (1) in the direction of I_1.

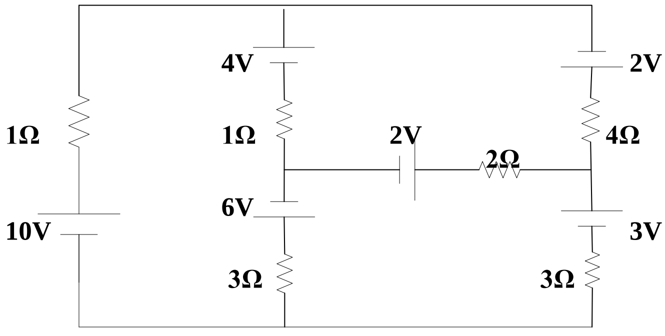
R_{11} : sum of resistances in loop (1)

R_{12} : total resistance common to loops (1) and (2).

Example: Write the loop equations for the network shown in the figure below:

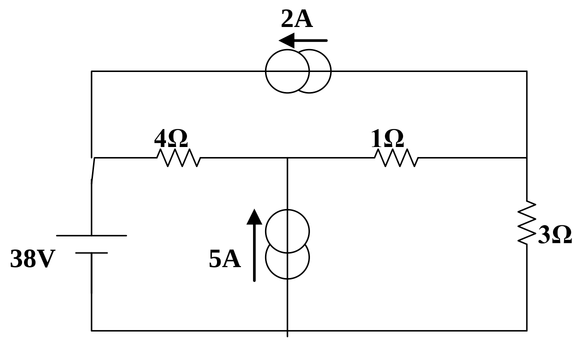


Example: Using Maxwell's loop current method find the current in the 2Ω resistor in the network below:

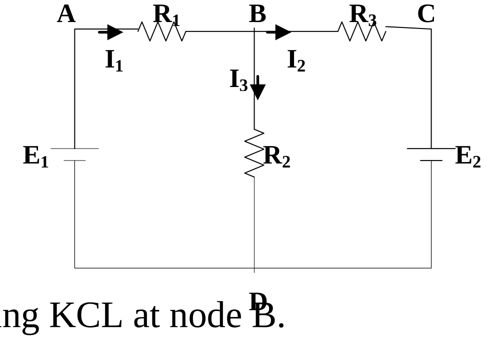


Note: If the network contains current sources, then the first step in applying Maxwell's loop currents is to convert all current sources to voltage sources.

Example: In the circuit shown below, find the current in the 3Ω resistor using Maxwell's loop currents theorem.



2.11: Nodal Analysis: In this method, one of the nodes is taken as a reference node and the potential of all other points in the circuit are measured w.r.t. the reference point.



If node D is chosen to be this reference node, then,

$V_A = E_1$
and $V_C = E_2$

The value of V_B can be found by applying KCL at node B.

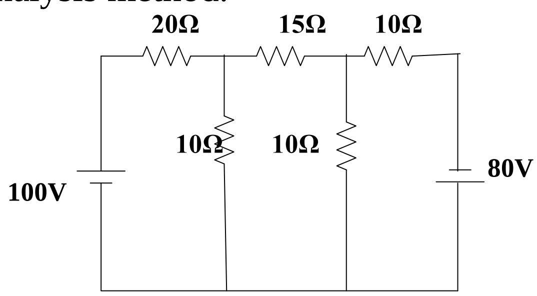
$I_1+I_2=I_3$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_3} = \frac{V_B}{R_2}$$

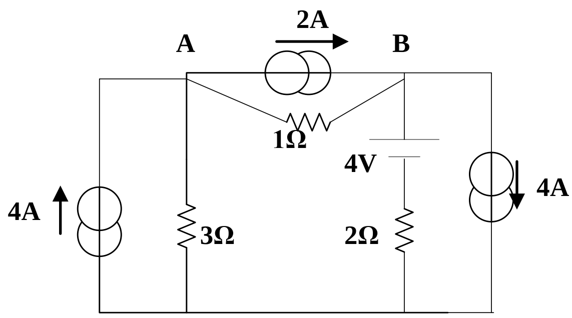
$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2}$$

Once V_B is known, all branch currents can be calculated.

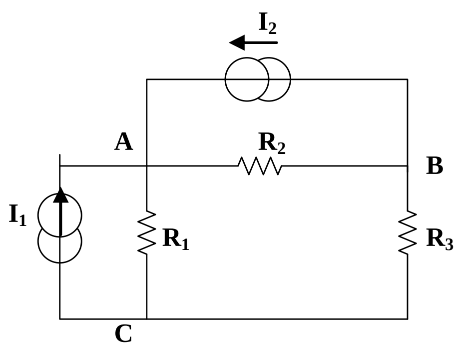
<u>Example:</u> Find all branch currents using nodal analysis method.



Example: Calculate the quantities V_A and V_B for the following network using Nodal analysis method.



Example:



At node A:

$$\sum I = 0$$

$$I_1 + I_2 - \frac{V_A}{R_1} - \frac{V_A - V_B}{R_2} = 0 \qquad ..(1)$$

At node B:

$$-I_2 - \frac{V_B}{R_3} + \frac{V_B - V_A}{R_2} = 0 \qquad ..(2)$$

$$\left(\frac{1}{R_1} + \frac{1}{R2}\right)V_A - \frac{1}{R_2}V_B = I_1 + I_2 \qquad ..(3)$$

$$-\frac{1}{R2}V_A + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_B = -I_2 \qquad ..(4)$$

$$(G_1 + G_2)V_A - G_2 V_B = (I_1 + I_2) \qquad ..(5)$$

$$-G_2V_A+(G_2+G_3)V_B=-I_2 \qquad ..(6)$$

$$G_{11}V_1-G_{12}V_2-G_{13}V_3.......-G_{1n}V_n=I_1$$

$$-G_{21}V_1+G_{22}V_2-G_{23}V_3.......-G_{2n}V_n=I_2$$

$$-G_{31}V_1-G_{32}V_2+G_{33}V_3.......-G_{3n}V_n=I_3$$

:

:

$$-G_{n1}V_1-G_{n2}V_2-G_{n3}V_3.......+G_{nn}V_n=I_n$$

where:

G_{11} = Sum of all conductances connected to node 1.

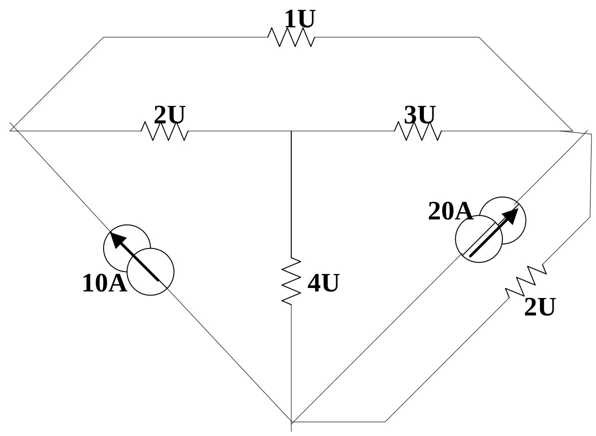
G_{12} = Sum of all conductances connected between node 1 and 2.

G_{nn} = Sum of all conductances connected to node n.

V_1 = potential of node 1.

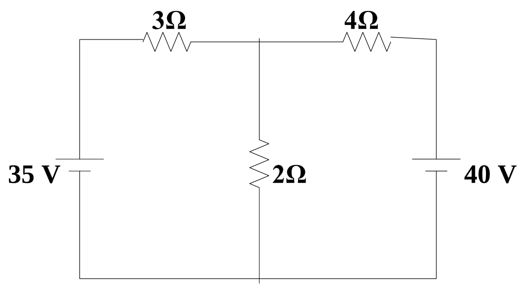
I_1 = sum of all current sources feeding node 1.

<u>Example:</u> Find the power supplied by the two current sources shown in the circuit:

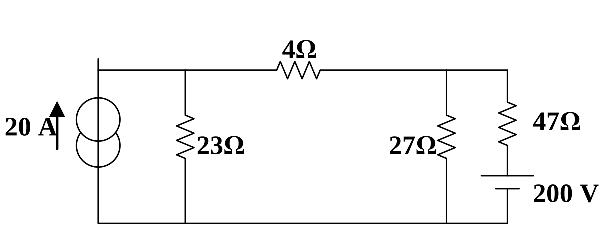


2.12: Superposition Theorem: In a linear network containing more than one source of emf, the resultant current in any branch is the algebraic sum of the currents that would be produced by each emf acting alone, all other sources of emf being replaced by their internal resistance.

Example: In the network shown, find the current in the branch 2Ω by superposition theorem.



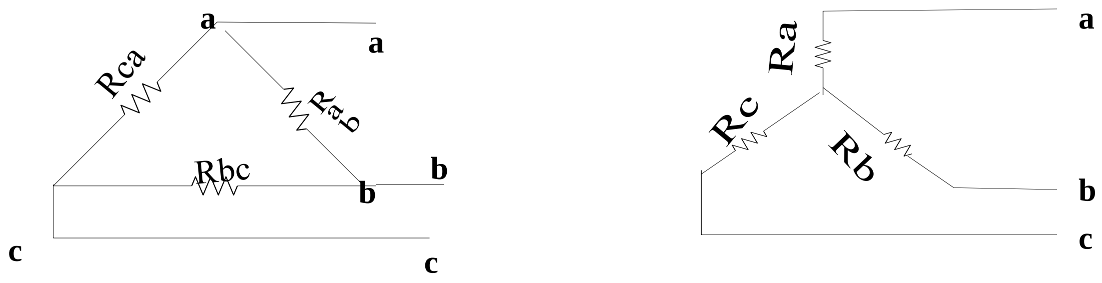
Example: Calculate the value of current in the 23 ohm resistor.



2.13: <u>Delta-Star and Star-Delta transformation:</u>

Converting delta network into star and vice versa often simplifies the network and makes it possible to apply series-parallel circuit technique.

a- Delta-Star transformation:



Resistance between a and b = $\frac{Rab(Rbc+Rca)}{Rab+Rbc+Rca}=Ra+Rb$

Resistance between b and c = $\frac{Rbc(Rab+Rca)}{Rab+Rbc+Rca}=Rb+Rc$

Resistance between c and a = $\frac{Rca(Rab+Rbc)}{Rab+Rbc+Rca}=Rc+Ra$

Solving these equations gives:

$$Ra=\frac{Rab*Rca}{Rab+Rbc+Rca}$$

$$Rb = \frac{Rab * Rbc}{Rab + Rbc + Rca}$$

$$Rc = \frac{Rbc * Rca}{Rab + Rbc + Rca}$$

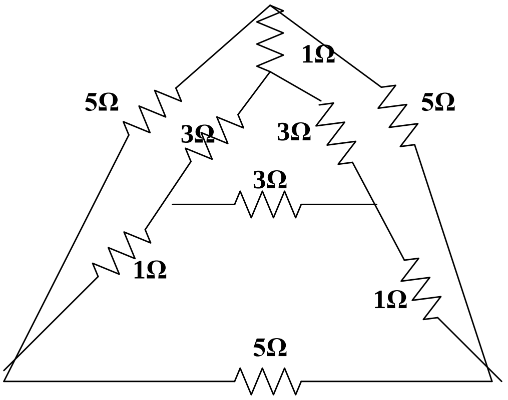
b- Star-Delta transformation:

$$Rbc = Rb + Rc + \frac{Rb * Rc}{Ra}$$

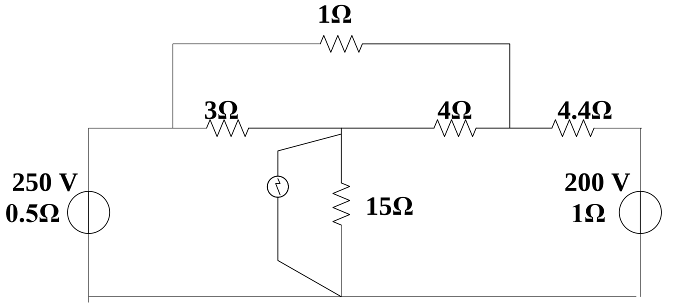
$$Rca = Rc + Ra + \frac{Rc * Ra}{Rb}$$

$$Rab = Ra + Rb + \frac{Ra * Rb}{Rc}$$

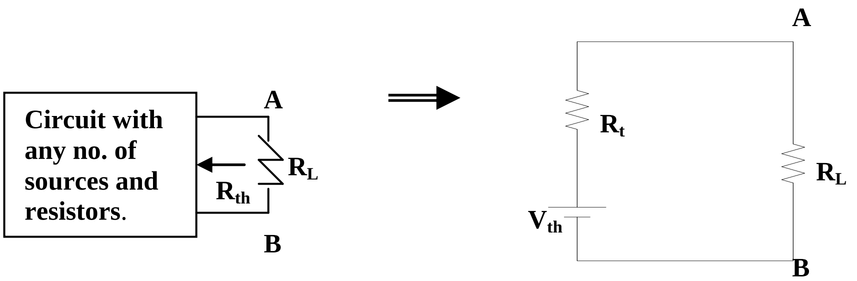
<u>Example: </u>Find Rab.



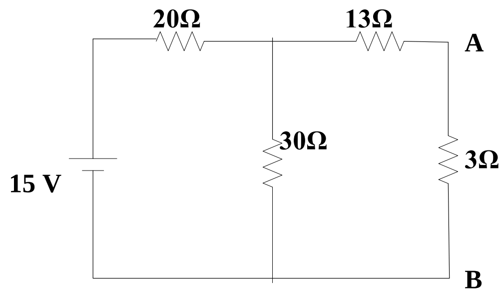
Example: Find the reading of the voltmeter for the circuit below:

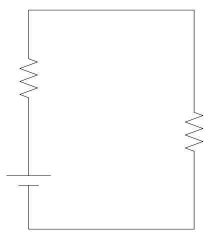


2.13: <u>Thevenin's theorem:</u> The current in any branch of a circuit is the same as that in the branch if it is connected across a source of electrical energy, the emf of which is equal to the potential difference which would appear across the branch if it is open circuited and with an internal resistance equal to the resistance which appears across the open circuited branch terminals. In calculating the internal resistance, sources of emf are treated as short circuits.

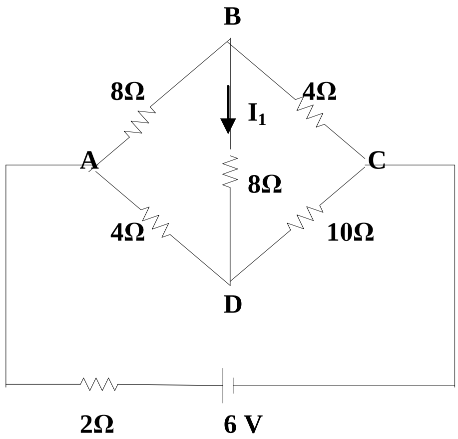


<u>Example:</u> Find Thevenin's equivalent circuit for the following circuit between points A and B.



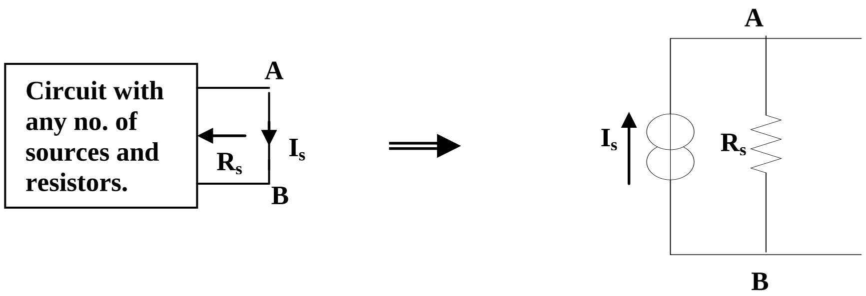


Example: Using Thevenin's theorem, find I_1.

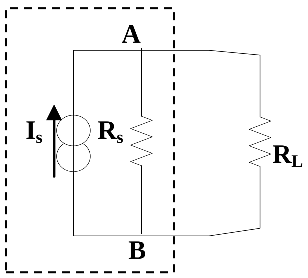


2.15: Norton's theorem:

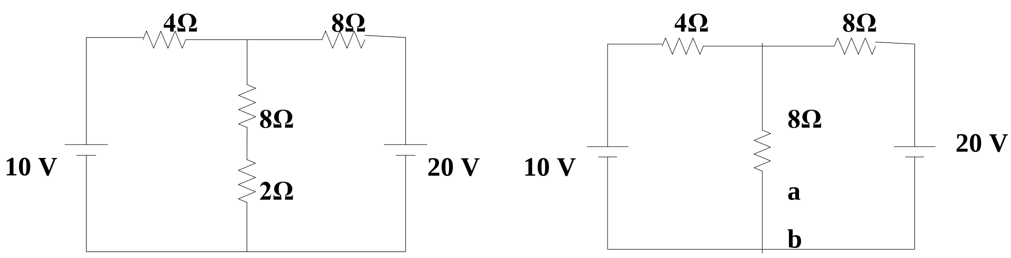
This theorem stats that: the current which flows in any branch of a network is the same as that which would flow in the branch if it were connected across a source of electrical energy, the short circuit current of which is equal to the current that would flow in a short circuit across the branch, and the internal resistance of

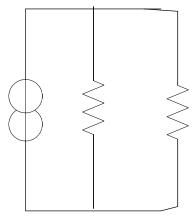


which is equal to the resistance which appears across the open circuit branch terminals.

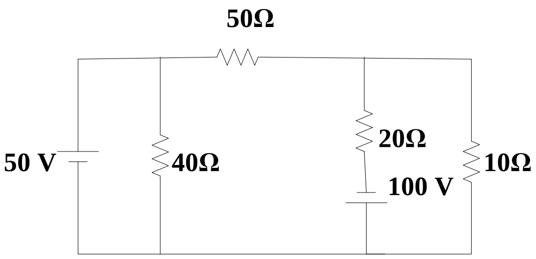


Example: Calculate the potential difference across the 2Ω resistor in the network shown below:



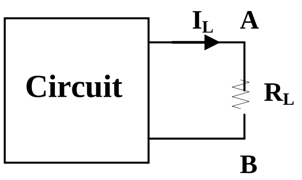


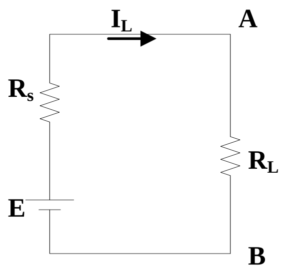
Example: Find the potential difference across the 10Ω resistor in the network shown below using Norton's theorem.



2.16: <u>Maximum power transfer theorem:</u>

This theorem states that in a dc circuit, maximum power is transferred from a source to load when load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all sources of emf are replaced by their internal resistance.





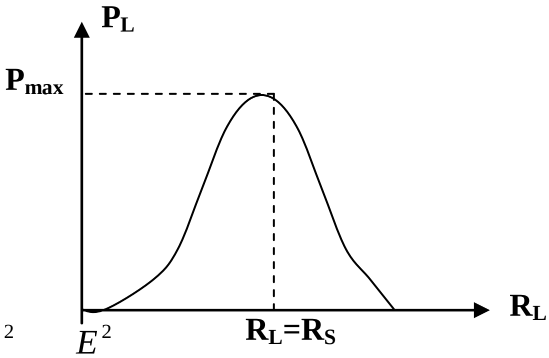
$$I_L = \frac{E}{R_s + R_L}$$

P_L = power delivered to the load;

$$P_L = I^2 * R_L = \frac{E^2}{(R_s + R_L)^2} * R_L$$

Power delivered to the load depends only upon R_L, i.e. E and R_s are constant.

$$\frac{dP_L}{dR_L} = \frac{E^2}{(R_s + R_L)^2} - \frac{2E^2 R_L}{(R_s + R_L)^3} = 0$$



R_L=R_s

Thus P_{max}=I_L^2 R_L

$$P_{\max} = I_L^{\,2} * R_L = \frac{E^2}{(R_L + R_L)^2} * R_L = \frac{E^2}{4R_L} = \frac{E^2}{4R_s}$$

At maximum power transfer condition:

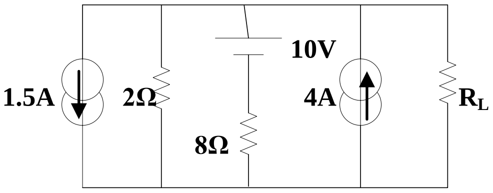
$$V_L = I_L * R_L = \frac{E}{(R_s + R_L)} * R_L = \frac{1}{2}E$$

i.e., the load voltage is one-half the open circuit voltage at the load terminal.

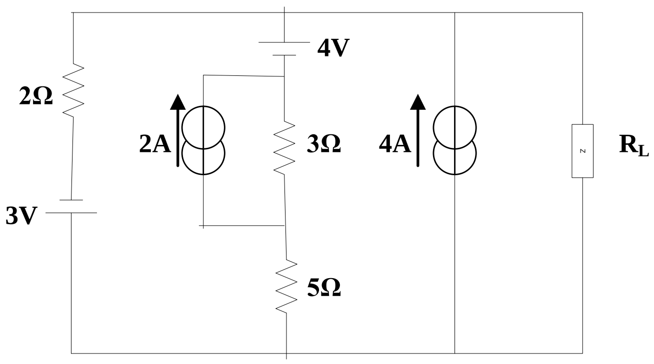
Efficiency=η=output power/total power

$$\eta = \frac{I_L^{\,2} R_L}{I_L^{\,2}(R_s + R_L)} = \frac{R_L}{R_s + R_L} = 50\%$$

Example: In the circuit below, calculate the value of R_L which absorbs maximum power then calculate this power.

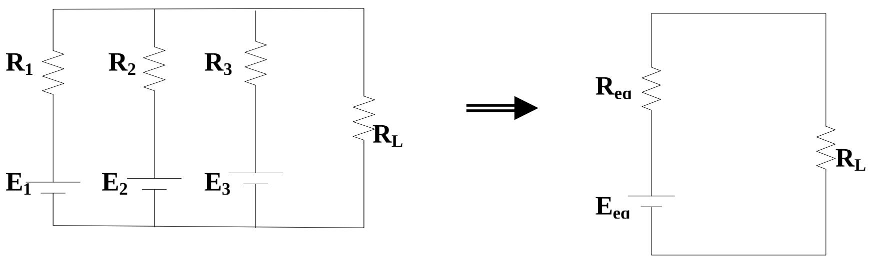


H.W. For the circuit shown below, calculate the value of R_L which absorbs maximum power and then calculate this maximum power.



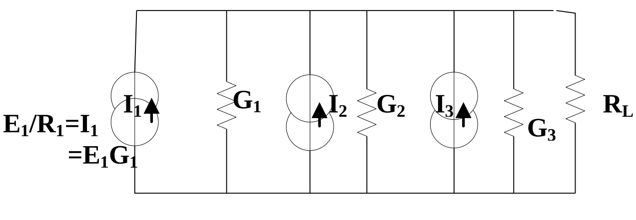
2.17: <u>Millman's theorem:</u>

Through the application of Millman's theorem, any number of parallel voltage sources can be reduced to one.



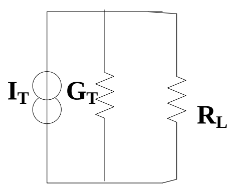
This can be done through performing the following steps:

Step 1:- Convert all voltage sources to current sources as follows:



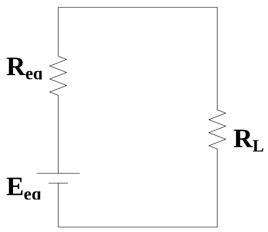
Step 2:- Combine parallel current sources

$$I_T=I_1+I_2+I_3+\ldots.+$$
$$G_T=G_1+G_2+G_3+$$



Step 3:- convert the resultant current source into voltage source.

$R_{eq}=1/G_{eq}$
$E_{eq}=I_T\,R_{eq}$
$=I_T/G_T$



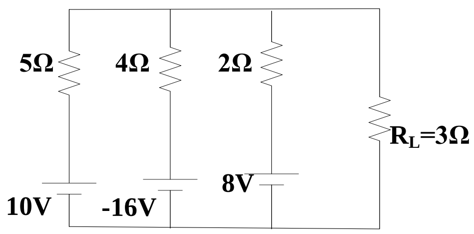
In general Millman's theorem states that for any number of parallel voltage sources:

$$E_{eq}=\frac{I_T}{G_T}=\frac{\pm I_1 \pm I_2 \pm I_3 \pm .... \pm I_n}{G_1+G_2+G_3+...+G_n}$$

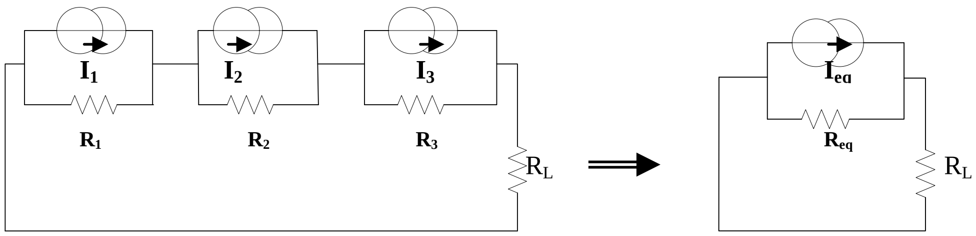
$$=\frac{\pm E_1G_1 \pm E_2G_2 \pm E_3G_3 \pm .... \pm E_nG_n}{G_1+G_2+G_3+...+G_n}$$

$$R_{eq}=\frac{1}{G_T}=\frac{1}{G_1+G_2+G_3+...+G_n}$$

<u>Example:</u> Using Millman's theorem, find the current through the resistor 3Ω also find its voltage.



The dual Millman's theorem is the combining of series current sources.

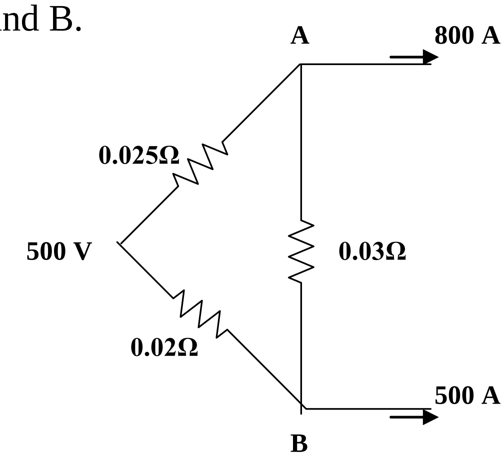


$$I_{eq} = \frac{\pm I_1 R_1 \pm I_2 R_2 \pm I_3 R_3}{R_1 + R_2 + R_3}$$

$$R_{eq} = R_1 + R_2 + R_3$$

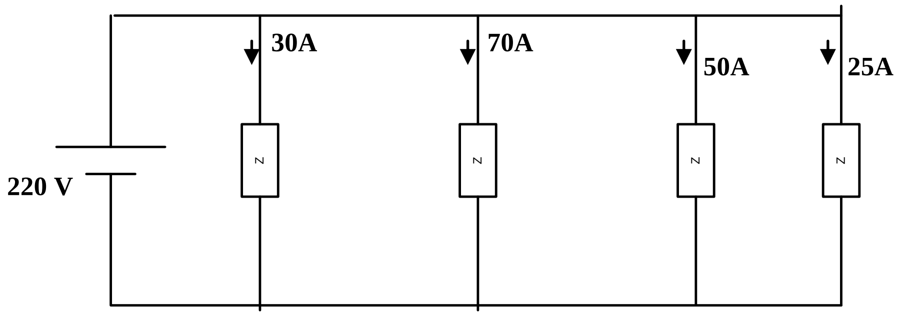
<u>Problems:</u>

P1- A 500 V dc generator supplies a load 800 A at point A through 0.025 Ω distributor and a load of 500 A at point B. Points A and B are joined by 0.03 Ω. Determine the current between A and B.

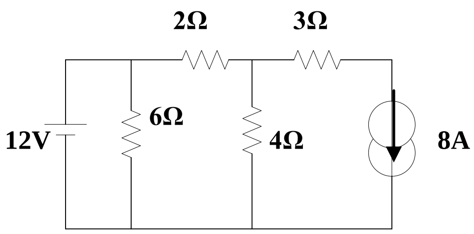


P2- A two wire distributor 500 m long is loaded as shown below. The maximum voltage drop allowed is 5% of the supply voltage. Calculate the cross sectional area of the conductor to be used. P=1/50 Ω/m/mm^2.

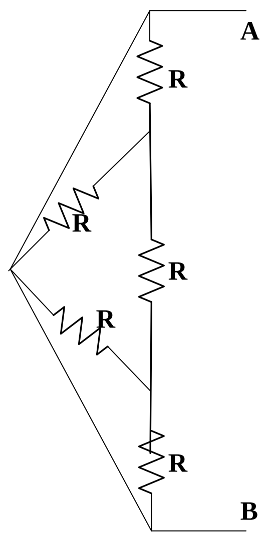
100m **150m** **150m** **100m**



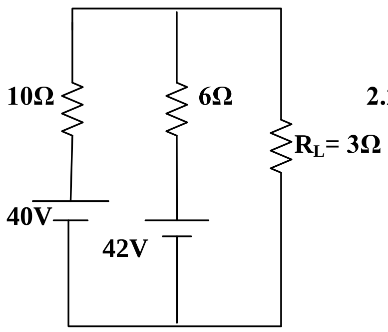
P3- Find the value and direction of the current in the 4Ω resistor using superposition theorem.

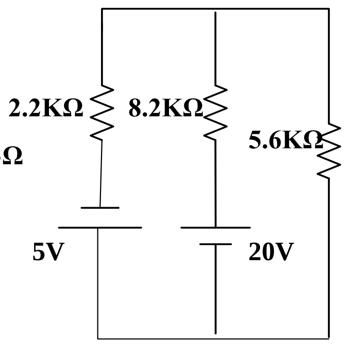


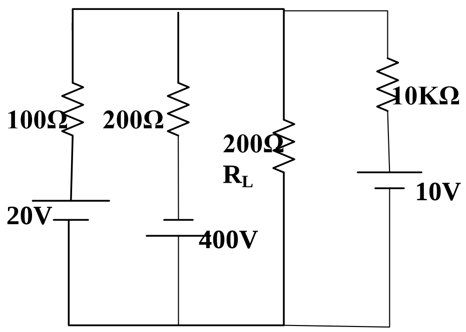
P4- Find R_{AB}.



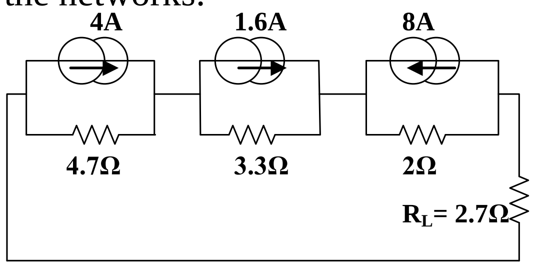
P5- Using Millman's theorem, find the current and voltage across the resistor R_L for the following networks:

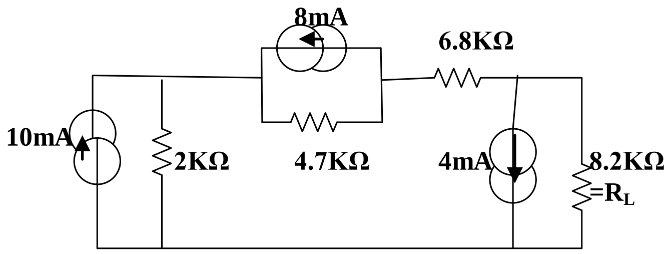






P6- Using the dual of Millman's theorem, find the current through R_L and its voltage for the networks:





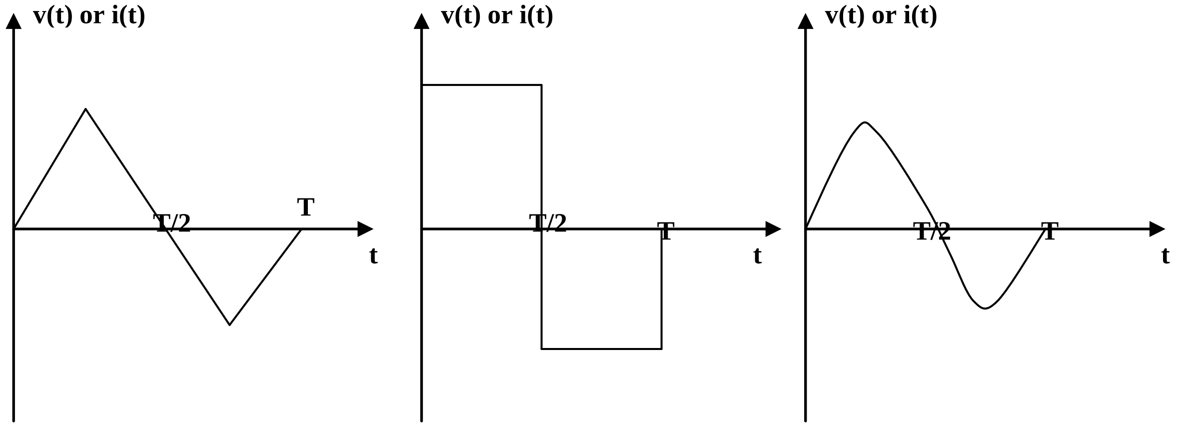
Chapter Three

A.C. Circuits

3.1 Introduction:

An alternating quantity is one that regularly acts first in one direction and then in the opposite direction and do not have constant magnitude with time. It's magnitude continuously vary with time.

3.2 A.C. Terminology:



a- Waveform: It is a graph showing the manner in which an alternating quantity changes with time. It is also called wave shape.

b- Instantaneous value: The value of the alternating voltage or current at any instant. It is taken as v(t) or e(t) for the voltage and i(t) for the current.

c- Period (cycle): It is one complete set of positive and negative values of an alternating quantity.

d- Periodic time: It is the time taken (in seconds) complete one cycle of an alternating quantity. It is generally represented by the symbol T.

e- Frequency : It is the number of cycles that occur in one second of the alternating quantity.

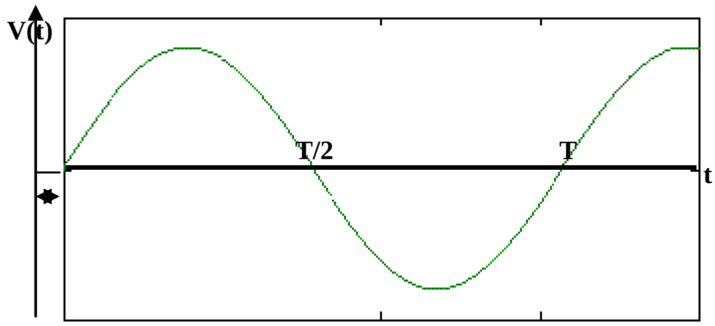
1 cycle/sec. = 1 Hz.

$f = \frac{1}{T}$ or $T = \frac{1}{f}$

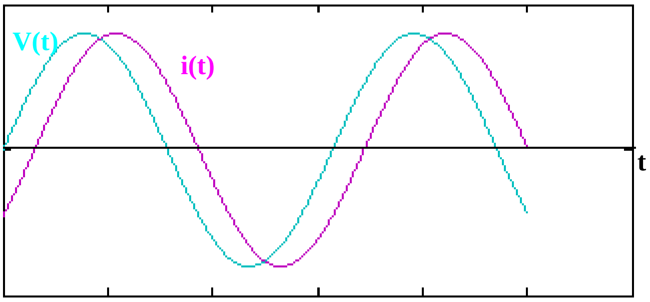
Where T is the time of one cycle.

f- Amplitude: The maximum value (positive or negative) attained by an alternating quantity. It is also called peak value.

g- Phase: The phase of a particular value of an alternating quantity is the fractional part of periodic time through which the alternating quantity has advanced or delayed from the selected zero position.



h- Phase-difference: when two alternating quantities with the same frequency have different zero points (reference) they are said to have phase difference.



The angle between the two zero points is the same angle of phase difference φ and it is measured in degrees or radians. The voltage waveform is called leading and the current waveform is called lagging.

3.3: **Generation of AC voltage:**

EMF can be induced by changing the flux linkage in two ways:

1- e.m.f α $N\frac{d\phi}{dt}$ (Faraday's Law)

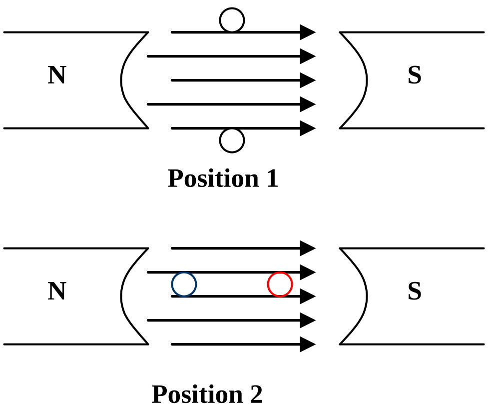
$$e = N\frac{d\phi}{dt}$$

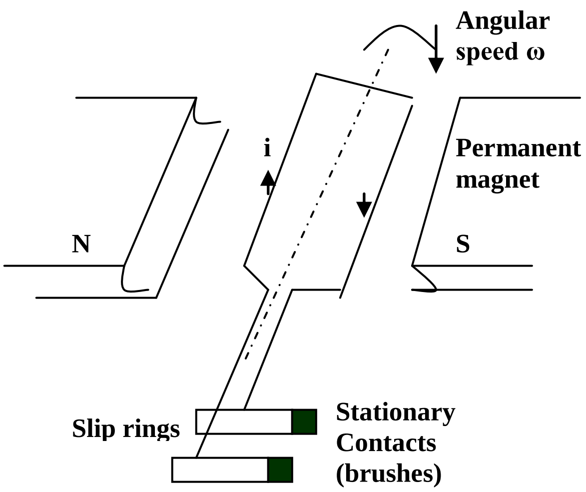
Where N : Number of turns

Φ : flux (wb)

In this case there are no movements for the conductors and the emf is induced due to change of the flux. The emf induced in this way is known as statically induced emf.

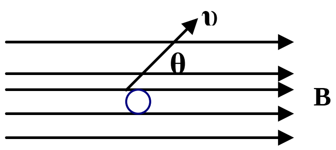
2- By moving a conductor in a uniform magnetic field, an emf will be induced. The emf produced in this way is known as dynamically induced emf.





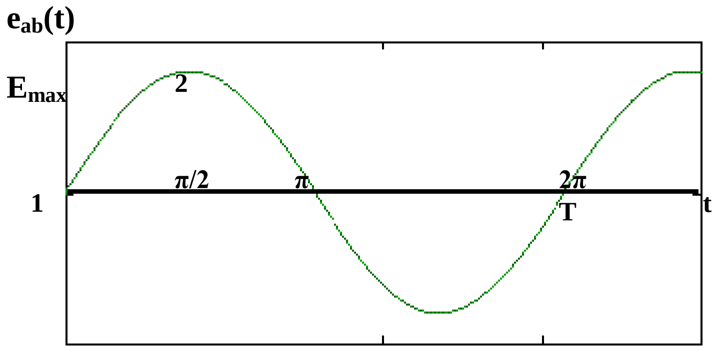
Consider a conductor with length "l" meters placed in a uniform magnetic field density "B" (wb/m^2). Let this conductor be moved with velocity υ m/sec. in the direction of the field.

Induced emf = e = B l υ sin θ (volt)



In the above figure, starting with the coil in position (2) perpendicular to the field

$\theta = 90^o$, $e_{ab} = E_{max}$. when the two sides rotate and come to be parallel movement to the field in position 1 resultant in no cutting action $e_{ab}(t)$=zero at position (1).
In between these two positions, the rate of change of flux is decreasing and the induced voltage reduces from E_{max} to zero.



The generated waveform can be expressed algebraically as:

$e_{ab}(t) = E_m \sin\theta$ (volt)

at $\theta = 90^o$ and $\theta = 270^o$ give the maximum or peak value of the waveform.

ω =angle/time = θ/t rad/sec

$\theta = \omega t$;

ω is always expresses in radians/second, while θ is expressed in radians.

$$e_{ab}(t). = E_m \sin \omega t$$

from the above waveform:

$\omega = \theta/t = 2\pi/T = 2\pi f$

3.4: Average value and effective value of AC quantity :

1- Average value: It is the arithmetical mean at all the values of an AC waveform.

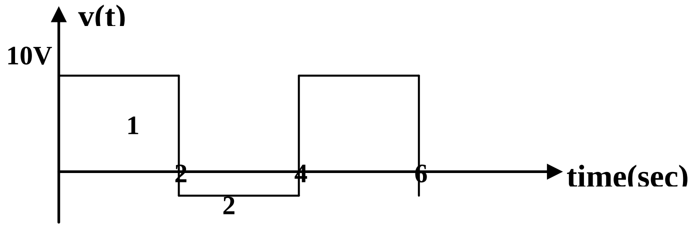
Average value=Area under the curve/Base

a- For unsymmetrical waveform:

Average value=(Area over one cycle)/(Base length of one cycle)

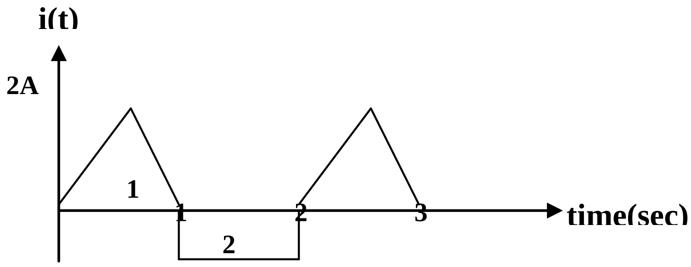
Example:

$$V_{average} = \frac{Area(1) + Area(2)}{4}$$



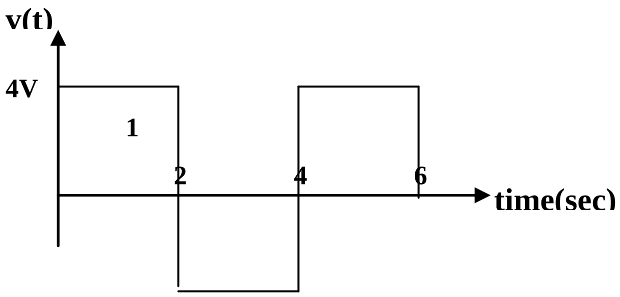
Example:

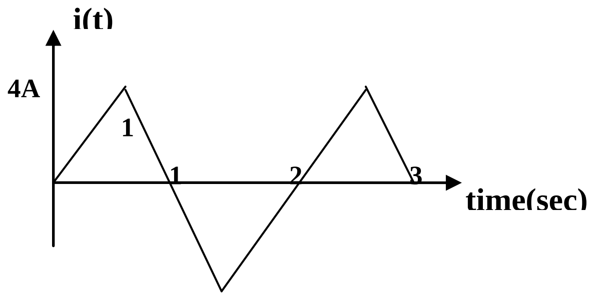
$$I_{average} = \frac{Area(1) + Area(2)}{2}$$



b- For symmetrical waveform:

Average value=Area over one alternation/Base length of one alternation.





$$V_{average} = \frac{Area(1)}{2}$$

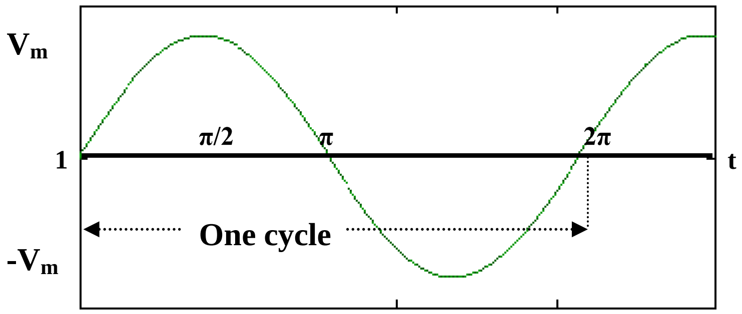
$$I_{average} = \frac{Area(1)}{1}$$

For an AC waveform represented by the function v(t) or i(t) the average value is:

$$V_{av} = \frac{1}{T}\int_0^T v(t)dt \quad \text{(Volt)}$$

$$I_{av} = \frac{1}{T}\int_0^T i(t)dt \quad \text{(Amp.)}$$

Example: Find the average value of the sinusoidal voltage shown below:



$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t d\omega t$$

$$= \frac{-Vm}{\pi}\cos\omega t]_0^{\pi}$$

$=0.6366\ V_m$

2- Effective value or root mean square (rms):

The effective or (rms) value of an AC quantity is that steady current DC which when pass through a given resistance for a given time produces the same amount of heat as produced by the AC current when passing through the same resistance for the same time.

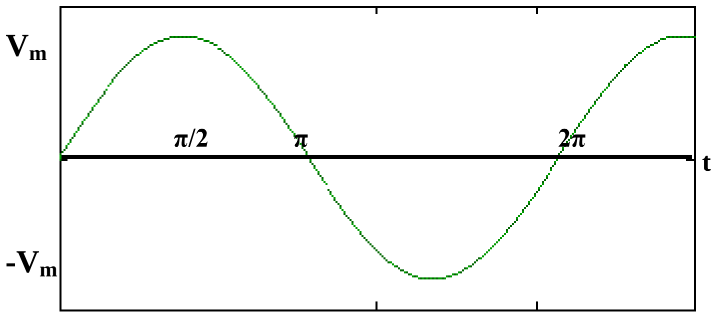
Instantaneous power produced by the alternating current=i^2R (watt)

Average power effective of the alternating current=$I^2_{rms}R$

$$Power = I^2_{rms}R = R\frac{1}{T}\int_0^T i^2(t)dt$$

$$I^2_{rms} = \frac{1}{T}\int_0^T i^2(t)dt \qquad I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

Example: Find the effective value of the sinusoidal voltage shown below:



$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(V_m \sin\omega t)^2\, dt}$$

$$= \frac{V_m^2}{2\pi}\int_0^{2\pi}\frac{1}{2}(1-\cos\omega t)\, d\omega t]^{0.5}$$

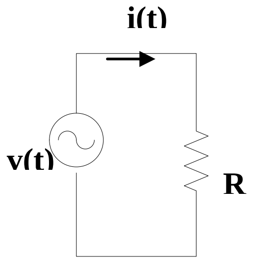
$=0.707\ V_m$

Form factor = (rms value)/(average value)

$$= \frac{0.707V_m}{0.636V_m} = 1.11$$

3.5: Simple signal element circuit:

1- Resistance:

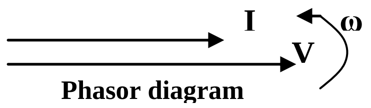


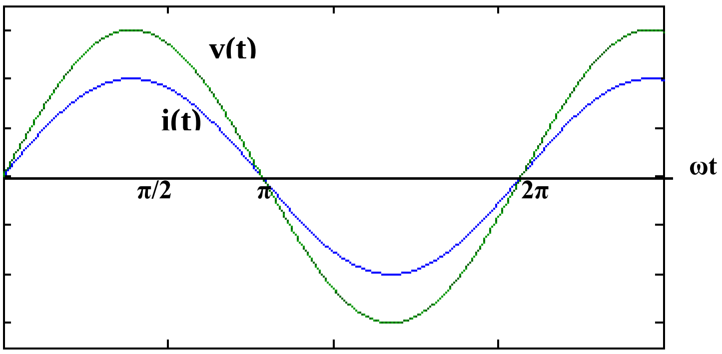
V(t)=Vm sin ωt . . . (1)
i(t)=v(t)/R
=Vm sin ωt /R=(Vm/R)sin ωt

i(t) =Im sin ωt . . . (2)
Im=Vm/R

It is clear from equations (1) and (2) that v(t) and i(t) are in phase.

In phase means that the difference between the two signals is zero.





P(t)= Instantaneous power= v(t)*i(t)

=Vm sin ωt *Im sin ωt = Vm Im $\sin^2$ωt

= 0.5 Vm Im (1-cos(2ωt)) = 0.5 Vm Im – 0.5 Vm Im cos(2ωt)

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\, d\omega t = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms}\, I_{rms} \quad \text{(watt)}$$

or $P_{av}=I^2_{rms}\,R$

2- Inductance:

$I(t) = Im \sin \omega t$ . . . (1)

$v(t) = L\, di(t)/dt$

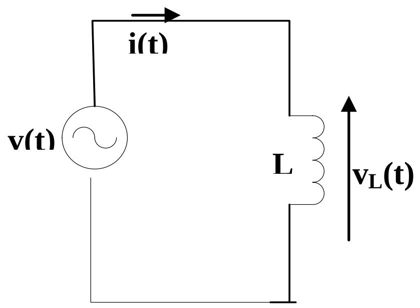
$= \omega L\, Im \cos\omega t = Vm \cos\omega t$

Or $v_L(t) = Vm \sin(\omega t + \pi/2)$ . . . (2)

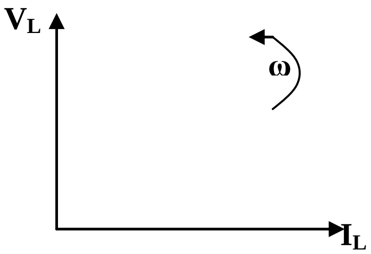
Where $Vm = \omega L\, Im = X_L\, Im$

$X_L = \omega L = 2\pi f L$

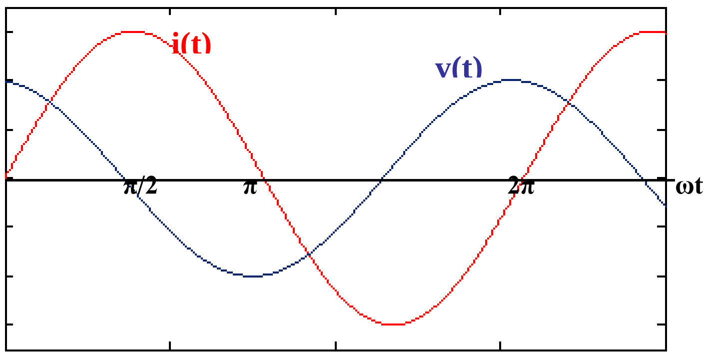
= Inductive reactance (Ω)



The current in a purely inductive circuit lags the voltage vector by 90°.



Phasor diagram



P(t) =Instantaneous power =v(t)*i(t)

$= Vm \cos\omega t * Im \sin\omega t = Vm\, Im \sin\omega t \cos\omega t$

$= 0.5\, Vm\, Im \sin(2\omega t)$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\, d\omega t = Zero$$

3- <u>Capacitance:</u>

Let $V(t) = Vm \sin\omega t = v_c(t)$ . . . (1)

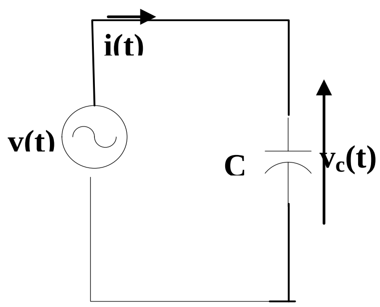
$i(t) = C\, dv_c(t)/dt$

$= \omega C\, Vm \cos\omega t = Vm/X_c \cos\omega t$

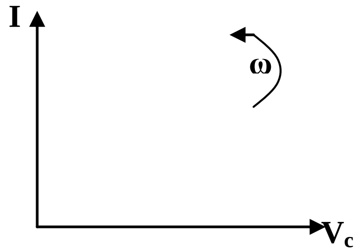
$i_L(t) = Im \sin(\omega t + \pi/2)$ . . . (2)

Where $X_c = 1/(\omega C) = 1/(2\pi f L)$

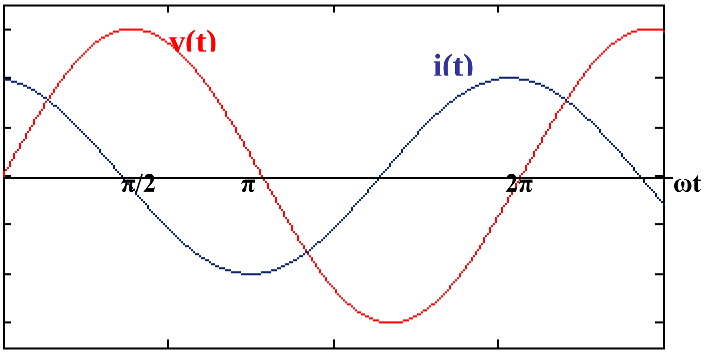
= Capacitive reactance (Ω)



The current in a purely capacitive circuit leads the voltage vector by 90°.



Phasor diagram



P(t) =Instantaneous power =v(t)*i(t)

= Vm cos ωt * Im sin ωt = Vm Im sin ωt cos ωt

= 0.5 Vm Im sin (2ωt)

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\,d\omega t = Zero$$

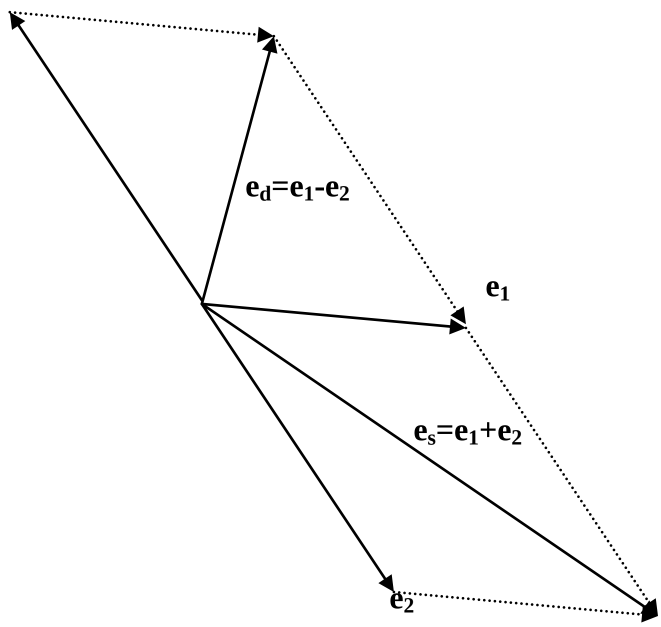
<u>Example:</u> Two sinusoidal emfs of the same frequency have maximum value of E_1=45V and E_2=60V respectively. E_2 lags E_1 by 45° . Plot the phasor diagram and then find $e_d=e_1-e_2$ and $e_s=e_1+e_2$.

$e_1(t)=45 \sin \omega t$

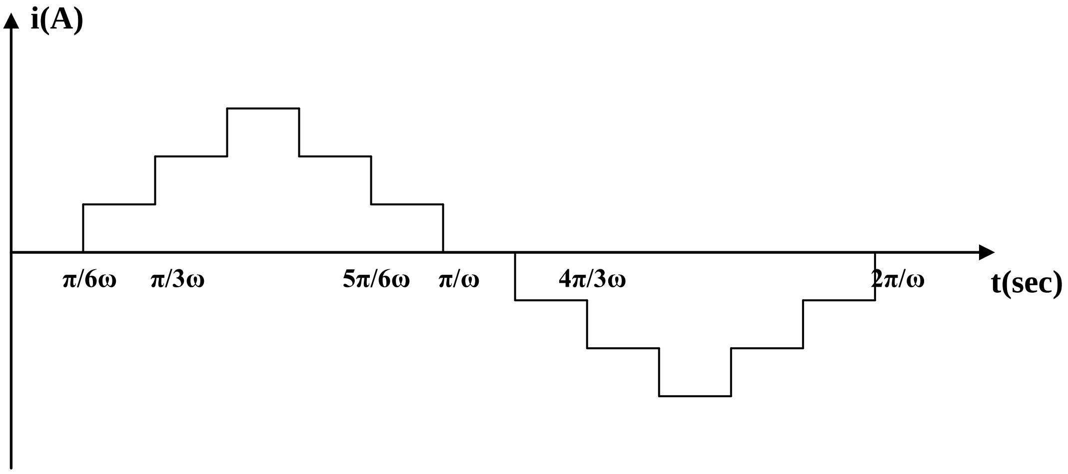
$e_2(t)=60 \sin (\omega t-45^o)$

E_s=97 V

E_d=43.75 V

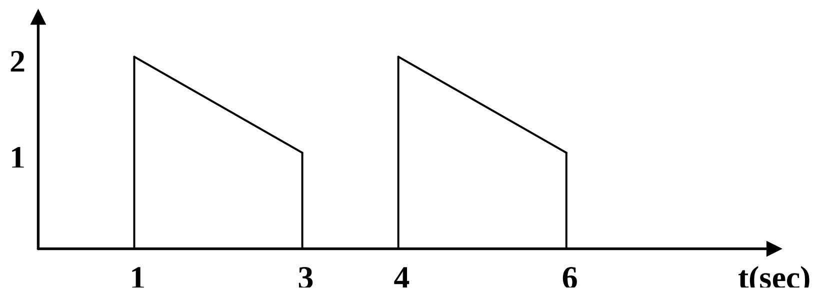


Example: A current has the waveform shown below. Calculate the rms value of the current and its form factor.



Example: The current in a circuit is given by: $i(t)=141.4 \sin 377t$ A. Find the value of: a- the rms current. b- the frequency c- the instantaneous value of the current when t=3 msec.

Example: Find the rms value for the current waveform shown below:



<u>Example:</u> Find the rms value for the voltage waveform given by $v(t)=50+30 \sin \theta$.