Some special continous distribution:

1- Uniform distribution:

A random variable X is said to be Uniformly distributed over the interval (a,b) if it's p.d.f is given by:

$$f(x)=\begin{cases}\frac{1}{b-a} & , \quad a<x<b \\ 0 & , \quad o.w\end{cases}$$

and writes briefly : $X \sim un(a,b)$

IS clearly that :

1-f(x)≥0 for all a<x<b

2-$\int_{-\infty}^{\infty} f(x)dx = 1 \rightarrow \int_{a}^{b} \frac{1}{b-a} dx = \frac{X}{b-a} |_{a}^{b} = \frac{b-a}{b-a} = 1$

The cumulative distribution function of a uniform random variable on (a, b) is given by :

$$F(X)=\begin{cases}0 & , \quad x \le a \\ \frac{x-a}{b-a} & , a<x<b \\ 1 & , \quad x \ge b\end{cases}$$

The mean :

$$\mathbf{E(X) = \int x\, f(x)\, dx = \int_{a}^{b} \frac{x}{b-a} dx = \frac{x^2}{2(b-a)} |_{a}^{b} = \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{(b+a)}{2}}$$

$$\mu_x = E(X) = \frac{b+a}{2}$$

The variance :

E(X^2)=$\int_{a}^{b} \boldsymbol{X^2 f(X) dx = \int_{a}^{b} \frac{X^2}{b-a} dx = \frac{X^3}{3(b-a)} |_{a}^{b} = \frac{b^3-a^3}{3(b-a)}}$

V(X)=E(X^2)- [E(X)]^2 =$\frac{b^3-a^3}{3(b-a)} - [\frac{b+a}{2}]^2 = \frac{(b-a)^2}{12}$

$$\sigma^2{}_x = \frac{(b-a)^2}{12}$$

<u>The M.G.F. :</u>

$$M_x(t) = E(e^{tx}) = \int_a^b e^{tx} \frac{1}{b-a} dx = \frac{e^{bt} - e^{at}}{t(b-a)}$$

Example:

If x is Uniformly distributed over the interval (0,10) find :

1-p(x<3) 2-p(3<x<8) 3-C.D.F OF X 4-$\mu_x =$ and $\sigma^2{}_x$

<u>Solve:</u>

$$f(x) = \begin{cases} \dfrac{1}{b-a} & , \; a < x < b \\ 0 & , \quad o.w \end{cases}$$

then:

$$f(x) = \begin{cases} \dfrac{1}{10} & , \; 0 < x < 10 \\ 0 & , \quad o.w \end{cases}$$

1-$p(x < 3) = \int_0^3 f(x)dx = \int_0^3 \frac{1}{10} dx = \frac{x}{10}\Big|_0^3 = \frac{3-0}{10} = \frac{3}{10}$

2-$P(3 < X < 8) = \int_3^8 f(x)dx = \int_3^8 \frac{1}{10} dx = \frac{x}{10}\Big|_3^8 = \frac{8-3}{10} = \frac{5}{10}$

3-CDF of X is:

$$F(X) = \begin{cases} 0 & , \quad x \le a \\ \dfrac{x-a}{b-a} & , a < x < b \\ 1 & , \quad x \ge b \end{cases}$$

then:

$$F(X)=\begin{cases}0 & , \quad x\le 0\\ \frac{x}{10} & ,0<x<10\\ 1 & , \quad x\ge 10\end{cases}$$

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$$\mu_x = \mathrm{E(X)} = \frac{\mathrm{b+a}}{2} = \frac{10+0}{2} = 5$$

$$\sigma^2{}_x = \frac{\mathbf{(10-0)^2}}{\mathbf{12}} = 8.333$$

<u>Homework:</u>

suppose X is continues random variable with uniform distribution having mean 1 and variance 4/3 <what is p(x>0).